Lafayette College
Department of Civil and Environmental Engineering

CE 321: Introduction to Environmental Engineering and Science **Fall 2019**

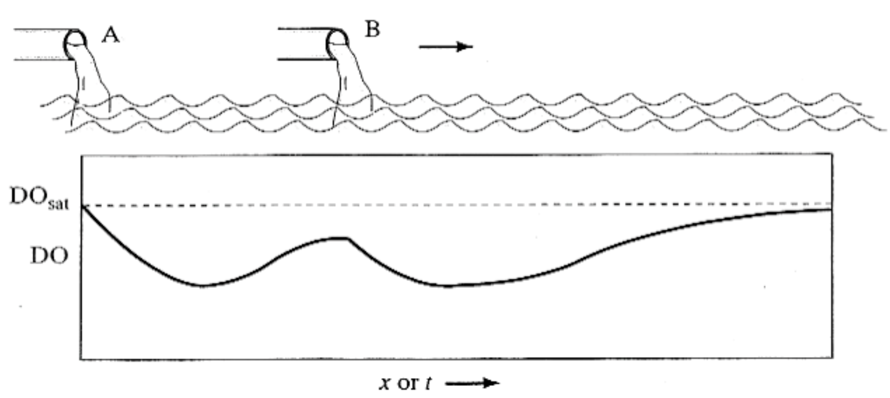
Homework #11
Due: Wednesday, December 5, 2019
SOLUTIONS

1) Suppose some wastewater had a BOD_5 equal to a 180 mg/L and a reaction rate k equal to 0.22/day. It also has total Kjedahl nitrogen content (TKN) of 30 mg/L.
    a. Find the ultimate carbonaceous oxygen demand (CBOD).
    b. Find the ultimate nitrogenous oxygen demand (NBOD).
    c. Find the remaining BOD (nitrogenous plus carbonaceous) after five days have elapsed.

2) A wastewater treatment plant discharges 1.0 m^3/s of effluent having an ultimate BOD of 40 mg/L into a stream flowing at 10.0 m^3/s. Just upstream from the discharge point, the stream has an ultimate BOD of 3.0 mg/L. The deoxygenation constant k_d is estimated at 0.22/day.
    a. Assuming complete and instantaneous mixing, find the ultimate BOD of the mixture of waste and river just downstream from the outfall.
    b. Assuming a constant cross-sectional area for the stream equal to 55 m^2, what BOD would you expect to find at a point 10,000 m downstream.

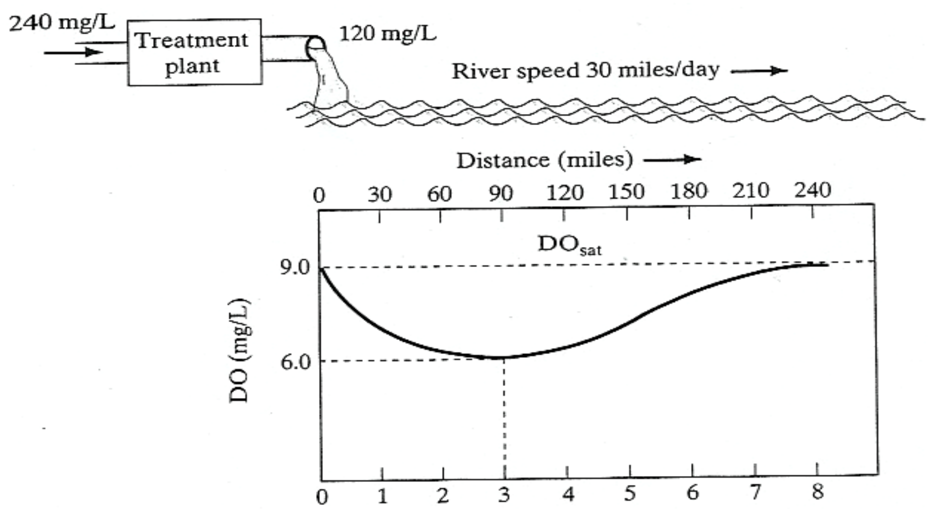
3) The wastewater in Problem 2 has DO equal to 4.0 mg/L when it is discharged. The river has its own DO, just upstream from the outfall, equal to 8.0 mg/L. Find the initial oxygen deficit of the mixture just downstream from the discharge point. The temperatures of sewage and river are both 15°C.

4) Two point sources of BOD along a river (A and B) cause the oxygen sag curve shown in the following image.



    a. Sketch the rate of reaeration vs. distance downriver.
    b. Sketch L_t (that is, the BOD remaining) as a function of distance downriver.

5) Untreated sewage with a BOD of 240 mg/L is sent to a wastewater treatment plant where 50 percent of the BOD is removed. The river receiving the effluent has the oxygen sag curve as shown in the following figure (the river has no other sources of BOD). Notice that downstream is expressed in both miles and days.



   a. Suppose the treatment plant breaks down and it no longer removes any BOD. Sketch the new oxygen sag curve starting just after the breakdown. Label the point which represents the critical distance downriver.
   b. Sketch the oxygen sag curve, as it would have appeared four day after the breakdown of the treatment plant.

6) The ultimate BOD a river and sewage outfall, after mixing, is found to be 50 mg/L. Also the DO is found to be at a saturation value of 10.0 mg/L after mixing, therefore no initial deficit. The deoxygenation rate coefficient k_d is 0.30/day and the reaeration rate coefficient k_r is 0.90/day. The river is flowing at the speed of 48.0 miles per day. The only source of BOD in this river is the single outfall.
   a. Find the critical distance downstream at which DO is minimum.
   b. Find the minimum DO.
   c. If a wastewater treatment plant is to be build, what fraction of the BOD would have to be removed from the sewage to assure a minimum DO concentration of 5.0 mg/L everywhere downstream?

7) A city of 200,000 people deposits 37 cubic feet per second (cfs) of sewage having a BOD of 28.0 mg/L and 1.8 mg/L of DO into a river that has a flow rate of 250 cfs and a flow speed of 1.2 ft/s. Just upstream of the release point, the river has a BOD of 3.6 mg/L and a DO of 7.6 mg/L. The saturation value of DO is 8.5 mg/L. The deoxygenation coefficient k_d is 0.61/day and the reaeration coefficient k_r is 0.76/day. Assuming complete and instantaneous mixing of the sewage and river find
   a. The initial oxygen deficit and ultimate BOD just downstream of the outfall
   b. The time and distance to reach the minimum DO
   c. The minimum DO
   d. The DO that could be expected 10 miles downstream

8) The town of Martins Creek, PA, has filed a complaint with the Department of Environmental Protection (DEP) citing the town of Portland, PA, for the discharge of raw sewage into the Delaware River. The raw sewage is considered to be the cause of high fecal coliform counts and reduced levels of dissolved oxygen (DO), which have lead to foul odors along the river between Portland and Martins Creek. The coliform counts and reduced DO levels have lead to restrictions of recreational areas within the Portland/Martins Creek reach of the Delaware River.

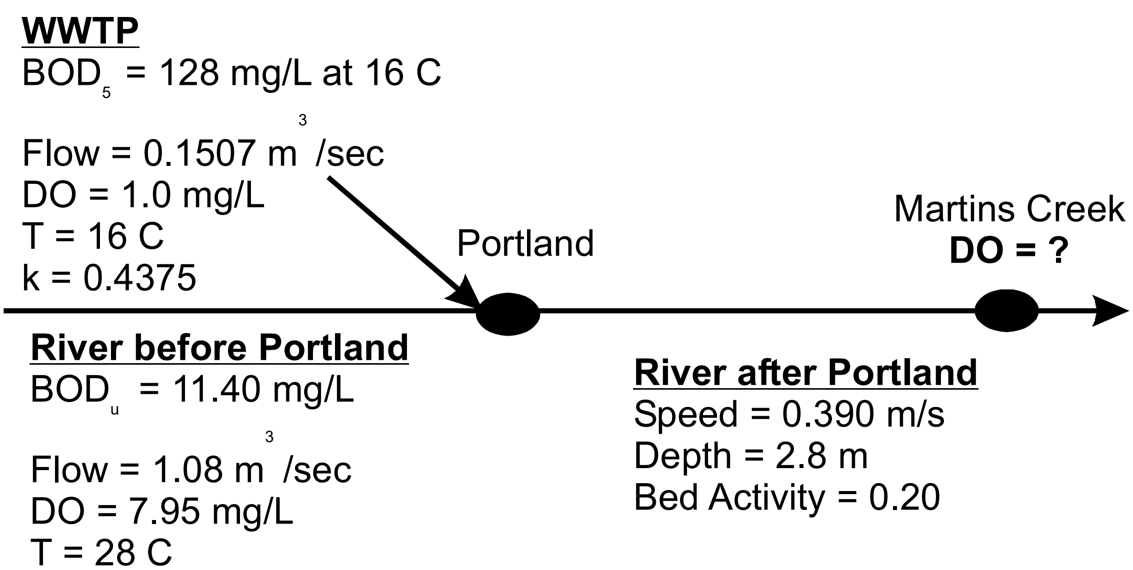
The DEP water quality criterion for the Delaware River is 5 mg/L of DO (i.e. at no point shall the DO concentration drop below 5 mg/L).

Martins Creek is 15.55 km down stream of Portland.

The following data pertain to the 7-year, 10-day low flow at Portland

Parameter	Wastewater	Delaware River just above Portland
Flow (m^3/sec)	0.1507	1.08
BOD_5 at 16^{o}C (mg/L)	128.00	Not provided
BOD_u at 28^{o}C (mg/L)	Not provided	11.40
DO (mg/L)	1.00	7.95
k at 20^{o}C (day^{-1})	0.4375	k of BOD in river is based on WW
Temperature (^{o}C)	16.00	28.00
Data of river after WWTP		
Average Speed (m/sec)	N/A	0.390
Average Depth (m)	N/A	2.80
Bed-activity coefficient	N/A	0.200

A) What is the DO concentration as mg/L at Martins Creek?



Distance between Portland and Martins Creek is 15.55 km

B) Does your answer make sense based on the complaint received?

✓+

1 GIVEN SOME WASTEWATER HAS A BOD_5 EQUAL TO 180 mg/L AND A REACTION RATE $k = 0.22/\text{day}$. IT ALSO HAS A TKN OF 30 mg/L

FIND THE ULTIMATE CBOD
THE ULTIMATE NBOD
REMAINING TOTAL BOD AFTER 5 DAYS

ASSUMPTIONS CONSTANT TEMPERATURE

SOLUTIONS $CBOD_{ULT} = L_0$ $180\,mg/L = L_0\left(1 - e^{-5\,days \cdot 0.22/day}\right)$ $L_0 = 269.81\,mg/L$

$NBOD_{ULT} = 30\,mg/L \cdot 4.5714\,mg\,O_2/mg\,N = 137.1\,mg\,O_2/L$

BOD REMAINING = TBOD − BOD_5 = $269.8\,mg\,O_2/L + 137.1\,mg\,O_2/L - 180\,mg\,O_2/L = 226.9\,mg/L\,O_2$

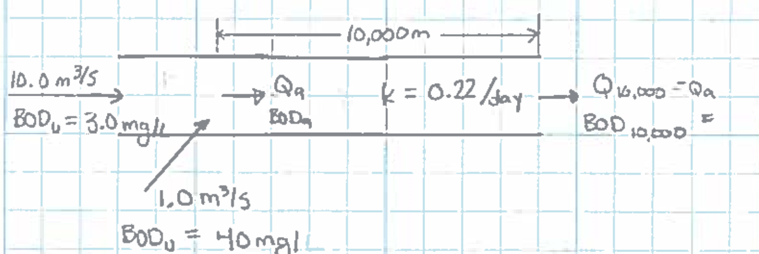
$CBOD_u$ = 269.8 mg/L O_2
$NBOD_u$ = 137.1 mg/L O_2
BOD REMAINING$_5$ = 226.9 mg/L O_2

2 GIVEN A WASTEWATER TREATMENT PLANT DISCHARGES 1.0 m^3/s OF EFFLUENT W/ BOD_u = 40 mg/L INTO A STREAM W/ FLOW = 10.0 m^3/s. JUST UPSTREAM, OF THE DISCHARGE POINT, THE STREAM HAS BOD_u = 3.0 mg/L. DEOXYGENATION CONSTANT $k_d = 0.22/\text{day}$. CROSS-SECTIONAL A OF STREAM = 55 m^2

FIND THE BOD_u OF THE MIX OF WASTEWATER AND RIVER JUST DOWNSTREAM FROM OUTFALL
THE BOD AT A PT 10,000 m DOWNSTREAM

ASSUMPIONS CONSTANT TEMP
INSTANTANEOUS, COMPLETE MIXING
BALANCE OF INS & OUTS (WATER & BOD)

SOLUTIONS



$Q_a = 10.0\,m^3/s + 1.0\,m^3/s$

$10.0\,m^3/s \cdot 3.0\,mg/L + 1.0\,m^3/s \cdot 40\,mg/L = Q_a\,BOD_a$

$BOD_a = 6.3636\,mg/L\,O_2$

$v = \frac{11\,m^3/s}{55\,m^2} = 0.2\,m/s$

$\frac{10{,}000\,m}{0.2\,m/s} \cdot \frac{1\,day}{86{,}400\,s} = 0.578704\,days$

2 (con't)

$$BOD_{10,000\,m} = 6.3636\ mg/L\ (1 - e^{-0.578704\,day \cdot 0.22\,/day})$$

$BOD_{10,000\,m}$ = **0.77** mg/L O_2

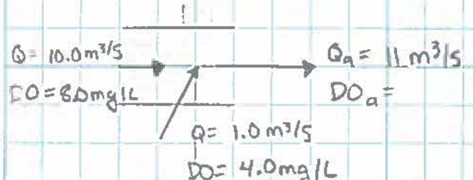
BOD_a = 6.36 mg/L O_2
$BOD_{10,000\,m}$ = **0.77** mg/L O_2

3 GIVEN THE WASTEWATER IN PROBLEM 2 HAS DO = 4.0 mg/L WHEN IT IS DISCHARGED. THE UPSTREAM DO OF THE RIVER IS 8.0 mg/L

FIND THE INITIAL OXYGEN DEFICIET OF THE MIX JUST DOWNSTREAM OF THE DISCHARGE POINT

ASSUMPTIONS CONSTANT TEMP OF RIVER & SEWAGE AT 15°C
INSTANTANEOUS MIXING
BALANCE OF BOD & WATER IN & OUT
CHLORIDE CONCENTRATION = 0

SOLUTION



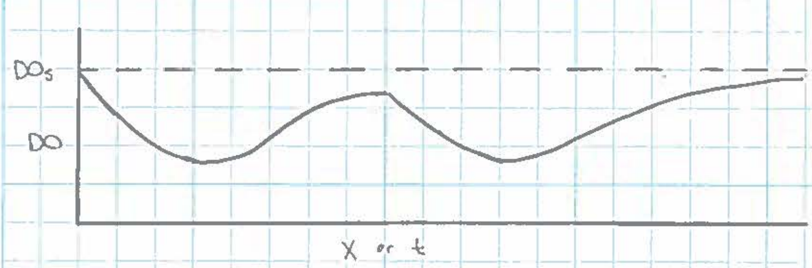
$$10.0\ m^3/s \cdot 8.0\ mg/L + 1.0\ m^3/s \cdot 4.0\ mg/L = 11\ m^3/s \cdot DO_a$$

$$DO_a = 7.63636\ mg/L$$

$$D_a = DO_{s\,(15°C)} - DO_a = (10.08 - 7.63636)\ mg/L = 2.443636\ mg/L$$

D_a = 2.44 mg/L

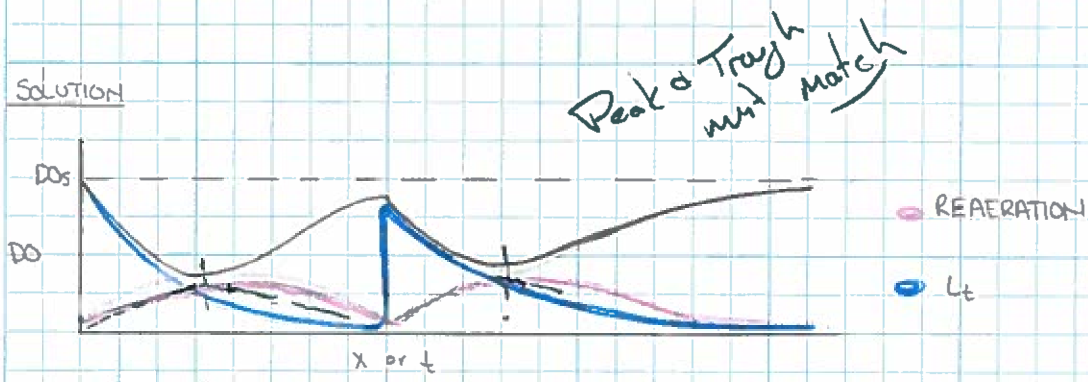
4 GIVEN TWO POINT SOURCES OF BOD ALONG A RIVER CAUSE THE OXYGEN SAG CURVE BELOW



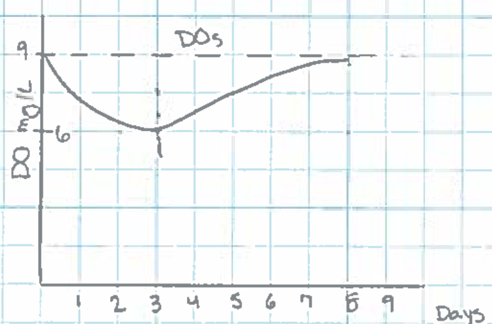
FIND SKETCH RATE OF REAERATION VS DISTANCE DOWNSTREAM
SKETCH L_t AS A FUNCTION OF DISTANCE DOWNRIVER

4 (con't) ASSUMPTIONS CONSTANT TEMP
WELL MIXED
LITTLE TO NO BOD PRESENT AT START

SOLUTION



5 GIVEN: UNTREATED SEWAGE W/ BOD = 240 mg/L IS SENT TO A WASTEWATER TREATMENT PLANT WHERE 50% OF BOD IS REMOVED. THE RIVER RECEIVING THE EFFLUENT HAS NO OTHER SOURCES OF BOD & THE OXYGEN SAG CURVE SHOWN.
RIVER v = 30 mi/day



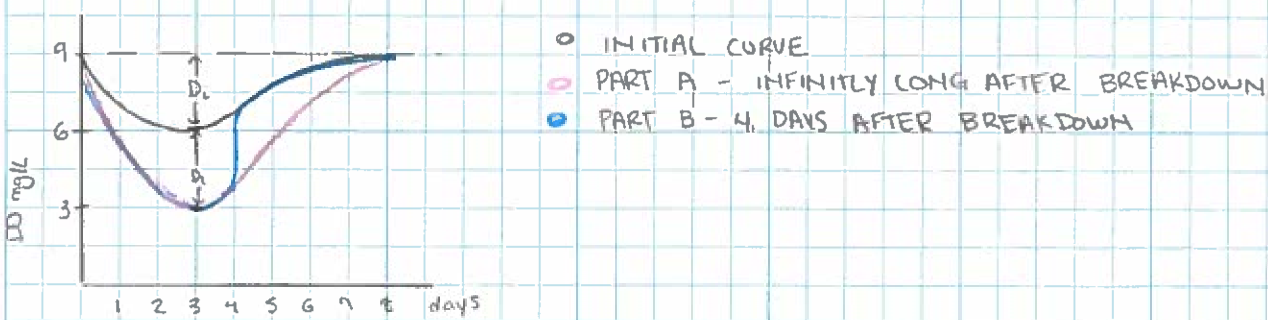
FIND THE NEW SAG CURVE IF THE TREATMENT PLANT WERE TO STOP FUNCTIONING & NO LONGER REMOVES BOD
THE SAG CURVE 4 DAYS AFTER BREAKDOWN OF THE PLANT

ASSUMPTIONS WELL MIXED (INSTANTANEOUSLY)
PLUG FLOW
CONSTANT TEMPERATURE
CONSTANT RIVER SPEED

SOLUTION A- BOD x2 SO D x2 SINCE $DO_1 = DO_S$, t_c IS SAME

B- UNTIL DAY 4, WILL RESEMBLE GRAPH FROM PART A. AT 4 days, JUMPS UP TO INITIAL SAG CURVE

5 (con't)



6 GIVEN THE BOD_u OF A RIVER JUST BELOW A SEWAGE OUTFALL = 50.0 mg/L AND THE DO = DO_s = 10.0 mg/L O_2. THE DEOXYGENATION RATE COEFFICIENT k_d = 0.30/day AND REAERATION RATE k_r = 0.90/day. THE RIVER FLOWS AT 48.0 mi/day.

FIND CRITICAL DISTANCE DOWNSTREAM DISTANCE WHERE DO IS AT A MINIMUM.
DO MINIMUM
IF A WWTP IS TO BE BUILT, WHAT FRACTION OF BOD WOULD HAVE TO BE REMOVED FROM THE SEWAGE TO ASSURE DO IS ALWAYS > 5.0 mg/L ANYWHERE DOWNSTREAM

ASSUMPTIONS THE SEWAGE OUTFALL IS THE ONLY BOD SOURCE
NO NET LOSS OR GAIN OF WATER
EVEN, CONSISTENT MIXING
CONSTANT TEMPERATURE
CONSTANT RIVER SPEED

SOLUTION $k_d = 0.30$ /day $k_r = 0.90$ /day $D_a = 0$ mg/L

$$t_c = \frac{1}{0.90-0.30} \ln\left\{\frac{0.90}{0.30}\left[1-\frac{0\,(0.90-0.30)}{0.30\cdot 50}\right]\right\} = 1.83102 \text{ days} \cdot \frac{48 \text{ mi}}{1 \text{ day}} = 87.889 \text{ mi}$$

$DO_{1.83} = DO_s - D_{1.83}$ $DO_s = 10.0$ mg/L

$$D_{1.83} = \frac{0.30\cdot 50}{0.90-0.30}\left(e^{-0.3\cdot 1.83} - e^{-0.9\cdot 1.83}\right) + 0e^{-0.90\cdot 1.83} = 9.6225 \text{ mg/L } O_2$$

$$DO_{1.83} = 10.0 \text{ mg/L} - 9.6225 \text{ mg/L} = 0.3775 \text{ mg/L}$$

t_c independent of BOD ADDED ∴ WE WANT $DO_{1.83} = 5$ mg/L

$D_{1.83} = 10 - 5 = 5$ mg/L

6 (con't)

$D_t = 5\ mg/L = \frac{0.3 \cdot BOD}{0.9-0.3}\left(e^{-0.3\cdot 1.83} - e^{-0.9\cdot 1.83}\right)$ BOD = 25.98 mg/L

% REMOVED = $\frac{50\ mg/L - 25.98\ mg/L}{50\ mg/L} \cdot 100\%$ = 48.04%

$Dist_c$ = 87.9 mi
$DO_{1.83}$ = 0.377 mg/L O_2
48.0% REDUCTION IN BOD REQUIRED

7 GIVEN A CITY OF 200,000 PEOPLE DEPOSITS 37 cfs OF SEWAGE OF SEWAGE WITH BOD = 28.0 mg/L AND 1.8 mg/L DO INTO A RIVER WITH A FLOW RATE OF 250 cfs AND A FLOW SPEED OF 1.2 ft/s. JUST UPSTREAM OF THE RELEASE POINT, THE RIVER HAS A BOD = 3.6 mg/L AND A DO = 7.6 mg/L. THE SATURATION VALUE OF DO = 8.5 mg/L. THE DEOXYGENATION COEFFICIENT k_d = 0.61 /day AND THE REAERATION COEFFICIENT k_r = 0.76 /day.

FIND INITIAL OXYGEN DEFICIT (D_a) AND ULTIMATE BOD JUST DOWNSTREAM OF OUTFALL

THE TIME & DISTANCE TO REACH MINIMUM DO

THE MINIMUM DO

THE EXPECTED DO 10 mi DOWNSTREAM

ASSUMPTIONS COMPLETE, INSTANTANEOUS MIXING OF THE SEWAGE & RIVER

CONSTANT TEMPERATURE

CONSTANT RIVER SPEED & FLOW RATES

NO OUTSIDE ADDITION/SUBTRACTION OF WATER OR BOD

SOLUTION

Q_R = 250 cfs, BOD = 3.6 mg/L, DO = 7.6 mg/L → $Q_a = Q_R + Q_w$, $BOD_a = L_a$ =, DO_a =, D_a = → k_r = 0.76/day, k_d = 0.61/day → $Q_{10mi} = Q_a$, $BOD_{10\,mi}$, $DO_{10\,mi}$

Q_w = 37 cfs, BOD = 28.0 mg/L, DO = 1.8 mg/L

D_a $DO_a = \frac{250\ cfs \cdot 7.6\ mg/L + 37\ cfs \cdot 28.0\ mg/L}{250\ cfs + 37\ cfs} = 6.85226\ mg/L$

$D_a = DO_s - DO_a = 8.5\ mg/L - 6.85\ mg/L = 1.65\ mg/L\ O_2$

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7 (con't)

$$L_a = \frac{3.4\,mg/L \cdot 250\,cfs + 37\,cfs \cdot 28.0\,mg/L}{250\,cfs + 37\,cfs} = 6.7456\,mg/L$$

$$t_c = \frac{1}{0.76/day - 0.61/day} \ln\left[\frac{0.76/day}{0.61/day}\left[1 - \frac{1.6477\,mg/L\,(0.76/day - 0.61/day)}{0.61/day \cdot 6.7456\,mg/L}\right]\right] = 1.052771\ days$$

$$t_c = 1.052771\ days \cdot \frac{86{,}400\,s}{1\ day} = 90959.425\,s \cdot 1.2\,f/s = 109151.31\ ft$$

$$DO_{1.05d} = 8.5\,mg/L - D_{1.05d}$$

$$D_{1.05d} = \frac{0.61/day \cdot 6.7456\,mg/L}{0.76/day - 0.61/day}\left(e^{-0.61/day \cdot 1.053day} - e^{-0.76/day \cdot 1.053day}\right) + 1.6477\,mg/L\, e^{-0.76/d \cdot 1.053d}$$

$$D_{1.05d} = 2.84859\ mg/L \qquad DO_{MIN} = 8.5\,mg/L - 2.84859\,mg/L = 5.65141\ mg/L$$

$$\frac{10\,mi}{1.2\,f/s} \cdot \frac{5280\,ft}{1\,mi} \cdot \frac{1\,d}{86{,}400\,s} = 0.509259\,d$$

$$D_{0.509d} = \frac{0.61/d \cdot 6.7456\,mg/L}{0.76/d - 0.61/d}\left(e^{-0.61/d \cdot 0.509d} - e^{-0.76/d \cdot 0.509d}\right) + 1.6477\,mg/L\, e^{-0.76/d \cdot 0.509d}$$

$$D_{0.509} = 2.5976788\ mg/L$$

$$DO_{10mi} = DO_S - D_{0.509} = 5.902\,mg/L\ O_2$$

$D_a = 1.65\ mg/L$
$L_a = 6.75\,mg/L$
$t_c = 1.05\ days$
$Dist_c = 109151\ ft$
$DO_{MIN} = 5.65\,mg/L$
$DO_{10mi} = 5.90\,mg/L$

8 PROBLEM MARTIN'S CREEK, PA HAS FILED A COMPLAINT CITING PORTLAND, PA FOR RELEASING UNTREATED SEWAGE INTO THE DELAWARE, WHICH THEY BELEIVE IS CAUSING LOW DO LEVELS & OTHER EFFECTS IN MARTIN'S CREEK.

GIVEN DEP CRITERION IS 5 mg/L DO. MARTIN'S CREEK IS 15.55 km DOWNSTREAM FROM PORTLAND. THE FOLLOWING DATA IS GIVEN FOR 7 yr 10 day FLOW AT PORTLAND.

	WASTEWATER	ABOVE PORTLAND DELAWARE RIVER
FLOW (m^3/s)	0.1507	1.08
BOD_5 @ 16° C (mg/L)	128.00	—
BOD_u @ 28°C (mg/L)	—	11.40
DO (mg/L)	1.00	7.95
K @ 20° (/day)	0.4375	Based on WW
TEMPERATURE (°C)	16.00	28.00

AFTER WWTP ENTERS RIVER	
AVG SPEED (m/s)	0.390
AVG DEPTH (m)	2.80
BED ACTIVITY COEFFICIET	0.200

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Name SCHULTHEIS, KARI | Course CE 321-01
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8 (con't) FIND DO AT MARTINS CREEK
DOES THIS MAKE SENSE BASED ON THE COMPLAINT

ASSUMPTIONS
- INSTANTANEOUS, FULL MIXING
- CONSTANT TEMP AFTER MIXING
- k IN RIVER BASED ON k in WW
- PLUG FLOW AFTER MIXING
- NO WATER OR BOD ENTERS OR LEAVES SYSTEM BESIDES WHAT IS SPECIFIED
- STEADY STATE
- NO CHLORIDE CONTENT

SOLUTION

$k_{16} = 0.4375 \cdot 1.135^{(16°-20°)} = 0.26363\ /day$

$L_{0\,ww}\left(1 - e^{-5day \cdot 0.26363/day}\right) = 128\,mg/L \qquad L_{0\,ww} = 174.775\,mg/L$

RIVER
$L_0 = 11.40\,mg/L$
$Q = 1.08\,m^3/s$
$DO = 7.95\,mg/L$
$T_R = 28°C$

WWTP
$L_0 = 174.775\,mg/L$
$Q = 0.1507\,m^3/s$
$DO = 1.0\,mg/L$
$T_{ww} = 16°C$
$k = 0.4375$ (@ 20°C) /day

After mixing:
$L_a = 31.41\,mg/L$ @ T_a
$Q_a = 1.2307\,m^3/s$ $\quad k_d = 0.6642/d$
$DO_a = 7.10\,mg/L$ $\quad k_r = 0.6069/d$
$T_a = 26.5°C$
$k_{T_a} = 0.6244\,/day$

$Q = 1.2307$
$DO_{15.5mi} =$

$T_a = \dfrac{0.1507\,m^3/s \cdot 16°C + 1.08\,m^3/s \cdot 28°C}{1.2307\,m^3/s} = 26.5306\,°C \qquad k_{T_a} = 0.4375 \cdot 1.056^{(26-20)} = 0.62447846/d$

$DO_a = \dfrac{1.08\,m^3/s \cdot 7.95\,mg/L + 0.1507\,m^3/s \cdot 1.0\,mg/L}{1.2307\,m^3/s} = 7.09897\,mg/L$

$DO_{s_a} \quad \dfrac{30° - 26.53°}{30° - 25°} = \dfrac{7.56\,mg/L - DO_{s_a}}{7.56\,mg/L - 8.26\,mg/L} \qquad DO_{s_a} = 8.0458$

$D_a = DO_{s_a} - DO_a = 0.94683$

$t_{MC} = \dfrac{15.55\,km}{0.39\,m/s} \cdot \dfrac{1000\,m}{1\,km} \cdot \dfrac{1\,d}{86{,}400\,s} = 0.46147910 7$

$k_{d_{20}} = 0.4375 + \dfrac{0.390\,m/s}{2.90\,m} \cdot 0.200 = 0.465357\,/day \qquad k_{d_{26.5}} = 0.465357 \cdot 1.056^{(26.5-20)} = 0.66424/d$

$k_{r_{20}} = \dfrac{3.9\,(0.390\,m/s)^{1/2}}{(2.8\,m)^{3/2}} = 0.51982823\,/day \qquad k_{r_{26.5°}} = 0.519828 \cdot 1.024^{(26.5-20)} = 0.606910/d$

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8 (con't)

$$L_a = \frac{174.77\,mg/L \cdot 0.1507\,m^3/s + 1.08\,m^3/s \cdot 11.40\,mg/L}{1.2307\,m^3/s} = 31.40538\,mg/L$$

$$D_{15.55km} = \frac{0.6642/d \cdot 31.41\,mg/L}{0.6069/d - 0.6642/d}\left(e^{-0.66 \cdot 0.4615} - e^{-0.607 \cdot 0.4615}\right) + 0.94683e^{-0.6069/d \cdot 0.4615d}$$

$$D_{15.55\,km} = 7.896067\,mg/L$$

$$DO_{MC} = 8.05\,mg/L - 7.90\,mg/L = 0.15\,mg/L$$

$$DO_{MC} = 0.15\,mg/L$$

THIS MAKES SENSE. A DO BELOW 5 IS UNDESIRABLE, AND THIS DO IS NEARLY ZERO. THIS WOULD RESULT IN A NEW TYPE OF ECOSYSTEM AND WOULD CAUSE THE COMPLAINTS OF MARTAIN'S CREEK