

Lafayette College
Department of Civil and Environmental Engineering

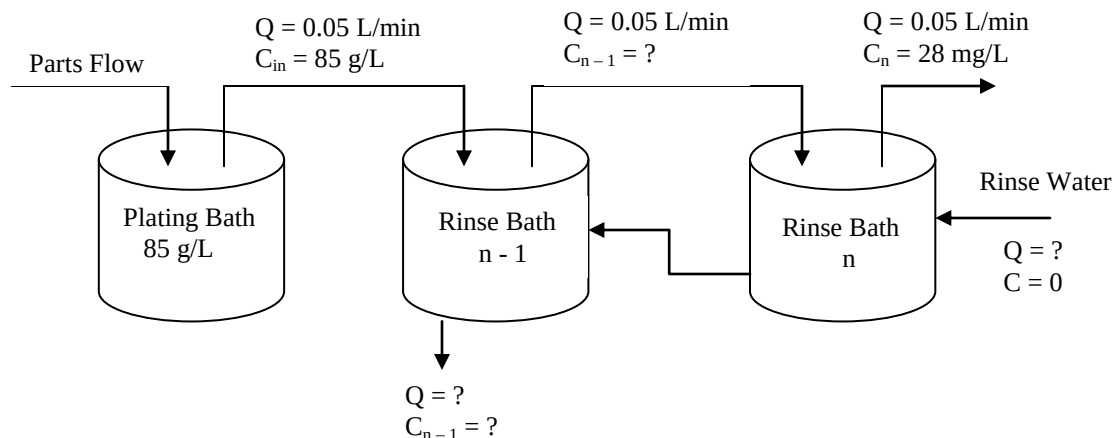
CE 321: Introduction to Environmental Engineering

Fall 2010

Homework #4

SOLUTIONS Due: Monday, 9/27/10 **SOLUTIONS**

1. A completely mixed reactor has an influent flow rate of 100 gal/min (380 l/min). The BOD concentration (nonconservative pollutant) of the wastewater entering the reactor is 150 mg/L. The detention time of the reactor is 2 hours. The reaction is first order, therefore the rate equation is $\frac{dC}{dt} = -KC$ and the rate constant is 0.40 hr^{-1} .
- What is the volume of the reactor? Report your answer in gallons.
 - Assuming the initial concentration of the reactor is 0 mg/L and a BOD of concentration 150 mg/L added at time 0, what is the concentration of the pollutant in the reactor after 2 hours?
 - What is the concentration of the BOD leaving the reactor after 2 hours?
 - What is the concentration of the BOD within the reactor assuming it reaches steady state?
 - Assume that there was no flow in or out of the reactor and the BOD was initially 150 mg/L. What is the concentration of the BOD after 2 hours?
2. It is found that the flow rate of a nickel plating system is too high. A countercurrent rinse system, shown below, is recommended in order to reduce the flow rate. Assuming that the C_n concentration remains at 28 mg/L, just as it was in the original plating system, estimate the new flow rate of the rinse water. Assume that the rinse tank is completely mixed and that no reactions take place in the rinse tank.



3. A sewage lagoon that has a surface area of 10 ha and a depth of 1 m is receiving 8640 m³/d of sewage containing 100 mg/L of biodegradable contaminant. At steady state, the effluent from the lagoon must not exceed 20 mg/L of biodegradable contaminant. Assuming the lagoon is well mixed and that there are no losses or gains of water in the lagoon other than the sewage input, what biodegradation reaction rate coefficient (d⁻¹) must be achieved for a first order reaction. (Answer: $k = 0.35 \text{ d}^{-1}$)

4. A CMFR is used to treat an industrial waste product, using a reaction that destroys the pollutant according to first-order kinetics, with $k=0.216 \text{ d}^{-1}$. The reactor volume is 500 m³, the volumetric flow rate of the single inlet and exit is 50 m³ day⁻¹, and the inlet pollutant concentration is 100 mg L⁻¹.
 - a. Assuming the CMFR has reached steady state, what is the outlet concentration after treatment? State your answer in units of mg/L.
 - b. The manufacturing process that generates the waste in Part A has to be shut down, and, starting at $t=0$, the concentration of C_{in} entering the CMFR is set to 0.
 - i. How many hours does it take the tank concentration to reach 10% of its initial, steady-state value?
 - ii. How many hours does it take to reach steady state?
 - c. A plug-flow reactor (PFR) is used to model the chemical transformation of the waste product presented in Part A. Assume that the flow rate and the first-order decay rate constant are unchanged ($Q = 50 \text{ m}^3 \text{ day}^{-1}$, $k = 0.216 \text{ day}^{-1}$). Based on this information answer the following.
 - i. What is the volume required for the PFR to obtain the same degree of pollutant reduction as the CMFR of Part A? Report your answer in m³.
 - ii. What is the % reduction in volume as compared to the volume of the CMFR for the same steady-state conditions?

5. For the following conditions determine whether a CMFR or a PFR is more efficient in removing a reactive compound from the waste stream under steady-state conditions with a first-order reaction: reactor volume = 280 m³, flow rate 14 m³/day, and reaction rate coefficient = 0.05 d⁻¹.

6. You are working for the Michigan Department of Environmental Quality (DEQ) and must issue a discharge permit to a certain industry. The industry wishes to discharge “green muck” into the Ickold River. The flow of waste water from the industry is 0.15 m³/s. The flow of water in the Ickold River (upstream from the waste water outfall) is 0.25 m³/s. Upstream of the outfall, the concentration of “green muck” in the river is 0.1 mg/L. *What is the maximum concentration the industry can discharge if the concentration of “green muck” in the river (after mixing) cannot exceed 2.5 mg/L?*

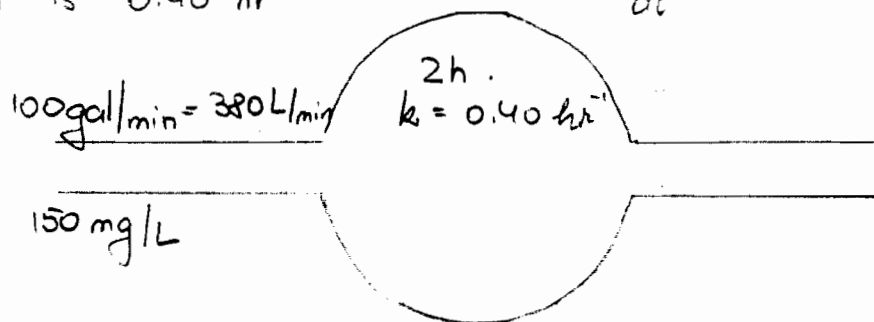
HW # 4

100% Done

CE 321

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1. A completely mixed reactor has an influent flow of 100 gal/min (380 L/min). The BOD concentration (nonconservative pollutant) of the waste water entering the reactor is 150 mg/L. The detention time of the reactor is 2 hours. The reaction is first order, therefore the rate equation is $\frac{dC}{dt} = -kC$ and the rate constant is 0.40 hr^{-1} .



Known:

- flow rate coming in $Q_{in} = 100 \text{ gal/min} = 380 \text{ L/min}$
- concentration of incoming BOD $C_{in} = 150 \text{ mg/L}$
- detention time of the reactor $\theta = 2 \text{ hours}$
- rate constant $k = 0.40 \text{ hr}^{-1}$

Find:

- volume of the reactor (gallons)
- assume $C_{reactor} = 0 \text{ mg/L}$, C_{in} at $t=0$ 150 mg/L. What is the concentration after 2 hours.
- concentration of BOD leaving reactor after 2 hours
- concentration of BOD at steady state.
- assume no flow in or out of the reactor and a concentration of BOD of 150 mg/L, what is the concentration of the BOD after 2 hours?

Assumptions:

- BOD is a nonconservative pollutant,
- no outside gains/losses due to percolation, evaporation
- constant flows and concentrations

$$a. \theta = \frac{V}{Q} \Rightarrow V = \theta \cdot Q = 2 \text{ hours} \cdot 100 \text{ gal/min} = 12,000 \text{ gal} = V$$

$$b. C_{2h} = C_{\infty} + (C_0 - C_{\infty}) \cdot e^{-\left[k + \frac{1}{\theta}\right]t}$$

$$C_{\infty} = \frac{C_{in} \cdot Q_{in}}{Q_{in} + k \cdot V} = \frac{150 \text{ mg/L} \cdot 380 \text{ L/min}}{380 \text{ L/min} + 0.4 \text{ /hr} \cdot 45,600 \text{ L}} =$$

$$C_{\infty} = 83.33 \text{ mg/L}$$

$$C_{2h} = 83.33 \text{ mg/L} + [0 \text{ mg/L} - 83.33 \text{ mg/L}] \cdot e^{-\left[0.4/\text{hr} + \frac{1}{2h}\right] \cdot 2h}$$

$$C_{2h} = 69.56 \text{ mg/L}$$

c. The concentration of BOD leaving the reactor after 2 hours is the same as the concentration after 2 hours.

d. Concentration at steady-state is C_{∞} calculated in point b.

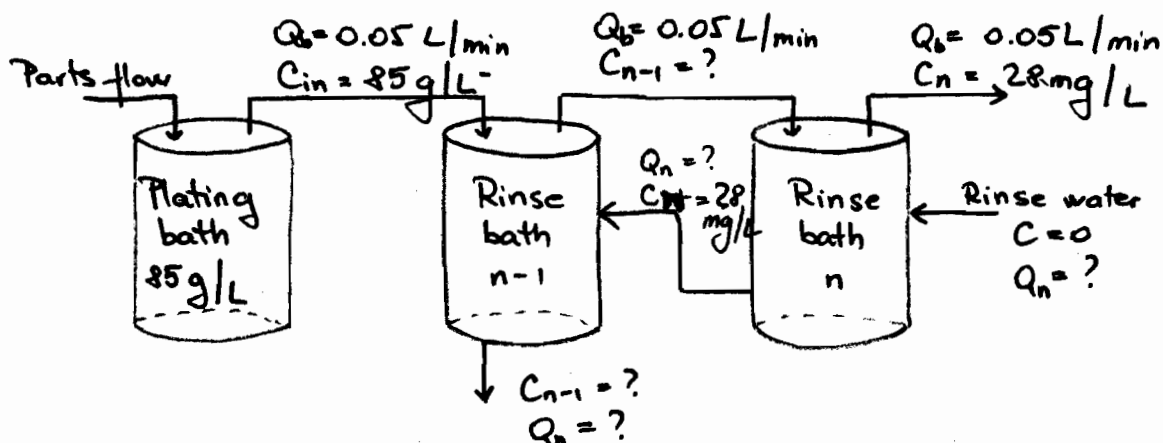
$$C_{\infty} = 83.33 \text{ mg/L}$$

e. no flow in or out $\Rightarrow$ batch reactor.

$$C_t = C_0 \cdot e^{-k \cdot t} = 150 \text{ mg/L} \cdot e^{-(0.4/\text{hr}) \cdot 2\text{hr}} =$$

$$\Rightarrow C_t = 67.4 \text{ mg/L}$$

2 Because rinse water flow rate for nickel plating bath is quite high, it is proposed that the countercurrent rinse system shown in the Figure be used to reduce the flow rate. Assuming that the C_n concentration remains the same at 28 mg/L , estimate the new flow rate. Assume that the rinse tank is completely mixed and that no reactions take place in the rinse tank.



Known:

- concentration for incoming stream from plating bath $C_{in} = 85 \text{ g/L}$
- flow rate between baths $Q_b = 0.05 \text{ L/min}$
- concentration of rinse water is zero
- concentration of water going out of rinse bath n $C_n = 28 \text{ mg/L}$

Find:

- the new flow rate $Q_n = ?$
- the new concentration in rinse bath n-1 $C_{n-1} = ?$

Assumptions:

- rinse tanks are completely mixed
- no reactions take place in the rinse tank
- steady state system, no accumulation ($\frac{dM}{dt} = 0$)
- constant flows, concentrations and temperatures
- the metal is a conservative chemical
- no outside gains/losses due to percolation or evaporation

For rinse bath n-1:

$$Q_b + Q_n = Q_b + Q_n \quad (\text{True})$$

$$Q_b \cdot C_{in} + Q_n \cdot C_n = Q_b \cdot C_{n-1} + Q_n \cdot C_{n-1}$$

$$\begin{aligned} 0.05 \text{ L/min} \cdot 85 \text{ g/L} + Q_n \cdot 0.028 \text{ g/L} &= \\ &= 0.05 \text{ L/min} \cdot C_{n-1} + Q_n \cdot C_{n-1} \end{aligned}$$

$$4.25 \text{ g/min} + Q_n \cdot 0.028 \text{ g/L} = (0.05 \text{ L/min} + Q_n) C_{n-1} \quad (1)$$

For rinse bath n:

$$Q_b \cdot C_{n-1} + \cancel{C_n \cdot Q_n} = Q_b \cdot C_n + Q_n \cdot C_n.$$

$$0.05 \text{ L/min} \cdot C_{n-1} = 0.05 \text{ L/min} \cdot 0.028 \text{ g/L} + Q_n \cdot 0.028 \text{ g/L}$$

$$0.05 \text{ L/min} \cdot C_{n-1} = 1.4 \times 10^{-3} \text{ g/min} + Q_n \cdot 0.028 \text{ g/L}$$

$$C_{n-1} = 0.028 \text{ g/L} + Q_n \cdot 0.56 \frac{\text{min} \cdot \text{g}}{\text{L}^2}$$

$$(1) \quad 4.25 \text{ g/min} + Q_n \cdot 0.028 \text{ g/L} = (0.05 \text{ L/min} + Q_n) C_{n-1}$$

$$4.25 \text{ g/min} + \cancel{Q_n \cdot 0.028 \text{ g/L}} = 0.05 \text{ L/min} \cdot 0.028 \text{ g/L} + \cancel{0.028 \text{ g/L} \cdot Q_n} + Q_n \cdot 0.028 \text{ g/L} + Q_n^2 \cdot 0.56 \frac{\text{min} \cdot \text{g}}{\text{L}^2}$$

$$4.2486 \text{ g/min} = (0.028 Q_n + 0.56 Q_n^2) \text{ g/L}$$

$$0.56 x^2 + 0.028 x - 4.2486 = 0$$

$$\Delta = 0.028^2 - 4 \cdot (0.56) \cdot (-4.2486) =$$

$$= 9.52 \Rightarrow \sqrt{\Delta} = 3.085$$

$$x_{1,2} = \frac{-0.028 \pm 3.085}{2 \cdot 0.56} = \begin{matrix} 2.73 \\ -2.78 < 0 \end{matrix}$$

$$\Rightarrow \boxed{Q_n = 2.73 \text{ L/min}}$$

$$C_{n-1} = 0.028 \text{ g/L} + 2.73 \text{ L/min} \cdot 0.56 \frac{\text{min} \cdot \text{g}}{\text{L}^2} \Rightarrow \boxed{C_{n-1} = 1.56 \text{ g/L}}$$

Green

3 A sewage lagoon that has a surface area of 10 ha and a depth of 1m is receiving 8640 m³/day of sewage containing 100 mg/L of biodegradable contaminant. At steady state, the effluent from the lagoon must not exceed 20 mg/L of biodegradable contaminant. Assuming the lagoon is well mixed and that there are no losses or gains of water in the lagoon other than the sewage input, what biodegradation reaction rate coefficient (d⁻¹) must be achieved for a first-order reaction?

Knowns:

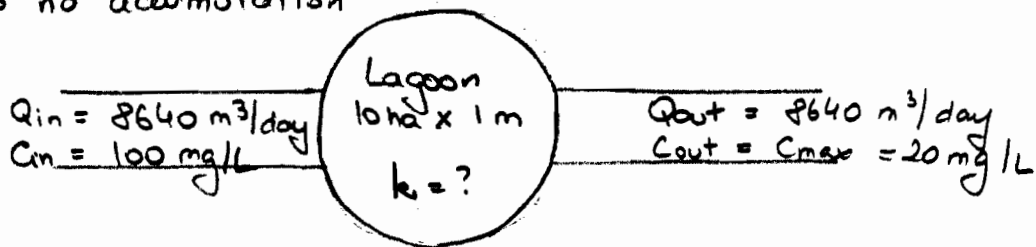
- lagoon surface area $A = 10 \text{ ha}$ and depth $d = 1 \text{ m}$
- flow rate of incoming stream $Q_{in} = 8640 \text{ m}^3/\text{day}$
- concentration of incoming contaminant $C_{in} = 100 \text{ mg/L}$
- maximum limit of contaminant concentration at steady state $C_{max} = 20 \text{ mg/L}$

Find:

- reaction rate coefficient $k = ? \text{ (d}^{-1}\text{)}$

Assume:

- first order reaction, non conservative, steady state
- not outside gains/losses due to percolation and evaporation
- constant flows and concentrations
- the lagoon is well mixed
- no accumulation



SS/Non Cons: $C_{out} = \frac{C_{in} \cdot Q_{in}}{Q_{out} + kV} \Rightarrow C_{out} \cdot Q_{out} + C_{out} \cdot kV = C_{in} \cdot Q_{in}$

$\Rightarrow C_{in} \cdot Q_{in} - C_{out} \cdot Q_{out} = C_{out} \cdot kV \quad \left(\Rightarrow \frac{Q(C_{in} - C_{out})}{C_{out} \cdot V} = k \right)$

$Q_{out} = Q_{in} \text{ (no accumulation)}$

$\Rightarrow k = \frac{8640 \text{ m}^3/\text{day} [100 \text{ mg/L} - 20 \text{ mg/L}]}{20 \text{ mg/L} \cdot (10 \times 10^4 \text{ m}^2 \times 1 \text{ m})} = 0.3456 / \text{day}$

$k = 0.35 \text{ day}^{-1}$

#4 A CMFR is used to treat an industrial waste product, using a reaction that destroys the pollutant according to first order kinetics, with $k = 0.216/\text{day}$. The reactor volume is 500 m^3 , the volumetric flow rate of the single inlet and exit is $50 \text{ m}^3/\text{day}$, and the inlet pollutant concentration is 100 mg/L .

Known:

- reaction rate coefficient $k = 0.216 \text{ day}^{-1}$
- reactor volume $V = 500 \text{ m}^3$
- flow rate in and out $Q = 50 \text{ m}^3/\text{day}$
- inlet pollutant concentration $C_{in} = 100 \text{ mg/L}$

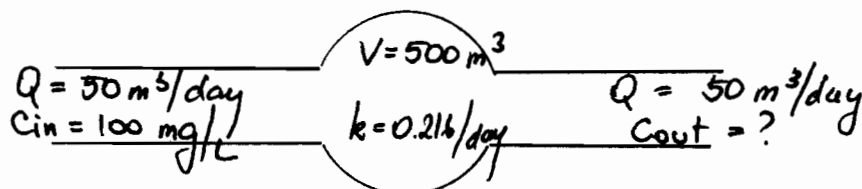
a. Assume the CMFR has reached steady state, what is the outlet concentration after treatment? (mg/L)

Assumptions:

- steady state (no accumulation)
- non conservative pollutant
- no outside gain/losses due to percolation or evaporation

Find:

- outlet concentration, $C_{out} = ?$



$$C_{out} = \frac{C_{in} \cdot Q}{Q + k \cdot V} = \frac{100 \text{ mg/L} \cdot 50 \text{ m}^3/\text{day}}{50 \text{ m}^3/\text{day} + 0.216/\text{day} \cdot 500 \text{ m}^3}$$

$$C_{out} = 31.65 \text{ mg/L}$$

b. The manufacturing process that generates the waste in Part A has to be shut down, and, starting at $t=0$, the concentration of C_{in} entering the CMFR is set to 0.

- How many hours does it take the tank concentration to reach 10% of its initial, steady state value.
- How many hours does it take to reach steady state?

Knowns:

- incoming concentration $C_{in} = 0 \text{ mg/L}$
- concentration at $t=0$ $C_0 = 31.65 \text{ mg/L}$
- concentration at time final $C_t = 3.165 \text{ mg/L}$
- reaction rate constant $k = 0.216/\text{day}$
- flow rate $Q = 50 \text{ m}^3/\text{day}$
- reactor volume $V = 500 \text{ m}^3$

Assumptions:

- non steady state, non conservative

Find:

- time when $C_t = 10\% C_0$ $t_{10\%} = ?$
- t when it reaches steady state $t_{ss} = ?$

$$C_t = C_{\infty} + (C_0 - C_{\infty}) e^{-(k + \frac{Q}{V}) \cdot t}$$

$$C_{\infty} = \frac{C_{in} \cdot Q}{Q + kV} = 0$$

$$C_t = C_0 \cdot e^{-(k + \frac{Q}{V}) t} \Rightarrow$$

$$\frac{3.165 \text{ mg/L}}{31.65 \text{ mg/L}} = e^{-\left[0.216/\text{day} + \frac{50 \text{ m}^3/\text{day}}{500 \text{ m}^3}\right] \cdot t}$$

$$0.1 = e^{-0.316 t}$$

$$\Rightarrow \ln(0.1) = -0.316 t \Rightarrow$$

$$\Rightarrow t = \frac{\ln(0.1)}{-0.316} \Rightarrow t = 7.27 \text{ days} = 174.9 \text{ hours} = t$$

$$C_{\infty} = C_0 + (C_0 - C_{\infty}) e^{-(k + \frac{Q}{V}) t}$$

$$0 = C_0 \cdot e^{-0.316 t}$$

$$\Rightarrow \ln 0 \text{ does not exist.}$$

steady state is reached only at infinity.

c. A plug-flow reactor (PFR) is used to model the chemical transformation of waste product presented in Part A. Assume that the flow rate and the first-order decay rate constant are unchanged ($Q = 50 \text{ m}^3/\text{day}$, $k = 0.216/\text{day}$). Based on this information answer the following.

- What is the volume required for the PFR to obtain the same degree of pollutant reduction as the CMFR of Part A? Report your answer in m^3 .
- What is the % reduction in volume as compared to the volume of the CMFR for the same steady-state conditions?

$$C_\theta = C_0 \cdot e^{-k \cdot \theta} \quad \left\{ \Rightarrow C_\theta = C_0 \cdot e^{-\left(k \cdot \frac{V}{Q}\right)} \right.$$

$$\theta = \frac{V}{Q}$$

$$C_\theta = C_{\text{out}} = 31.65 \text{ mg/L} \quad k = 0.216/\text{day}$$

(from A)

$$Q = 50 \text{ m}^3/\text{day}$$

$$C_0 = 100 \text{ mg/L}$$

$$\Rightarrow \frac{31.65 \text{ mg/L}}{100 \text{ mg/L}} = e^{-\left(0.216/\text{day} \cdot \frac{V}{50 \text{ m}^3/\text{day}}\right)}$$

$$0.3165 = e^{-\left(4.32 \times 10^{-3} \frac{1}{\text{m}^3} \cdot V\right)}$$

$$\frac{\ln(0.3165)}{-4.32 \times 10^{-3}} \text{ m}^3 = V \Rightarrow \boxed{V = 266.3 \text{ m}^3}$$

$$\text{ii. } \left(1 - \frac{V_{\text{PFR}}}{V_{\text{CMFR}}}\right) 100 = \left(1 - \frac{266.3 \text{ m}^3}{500 \text{ m}^3}\right) 100 = 46.74\%$$

percent reduction in volume is 46.74%

5 For the following conditions determine whether CMFR or a PFR is more efficient in removing a reactive compound from the waste stream under steady-state conditions with a first-order reaction: reactor volume = 280 m^3 , flow rate = $14 \text{ m}^3/\text{day}$, and a reaction rate coefficient = $0.05/\text{day}$.

Knowns:

- reactor volume $V = 280 \text{ m}^3$
- flow rate $Q = 14 \text{ m}^3/\text{day}$
- reaction rate coefficient $k = 0.05/\text{day}$

Find:

- efficiency of a CMFR vs. a PFR

Assumptions:

- steady state conditions (no accumulation)
- first order reaction
- no outside gains/losses due to persolation/evaporation
- non conservative compound

efficiency: $\eta = \frac{C_{in} - C_{out}}{C_{in}} \cdot 100 = \left(1 - \frac{C_{out}}{C_{in}}\right) \cdot 100$

for CMFR: $C_{out} = \frac{C_{in} \cdot Q_{in}}{Q_{out} + kV}$ $\left\{ \Rightarrow \frac{C_{out}}{C_{in}} = \frac{Q}{Q + k \cdot V} \right.$
 $Q_{in} = Q_{out}$

$\Rightarrow \frac{C_{out}}{C_{in}} = \frac{14 \text{ m}^3/\text{day}}{14 \text{ m}^3/\text{day} + 0.05/\text{day} \cdot 280 \text{ m}^3} = 0.5 \Rightarrow \boxed{\eta_{CMFR} = 50\%}$ (1)

for PFR: $C_{\theta} = C_0 \cdot e^{-k\theta} \Rightarrow \frac{C_{out}}{C_{in}} = e^{-k\theta}$ $\left\{ \Rightarrow \right.$
 $\theta = \frac{V}{Q} = \frac{280 \text{ m}^3}{14 \text{ m}^3/\text{day}} = 20 \text{ days}$

$\frac{C_{out}}{C_{in}} = e^{-0.05/\text{day} \cdot 20 \text{ days}} = e^{-1} = 0.368 \Rightarrow \boxed{\eta_{PFR} = 63.2\%}$ (2)

(2) > (1) $\Rightarrow$ PFR is more efficient

#6 You are working for the Michigan Department of Environmental Quality (DEQ) and must issue a discharge permit to a certain industry. The industry wishes to discharge "green muck" into the Ickold River. The flow of waste water from the industry is $0.15 \text{ m}^3/\text{s}$. The flow of water in the Ickold River (upstream from the waste water outfall) is $0.25 \text{ m}^3/\text{s}$. Upstream of the outfall, the concentration of "green muck" in the river is 0.1 mg/L . What is the maximum concentration the industry can discharge if the concentration of "green muck" in the river (after mixing) cannot exceed 2.5 mg/L ?

Knowns:

- flow of waste water $Q_{ww} = 0.15 \text{ m}^3/\text{s}$
- flow of the river $Q_r = 0.25 \text{ m}^3/\text{s}$
- concentration of "green muck" in river $C_r = 0.1 \text{ mg/L}$
- concentration after mixing $C_{mix} = 2.5 \text{ mg/L}$

Find:

- maximum discharge concentration of "green muck" $C_{ww} = ?$

Assumptions:

- steady state (no accumulation)
- "green muck" is conservative, does not react/degrade
- no outside gains/losses due to percolation or evaporation
- water and the green muck mix completely and immediately

$$\begin{array}{l} Q_r = 0.25 \text{ m}^3/\text{s} \\ C_r = 0.1 \text{ mg/L} \end{array} \quad \begin{array}{l} Q_{mix} \\ C_{mix} \end{array}$$

$$\begin{array}{l} Q_{ww} = 0.15 \text{ m}^3/\text{s} \\ C_{ww} = ? \end{array}$$

$$Q_r + Q_{ww} = Q_{mix}$$

$$Q_{mix} = 0.4 \text{ m}^3/\text{s}$$

$$\text{SS / Cons: } \frac{dM}{dt} = 0 \Rightarrow$$

$$\Rightarrow C_r \cdot Q_r + C_{ww} \cdot Q_{ww} = Q_{mix} (C_{mix})$$

$$C_{ww} = \frac{Q_{mix} \cdot C_{mix} - C_r \cdot Q_r}{Q_{ww}} =$$

$$C_{ww} = \frac{0.4 \text{ m}^3/\text{s} \cdot 2.5 \text{ mg/L} - 0.1 \text{ mg/L} \cdot 0.25 \text{ m}^3/\text{s}}{0.15 \text{ m}^3/\text{s}}$$

$$C_{ww} = 6.5 \text{ mg/L}$$