Theoretical relation of the Formula Electric Car Physical Parameters of Load Torque, Supply Current and Motor Speed

Zainab Hussein

3-24-2017

Introduction

By power conservation, analysis of the mathematical relations between motor speed, load torque and supply current is possible. For an ideal DC motor, input electrical power is expected to equal output mechanical power for a lossless motor. The Formula Electric car project using an AC motor, characterization of that motor and motor controller system uses a basis of conservation of power as a mathematical model.

The parameters for analysis, load torque, motor speed and supply current are a function of throttle, load and supply voltage settings. There are two approaches for this characterization a physical model of the car and a user (driver) controlled model. The physical model considers what parameters have a physical meaning such as torque, rpm and current, while the user controlled model considers what parameters the driver can effect or access in situ, such is throttle controlled by the pedal press.

Using the DC model to analyze the AC motor, conservation of power still has electrical input power equal mechanical output power represented by Eq.1

$$vi = \tau\omega \qquad \text{Eq.1}$$

Where v is supply voltage (V), i is supply current (A), τ is load torque (Nm) and ω is motor speed in (rad/s)

Our setup supplies a constant supply voltage, therefore the electrical power is controlled by the supply current. The constant voltage acts as a coefficient of the supply current, effectively resulting to a direct proportionality relation between supply current and input power shown in Eq.2

$$P_{in} = vi \qquad \text{Eq.2}$$

Where P_{in} is input electrical power, v a constant supply voltage and i is supply current.

For mechanical power relation to the load and motor speed mechanical parameters, it is more complex because both torque and rpm are changing parameters whose coefficients can only be analyzed when one of the two parameters is held constant. For an ideal motor, input power equals output power in Eq.3, thus, relation of torque and rpm can be drawn for a given constant power shown in Eq.4, with a corresponding current relation for that same power shown in Eq.5

$$P_{in} = P\_out \qquad \text{Eq.3}$$

$$P_0 = \tau\omega \qquad \text{Eq.4}$$

$$P_0 = ki \qquad \text{Eq.5}$$

Where P_0 is a given power (Watts), k is constant value representing the constant supply voltage. Equating Eq.4 and 5 result to Eq.6

$$ki = \tau\omega \quad \text{Eq.6}$$

Eq.6 simplifies to Eq.7

$$i = \frac{1}{k}\tau\omega \quad \text{Eq.7}$$

Since motor speed and load torque change at different degrees, attribute of the constants associated with them have to be analyzed individually while holding one constant for changing values of the other. The goal is to show what effect changing any of the physical parameters of the car, has on the rest of the parameters.

Method and Results

To find the expected mathematical equations relating the physical parameters dictated by Eq.3-7, arbitrary values of power are used to generate the three physical parameters whose relation is being sought. The data for this found in Appendix.

Constant power

For set values of motor speed, figure shows the effect of constant current on torque. This shows an expected hyperbolic relation.

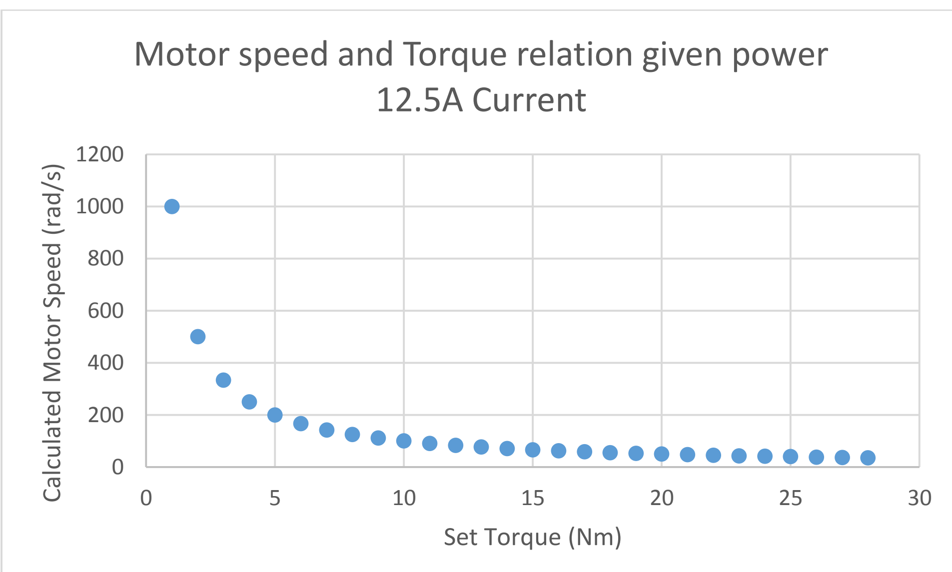

Figure 1torque and rpm relation with constant current

For set values of torque, figure 2 shows the effect of constant current on motor speed. This shows an expected hyperbolic relation.

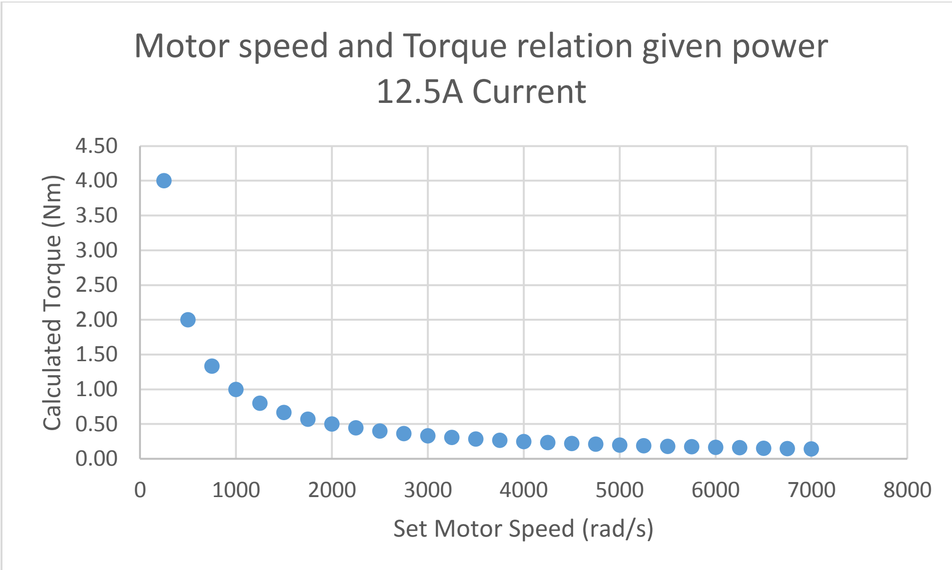

Figure 2 torque and rpm relation with constant current

Changing power

For set values of motor speed, figure 3 shows the effect of constant rpm on torque and current. This shows an expected linear relation.

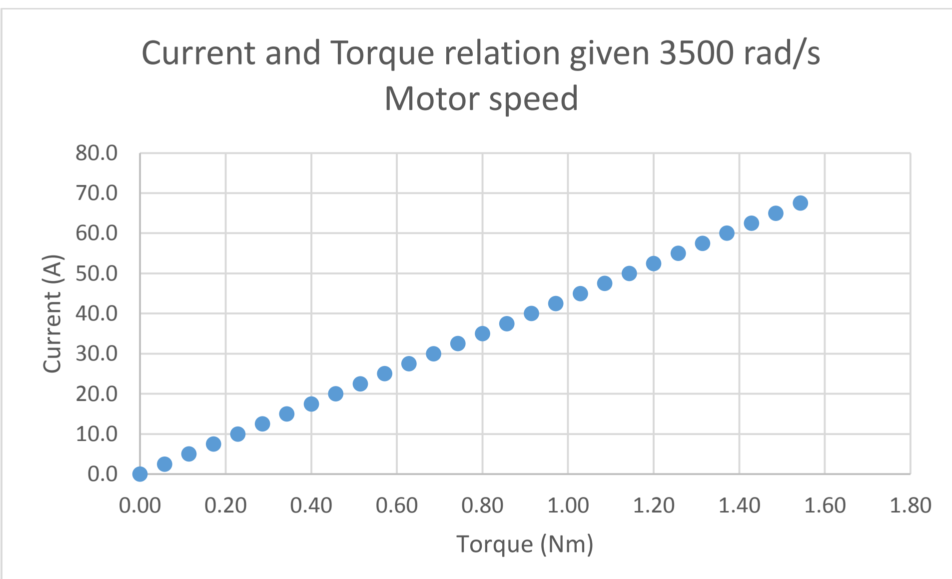

Figure 3 torque and current relation with constant rpm

For set values of torque, figure 3 shows the effect of constant torque on rpm and current. This shows an expected linear relation

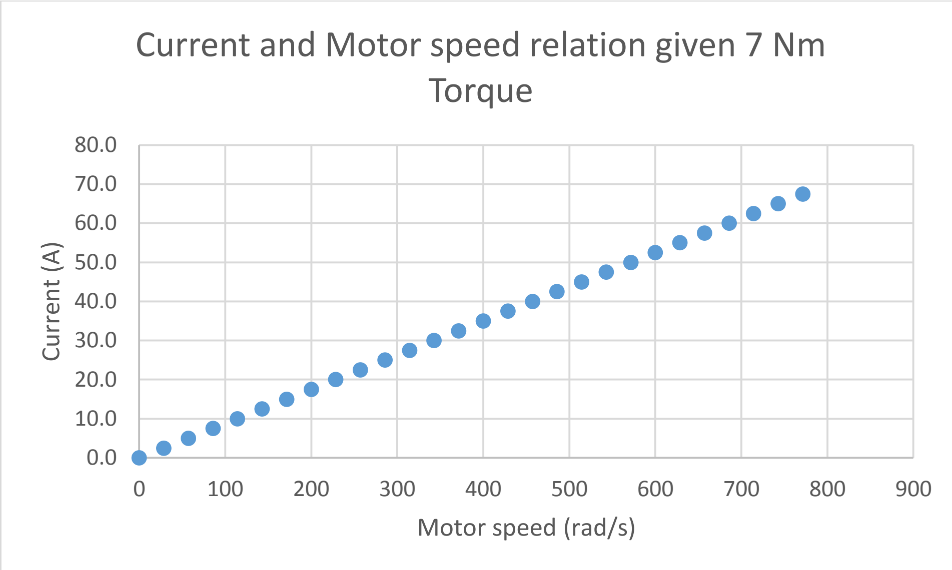

Figure 4 torque and current relation with constant rpm

Conclusion

Theoretically, both motor speed and torque have a linear relationship to current when either is held constant. Thus, experimental data should ideally show an array of linear relationships as shown in figure 3 and 4 for the case of changing power.

Also both motor speed and torque have a hyperbolic relationship to one another for constant value of current. Thus experimental data should also ideally show an array of hyperbolic relationships as shown in figure 1 and 2 for the case of constant power.

Appendix

Table 1

Constant power (constant Current)						
	set k		set RPM		set Torque	
P_0 (W)	k	Calculated Current	RPM	Calculated Torque	Torque	Calculated RPM
1000	80	12.5	250	4.00	1	1000
1000	80	12.5	500	2.00	2	500
1000	80	12.5	750	1.33	3	333
1000	80	12.5	1000	1.00	4	250
1000	80	12.5	1250	0.80	5	200
1000	80	12.5	1500	0.67	6	167
1000	80	12.5	1750	0.57	7	143
1000	80	12.5	2000	0.50	8	125
1000	80	12.5	2250	0.44	9	111
1000	80	12.5	2500	0.40	10	100
1000	80	12.5	2750	0.36	11	91
1000	80	12.5	3000	0.33	12	83
1000	80	12.5	3250	0.31	13	77
1000	80	12.5	3500	0.29	14	71
1000	80	12.5	3750	0.27	15	67
1000	80	12.5	4000	0.25	16	63
1000	80	12.5	4250	0.24	17	59
1000	80	12.5	4500	0.22	18	56
1000	80	12.5	4750	0.21	19	53
1000	80	12.5	5000	0.20	20	50
1000	80	12.5	5250	0.19	21	48
1000	80	12.5	5500	0.18	22	45
1000	80	12.5	5750	0.17	23	43
1000	80	12.5	6000	0.17	24	42
1000	80	12.5	6250	0.16	25	40
1000	80	12.5	6500	0.15	26	38
1000	80	12.5	6750	0.15	27	37
1000	80	12.5	7000	0.14	28	36

Table 2

Changing power (changing current)						
	set k		set RPM		set Torque	
P_0 (W)	k	Calculated Current	RPM	Calculated Torque	Torque	Calculated RPM
0	80	0.0	3500	0.00	7	0
200	80	2.5	3500	0.06	7	29
400	80	5.0	3500	0.11	7	57
600	80	7.5	3500	0.17	7	86
800	80	10.0	3500	0.23	7	114
1000	80	12.5	3500	0.29	7	143
1200	80	15.0	3500	0.34	7	171
1400	80	17.5	3500	0.40	7	200
1600	80	20.0	3500	0.46	7	229
1800	80	22.5	3500	0.51	7	257
2000	80	25.0	3500	0.57	7	286
2200	80	27.5	3500	0.63	7	314
2400	80	30.0	3500	0.69	7	343
2600	80	32.5	3500	0.74	7	371
2800	80	35.0	3500	0.80	7	400
3000	80	37.5	3500	0.86	7	429
3200	80	40.0	3500	0.91	7	457
3400	80	42.5	3500	0.97	7	486
3600	80	45.0	3500	1.03	7	514
3800	80	47.5	3500	1.09	7	543
4000	80	50.0	3500	1.14	7	571
4200	80	52.5	3500	1.20	7	600
4400	80	55.0	3500	1.26	7	629
4600	80	57.5	3500	1.31	7	657
4800	80	60.0	3500	1.37	7	686
5000	80	62.5	3500	1.43	7	714
5200	80	65.0	3500	1.49	7	743
5400	80	67.5	3500	1.54	7	771