# Social Security and Risk Sharing: The Role of Economic Mobility across Generations

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**Abstract**

How well does Social Security jointly insure lifetime earnings risk and longevity risk? We show that the answer to this fundamental question depends critically on the nature of economic mobility across generations. To show this result, we compare two economies. In our first economy, inheritances are uncorrelated with wage earnings, implying that an individual's earnings are unrelated to the wages and asset holdings of their predecessors. In our second economy, there is no such economic mobility; instead, low wage earners are stuck receiving small inheritances from their low-wage ancestral line while high wage earners enjoy large inheritances from their high-wage ancestral line. We make these comparisons in a variety of settings including both fixed and endogenous factor prices. Social Security causes large welfare losses in the first economy but can generate large welfare gains in the second economy. Given the apparent limits to economic mobility in the US, the welfare gains from collective risk sharing through Social Security are potentially large.

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# 1 Introduction

An individual making life-cycle saving decisions faces longevity risk and therefore must plan for every possible lifespan, including the possibility of living to be very old. This results in additional saving, which is costly from the perspective of the individual. If her lifespan were certain, she could plan to deplete her assets exactly at the end of life. However, facing an uncertain lifespan, she will consume less over the life cycle and pass away with unconsumed assets. From an ex-ante perspective, before entering the workforce, the individual also faces lifetime earning, or wage-type risk. Wage-type risk is the ex-ante risk that an individual will have a low wage or low lifetime earnings. Several exogenous factors such as zip code and parental earnings contribute to an individual's lifetime earnings and could contribute to an ex-ante risk of experiencing low wages. Prior to making any economic decisions an individual faces both wage-type and longevity risk. Adding an additional layer of complexity, these risks are largely uninsurable.[1]

Social Security is designed to provide insurance against these two risks (among other justifications). Social Security benefits are paid according to a progressive benefit earning rule, which redistributes wealth towards those with low lifetime wage earnings, thereby at least partially addressing the problem of low lifetime earnings. Benefits are also paid as a life annuity which provides partial longevity insurance. Both of these design features may help alleviate old age poverty and could improve the expected lifetime utility of participants. Social Security is a large program, providing retirement benefits to 47 million retirees (including dependents) totaling $844 billion in 2018 by taxing the wages of 176 million workers (SSA, 2019). With mounting fiscal pressure to reform Social Security, we are at an important inflection point where it is valuable to reflect and analyze this important program, as a guide for policymakers who are evaluating the necessity and future of this program.

We evaluate Social Security's effectiveness as a social insurance program in hedging lifetime earnings risk and longevity risk. We calculate the welfare benefit or cost of the system for individuals who face these risks, using an ex-ante perspective—that is, considering the individual's welfare before risk is resolved. We show that the welfare gains of Social Security are potentially quite large and depend crucially on the assumptions that we make about the nature of economic mobility across generations. While it is already well-understood that Social Security may play an important insurance role in the welfare of individuals in general, it is perhaps lesser known that the importance of Social Security's insurance role hinges on the degree to which individuals are able to break free from the poverty of their ancestral line (economic

[1]Although longevity risk can be insured using annuities, participation is low in private annuity markets and the market is characterized by asymmetric information and adverse selection (see Pashchenko (2013) for a discussion). Similarly, there is no insurance market to protect against low wage earnings over the life cycle.

mobility). In other words, we show that Social Security's ability to mitigate risk is best understood in the context of a model that explicitly factors economic mobility into the analysis. We find that the welfare effect of Social Security is quite negative in the presence of economic mobility, but the welfare effect is large and positive when there is a lack of economic mobility.

We consider three separate economies to illustrate our point: two equilibrium economies with different assumptions about economic mobility and a partial equilibrium benchmark economy for comparison. In each economy, individuals have full information and make rational decisions. While there are a variety of ways to model economic mobility, for our purposes economic mobility has to do with the degree to which the wages of a child are connected to the wage of their parent, with immobile societies featuring a strong correlation between the two and mobile societies featuring no correlation. All other assumptions and features are identical across the economies that we consider, and yet the level of economic mobility itself is sufficient to determine whether Social Security has a negative or a positive effect on individual welfare.

In the Perfect Mobility Economy (Economy I), we assume that inheritances are accidental bequests that are distributed evenly across the wage distribution as in most macroeconomic models. Wage income is mean-reverting in this economy and does not depend on the wages or asset holdings of one's ancestors. An individual's wage income is not linked to inheritance income and so inequality is not passed down from one generation to the next. In other words, inheritance income is not correlated with anything in the model. An individual is neither helped nor harmed by their ancestral line, since all individuals receive the same bequest and draw a random wage from the same distribution. Thus we are effectively assuming perfect economic mobility. In this economy we find that Social Security reduces welfare significantly because it crowds out private savings which reduces equilibrium bequests. Social Security is far worse than living with uninsured lifetime earning risk and longevity risk because the program reduces bequests which are a large source of income for low wage earners. For individuals with low wage earnings, the reduction in bequest income is much larger than the increase in lifetime income provided by a progressive Social Security system. Consequently, the reduction in bequest income for lower wage earners reduces their wellbeing by more than the amount that longevity and lifetime earning insurance improves wellbeing. In net, Social Security reduces ex-ante welfare in this economy.

In the No Mobility Economy (Economy II), we assume that accidental bequest income is tied directly to wage income. Individuals with low wages receive bequests from the deceased who also had low wages and individuals with high wages receive bequests from those with high wages. Essentially, the economy lacks intergenerational economic mobility, since individuals who receive larger bequests also receive higher wages. See De Nardi (2004) and Chetty et al. (2014) and the references therein for a discussion of intergenerational linkages and inequality.[2] Intergenerational inequality is more persistent in this framework, and Social Security is welfare enhancing. Social Security improves ex-ante lifetime wellbeing by 17%. The result is driven entirely by the progressivity of Social Security benefits and not by longevity insurance. If we assume instead that Social Security benefits are proportional to Social Security taxes paid, the welfare effect is zero. The longevity insurance aspect alone does not improve the welfare of individuals in this economy because it is exactly offset by a reduction in bequest income (as in Caliendo et al. (2014)). The wage-type insurance provided through a progressive Social Security benefit is welfare enhancing in this economy because there is no other source of intergenerational economic mobility.

As a final comparison, we model a partial equilibrium economy (Economy III) where wealth does not pass from one generation to the next. It is as if the government confiscates the assets of the deceased and throws them in the ocean. In this framework, the longevity insurance of Social Security comes without the cost of reduced bequest income since individuals do not receive bequests in the first place. As such, the welfare gains of Social Security are larger in this economy than in Economies I and II.

In our estimation, Social Security seems to be a powerful program when it comes to hedging combined lifetime earning and longevity risk. Social Security provides lifetime earning or wage-type insurance by paying progressive benefits and offers longevity insurance by paying benefits as an annuity. The program provides these benefits at the cost of reducing private savings and private intergenerational transfers. If individuals do not receive bequests, or if bequests are linked to wages and low wage individuals inherit small bequests while high wage individuals receive large bequests, then the benefits of redistribution through Social Security outweigh the costs of reduced private transfers. Although the actual relationship between wages and bequests across generations is more complex than any of our three model economies, a growing body of empirical evidence from Chetty et al. (2014) and others suggests that economic mobility is limited in the US (if not for all people, certainly for some) and we have shown that mobility has a first-order effect on the risk sharing role of Social Security. Hence, in addition to being an interesting and important topic in its own right, the nature of economic mobility is a key ingredient in our analysis of Social Security. If the limits to mobility are ignored, the welfare gains from risk sharing through Social Security can be seriously underestimated.

The main intuition for our result rests on the intergenerational transmission of wealth. In Economy I, individuals of all wage types receive the same bequest income, so in this sense the economy has a natural, built-in insurance mechanism to it: low wage earners get big bequests and high earners get small bequests, relative to their

[2] See Chetty and Hendren (2018a,b), Chetty et al. (2018a), Chetty et al. (2018b), and Chetty et al. (2020) for a discussion of the determinants of economic mobility.

earnings. Social Security harms welfare in this economy, because it unwinds some of the private insurance implied by intergenerational wealth transfers. In contrast in Economy II, bequests are tied directly to wages and income inequality is more pronounced. In Economy II, Social Security's redistributive benefit earning rule is the only insurance against low wage realizations, and thus the program is welfare enhancing. Economy II lacks any built-in insurance mechanism across generations—poverty and wealth are transmitted through rigid dynasties that can't be broken—and Social Security is welfare improving because its redistributive benefit earning rule helps to dampen the inequality that persists in the absence of the type of family insurance arising in Economy I.

We explore the robustness of our results in a variety of model settings including accidental vs intentional bequests as well as fixed vs endogenous factor prices. We confirm the intuition of our main results along these and other dimensions.

Starting with the seminal work of Yaari (1965), there is a long tradition in economic theory of studying longevity risk. Broadly speaking, papers in this large literature seek to understand the various factors affecting the decision to annuitize one's wealth, as well as the reasons why annuity markets are notoriously thin even when the welfare gains from annuitization appear larger. Just a few examples include Kotlikoff and Spivak (1981), Mitchell et al. (1999), Brown (2001, 2007), Brown et al. (2008, 2017, 2018), Davidoff et al. (2005), Finkelstein and Poterba (2004), Lockwood (2012), and Sheshinski (2008). In our paper, we assume individuals lack access to competitive annuity markets and have no mechanism—beyond self insurance—for hedging this risk. This opens the door for collective risk sharing through Social Security.[3]

The most closely related paper in this area, Caliendo et al. (2014), explores the role of Social Security in providing longevity insurance. The authors show that the benefits of annuitization provided by Social Security can be *completely* unwound in a general equilibrium framework with endogenous accidental bequests. Our paper builds on the intuition of Caliendo et al. (2014) to explore a model with heterogeneous wages and within cohort income redistribution through a progressive Social Security system. By adding wage heterogeneity, we are able explore different assumptions about the nature of economic mobility across generations and consider the dual insurance role of Social Security in mitigating multiple risks.

Fuster et al. (2003) also find that Social Security can improve welfare when individuals face uncertainty over lifetime income and longevity. They focus primarily on the consequences of two-sided altruism and find that Social Security increases wellbeing for most households. They also find that Social Security's welfare effect depends

[3] Hosseini (2015) studies the welfare effects on mandatory annuitization through Social Security in a model where private annuity markets do exist but suffer from adverse selection problems. We abstract from the adverse selection problem and instead assume annuity markets are closed and focus our attention on the issue of economic mobility.

on the parent-child wage correlation, which motivates a study like ours that focuses specifically on the role of economic mobility in determining the insurance properties of Social Security.

Considering the literature more broadly, several papers explore the insurance role of Social Security including Hubbard and Judd (1987), İmrohoroğlu et al. (1995), Conesa and Krueger (1999), Huggett and Ventura (1999), Hong and Rios-Rull (2007), Fehr and Habermann (2008), Cremer et al. (2008), Cremer and Pestieau (2011), Fehr et al. (2013), and Bagchi (2015), among others. These papers reach a variety of conclusions about Social Security's risk sharing role, but they do not focus directly on the role of economic mobility in assessing this question.[4] We are able to offer a set of new insights about the importance of economic mobility in assessing the insurance role of Social Security. If individuals lack economic mobility, then Social Security can improve lifetime wellbeing.[5]

Finally we diverge from the behavioral literature that explores the role of Social Security as a form a forced saving in models where individuals save "too little" on their own.[6] Instead, we focus on the insurance provided through Social Security in a fully rational neoclassical framework.

## 2 Model

The purpose of our model is to show that the ability of Social Security to jointly insure lifetime earnings risk and longevity risk depends critically on the nature of economic mobility across generations. As such, our analysis includes both lifetime earning and longevity risks and two different equilibrium assumptions about economic mobility. To illustrate the importance of the intergenerational transfer of wealth (economic mobility) as cleanly as possible, we abstract from other details that could

[4]A number of studies examine the degree to which differential mortality by wage type offsets the level of redistribution that is otherwise implied by the progressive benefit earning rule (Coronado et al. (1999, 2002, 2011), Bishnu et al. (2019), Liebman (2002), Goda et al. (2011), Gustman and Steinmeier (2001)), and a number of papers consider how this fact affects the desirability of different reform options (Pestieau and Racionero (2016), Bommier et al. (2011), and Sheshinski and Caliendo (2020)). We abstract from differential mortality considerations and focus directly on the role of intergenerational economic mobility in the study of risk sharing through Social Security.

[5]One implication of our analysis is that optimal reform policies would preserve the current benefit levels. A vast literature explores changes in Social Security taxes and benefits in response to changing demographics such as Coronado et al. (1999), De Nardi et al. (1999), Coronado et al. (2002), Diamond and Orszag (2005), Bommier et al. (2011), İmrohoroğlu and Kitao (2012), Kitao (2014), Pestieau and Racionero (2016), Bagchi (2016, 2017), and McGrattan and Prescott (2017) among many others. A closely related literature focuses on Social Security privatization in response to aging demographics, such as Feldstein (1996), Huggett and Ventura (1999), Kotlikoff et al. (2007), Nishiyama and Smetters (2007), and Conesa and Garriga (2008) among others.

[6]See Findley and Caliendo (2008) for a survey of behavioral models used to justify Social Security.

obscure this mechanism. For example we model bequests as accidental and hold factor prices fixed, although we relax both of these assumptions later. This keeps our initial model setting simple and focused on the point at hand.

We consider a continuous time life-cycle model with uncertain lifespan, wage heterogeneity, and a Social Security retirement system that mimics key features of the US program. Age is continuous and is indexed by $t$. At each moment in time an infinitely divisible cohort of unit mass is born. Individuals are born at $t = 0$ and die no later than $t = T$. The probability of surviving to age $t$ from the perspective of age 0 is $S(t)$, where $S(0) = 1$ and $S(T) = 0$. Retirement from the workforce occurs at $t = t_R$.

An individual's wage rate is drawn from the unit interval, $w \in [0, 1]$. Wages vary across the population according to the density function $g(w)$, and individuals have full information about their wage type from birth. Individuals must pay Social Security taxes at rate $\tau$ on wage income up to a wage cap $w_c < 1$. After retirement, individuals receive Social Security benefits $b(w)$ that depend on their wage earnings over the working period.

The individual's consumption is $c(t)$ and utility from consumption is $u(c(t))$, with $u_c > 0$ and $u_{cc} < 0$. Private annuity markets do not exist and all saving is done in a risk-free account that pays interest at rate $r$. The individual's savings balance at time $t$ is denoted $k(t)$. The individual starts the life cycle with nothing $k(0) = 0$ and must pay off all debts by the maximum survival age, $k(T) = 0$. Future utility is discounted at rate $\rho$.

The assets of the deceased are bequeathed to survivors. The individual does not have a bequest motive, rather any bequests are a result of lifespan uncertainty and are "accidental". We will consider three different assumptions for how bequests are distributed. In each case, the we denote the flow of bequest income received by an individual of wage type $w$ as $B(w)$.

The individual chooses $(c(t), k(t))_{t\in[0,T]}$ to solve

$$\max \int_0^T e^{-\rho t} S(t) u(c(t)) dt,$$

subject to

$$\dot{k}(t) = rk(t) + w - \tau \min(w, w_c) + B(w) - c(t), \text{ for } t \in [0, t_R],$$

$$\dot{k}(t) = rk(t) + b(w) + B(w) - c(t), \text{ for } t \in [t_R, T],$$

$$k(0) = k(T) = 0.$$

For CRRA utility, the solution is a path of consumption and saving:

$$c(t) = S(t)^{\frac{1}{\sigma}} e^{\frac{(r-\rho)t}{\sigma}} \left( \frac{\int_0^{t_R} e^{-rt}(w - \tau \min(w, w_c) + B(w))dt + \int_{t_R}^{T} e^{-rt}(b(w) + B(w))dt}{\int_0^T e^{(\frac{(r-\rho)}{\sigma}-r)t} S(t)^{\frac{1}{\sigma}} dt} \right),$$

$$k(t) = \int_0^t e^{r(t-v)}(w - \tau \min(w, w_c) + B(w) - c(v))dv, \qquad \text{for } t \leq t_R,$$

$$k(t) = e^{r(t-t_R)}k(t_R) + \int_{t_R}^{t} e^{r(t-v)}(b(w) + B(w) - c(v))dv, \qquad \text{for } t > t_R.$$

The optimal path of consumption declines over the life cycle based on the survival function $S(t)^{\frac{1}{\sigma}}$ and grows (or falls) at rate $(r-\rho)/\sigma$. When the interest rate and discount rate are equal $r = \rho$, the optimal consumption path declines over the life cycle. Assets earn interest at rate $r$ and grow if disposable income is greater than consumption and fall if disposable income is less than consumption. The asset path has a kink at retirement $t = t_R$.

We consider three types of economies that differ according to the way in which wealth is transmitted across generations.

In the first, accidental bequests are distributed equally to all of the living. This is the standard assumption in most macroeconomic models such as Hubbard and Judd (1987), İmrohoroğlu et al. (1995), Conesa and Krueger (1999), Huggett and Ventura (1999), Hong and Rios-Rull (2007), Nishiyama and Smetters (2007), and İmrohoroğlu and Kitao (2012). In this economy, each individual receives the same bequest, regardless of wage-type, and so bequests provide partial insurance against drawing a low wage-type.

In the second economy, accidental bequests are transmitted within wage type. That is, an individual with a given wage inherits bequests from those who had the same wage type. Income inequality is persistent across generations and is more severe than in first economy. This is similar to assuming that there is no intergenerational economic mobility.

In the third economy, we assume that the government captures all accidental bequests and spends the revenue on non-valued goods (or additively-valued goods). In the third economy, no one receives bequest income.

## 2.1 Economy I: Perfect Mobility

In this economy wealth is transmitted across generations in the traditional manner in the macroeconomics literature. Bequest income is spread evenly across all surviving individuals (of all ages and wage types). That is $B(w) = B$ for all $w$ and all $t$.[7]

Total bequest income received by survivors equals the total assets of the deceased according to the following aggregate resource constraint

$$\int_0^T S(t) B dt = \int_0^1 \int_0^T g(w) \left( -\dot{S}(t) \right) k(t|w, B) dt dw.$$

The left-hand side of the resource constraint shows bequests received; the right-hand side shows the assets of the deceased of each wage type and each possible date of death. Notice that everyone receives the same bequest regardless of wage type. In that sense, this economy has economic mobility, because no one is harmed (or helped) by receiving a small (or large) inheritance. The Social Security budget must balance

$$\int_0^{w_c} \int_0^{t_R} g(w) S(t) \tau w dt dw + \left( \int_{w_c}^1 g(w) dw \right) \int_0^{t_R} S(t) \tau w_c dt = \int_0^1 \int_{t_R}^T g(w) S(t) b(w) dt dw.$$

The left side of the equation is aggregate (average) taxes collected and the right side is benefits paid. Solving for the balanced budget Social Security tax we have

$$\tau = \frac{\int_0^1 \int_{t_R}^T g(w) S(t) b(w) dt dw}{\int_0^{w_c} \int_0^{t_R} g(w) S(t) w dt dw + \left( \int_{w_c}^1 g(w) dw \right) \int_0^{t_R} S(t) w_c dt}.$$

A **Stationary Equilibrium** is comprised of household allocations $(c(t|w), k(t|w))_{t \in [0,T]}$ for all $w$ that solve each individual's full information optimization problem given wage density $g(w)$ on $[0, 1]$ and survival probabilities $S(t)$ on $[0, T]$, bequest income $B$ that satisfies the aggregate resource constraint, and Social Security tax rate $\tau$, taxable wage cap $w_c$, and benefit earning rule $b(w)$ that jointly balance the Social Security budget.

The consumption and saving decisions of households and the bequest amount $B$ are jointly determined in equilibrium. Everyone in the economy receives the same bequest, regardless of their wage-type. Bequests depend on asset holdings at the date of death. Similarly, asset holdings and consumption decisions depend on bequests received. Consumption and asset holdings also depend on the Social Security program and survival probabilities.

While the equilibrium concept that we utilize here is conventional, there are other ways to model intergenerational dynamics and the flow of bequest income from parent to child. In the appendix, we entertain a setting that is quite different from our baseline analysis by including specific parent-child linkages, and yet we show that all of the welfare results continue to hold.

[7] We abstract from any timing uncertainty regarding the receipt of a bequest. Cottle Hunt and Caliendo (2021) find that Social Security is welfare enhancing if the timing of bequest income is risky.

## 2.2 Economy II: No Mobility

In this economy wealth is dynastic and is therefore transmitted within wage types. Individuals of a given wage type collect bequest income from deceased individuals of the same wage type. In other words, individuals inherit their wage type from an equilibrium ancestral line bearing the same wage type. Note that in this economy, the intergenerational transmission of wealth is no longer redistributive. In Economy I, all wage types receive the same bequest income, so in this sense the economy has a natural, built-in insurance mechanism to it: low wage earners get big bequests and high earners get small bequests, relative to their earnings. In Economy II, this insurance feature is removed, income inequality is more pronounced, and Social Security's redistributive benefit earning rule is the only insurance against low wage realizations.

This economy is composed of a continuum of equilibria, with bequest income differing by wage type $B(w)$, which satisfies the following

$$\int_0^T S(t)B(w)dt = \int_0^T \left(-\dot{S}(t)\right) k(t|w, B(w))dt \text{ for all } w \in [0,1].$$

Here the left-hand side of the equation shows total bequests received by all survivors of a particular wage type. The right-hand side shows the total assets of the deceased of the same wage-type at all dates of death. Note that bequests received depend directly on wage-type. This equation holds for each possible wage-type. The Social Security budget must balance in Economy II as in Economy I

$$\tau = \frac{\int_0^1 \int_{t_R}^T g(w)S(t)b(w)dtdw}{\int_0^{w_c} \int_0^{t_R} g(w)S(t)wdtdw + \left(\int_{w_c}^1 g(w)dw\right) \int_0^{t_R} S(t)w_c dt}.$$

A **Stationary Equilibrium** is comprised of household allocations $(c(t|w), k(t|w))_{t\in[0,T]}$ for all $w$ that solve each individual's full information optimization problem given wage density $g(w)$ on $[0,1]$ and survival probabilities $S(t)$ on $[0,T]$, a distribution of bequest income $B(w)$ that satisfies a continuum of aggregate resource constraints (one for each wage type), and Social Security tax rate $\tau$, taxable wage cap $w_c$, and benefit earning rule $b(w)$ that jointly balance the Social Security budget.

As in the previous economy, in this dynastic economy consumption, asset holdings, and bequests are jointly determined in equilibrium; however, in this economy bequests depend directly on wage-type. Bequests received by a particular wage-type depend on the asset holdings of the same wage-type at the date of death. Similarly, asset holdings and consumption of a particular wage-type depend on the bequests they receive, as well as on the Social Security system and survival function.

## 2.3 Economy III: Bequests Thrown in the Ocean

In this economy accidental bequests are confiscated by the government and used to purchase non-valued goods, or additively-valued goods. Equivalently, bequests are thrown in the ocean. The only intergenerational transfers occur within the Social Security system since there are no private intergenerational transfers. That is $B(w) = 0$ for all $w$ and all $t$. This serves as a partial-equilibrium benchmark for comparison. The Social Security budget must balance in Economy III as in Economy I and Economy II.

A **Stationary Equilibrium** is comprised of wage density $g(w)$ on $[0, 1]$, survival probabilities $S(t)$ on $[0, T]$, household allocations $(c(t|w), k(t|w))_{t\in[0,T]}$ for all $w$ that solve each individual's full information optimization problem given bequest income $B(w) = 0$ for all $w$ and all $t$, and Social Security tax rate $\tau$, taxable wage cap $w_c$, and benefit earning rule $b(w)$ that jointly balance the Social Security budget.

# 3 Calibration

We calibrate the model to the US economy. Following Cottle Hunt and Caliendo (2020), we calibrate the wage density function to match the distribution of wages reported by the Social Security Administration, who report that average individual wage earnings in 2018 were \$52,146. Likewise the maximum taxable wage income for 2018 was \$128,400, and over the last few decades the share of workers who earn more than the taxable maximum has held fairly steady at slightly above 6% (SSA, 2013). These facts give us quantitative targets to calibrate the model density function $g(w)$.

We assume wages follow a truncated beta distribution with density

$$g(w) = \frac{w^{\gamma-1}(1-w)^{\beta-1}}{\int_0^1 w^{\gamma-1}(1-w)^{\beta-1}dw}.$$

We assume the maximum wage in the economy is twice the taxable wage cap \$256,800, which is normalized to model wage $w = 1$. The choice of maximum wage influences the length of the right tail of our income distribution; however, given our calibration procedure, it does not impact the share of the population in the right tail. All earners above the tax cap share the same tax liability and receive the same benefit. With the maximum wage normalized to 1, the taxable wage cap in the model is $w_c = 0.5$.

We select $(\gamma, \beta)$ to approximate two targets. First, we calibrate the mean wage to $\$52,146/\$256,800 = 0.2031$. The mean wage from the beta distribution is $\mathbb{E}(w) = \gamma/(\gamma + \beta)$, and we use this fact to ensure that, for any choice of $\beta$, the parameter $\gamma$ is chosen such that $\mathbb{E}(w) = 0.2031$. That is, $\gamma = 0.2031\beta/(1 - 0.2031)$. Second, we can then choose $\beta$ to match the share of individuals above the taxable wage cap (6%): $\int_{w_c}^{1} g(w)dw = 0.06$. Doing this yields $\beta = 4.2377$ and $\gamma = 1.0798$. This calibrated density has 6% of individuals above the taxable wage cap, and the mean and taxable wage cap are in the correct positions relative to one another.

Social Security benefits (PIA) are a piecewise linear function of earnings that replace 90% of wages up to the first bend point, 32% of wages between the first and second bend points, and 15% of wages between the second and third bend points. Beyond the third bend point, the function is flat. We use a conventional estimate of the bend points relative to average wages, $0.2\mathbb{E}(w)$, $1.24\mathbb{E}(w)$, and $2.47\mathbb{E}(w)$ (as in, e.g., Alonso-Ortiz (2014)).

We assume individuals begin work at age 18 (model age 0), retire at age 67 (model age $t_R = 49$) and pass away no later than age 100 (model age $T = 82$). The baseline survival function $S(t)$ is calibrated to ensure that the ratio of workers to retirees is 3.3, which is approximately the average value in the US during the period 2000-2010. The survival function $S(t) = 1 - (t/T)^{2.58}$ produces a ratio of workers to retirees $\int_0^{t_R} S(t)dt / \int_{t_R}^{T} S(t)dt = 3.3$. We set the Social Security tax rate to balance the Social Security budget $\tau = 0.1199$.[8]

We assume CRRA preferences, $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 3$. We set the rate of time preference $\rho = 0$ and the risk free interest rate to $r = 0$. This corresponds to the golden rule level of capital and ensures the economy is (barely) dynamically efficient because we have abstracted away from population growth and wage growth. With this parameterization Social Security is, essentially, fully-funded. We prefer this parameterization because the return on Social Security is neutral compared to private savings. Any welfare gains or losses that we find will be attributable to the insurance roles of Social Security rather than to dynamic inefficiency (as in Diamond (1965)). In robustness exercises, we set $r = 2.9\%$ according to SSA projections. We summarize our baseline calibration in Table 1.

# 4 Welfare Analysis

## 4.1 Ex-ante Compensating Variation

To measure the effectiveness of Social Security in providing insurance against longevity risk and wage-type risk, we calculate a compensating variation that shows the percentage of lifetime consumption that an individual is willing to pay to live in a world with Social Security, before their wage-type is revealed

[8]This is a little larger than the full employer and employee tax of 10.6% (the Old-Age and Survivors Insurance (OASI) tax rate since 1990).

| **Calibration** | |
|---|---|
| **Wages:** | |
| $g(w) = \frac{w^{\gamma-1}(1-w)^{\beta-1}}{\int_0^1 w^{\gamma-1}(1-w)^{\beta-1}dw}$ | truncated beta pdf over wage type for $w \in [0,1]$ |
| $w_c = 0.5$ | taxable wage cap ($128,400 in 2018) |
| $\mathbb{E}(w) = 0.2031$ | mean individual wage ($52,146 in 2018) |
| $\int_{w_c}^1 g(w)dw = 0.06$ | share of individuals above taxable wage cap |
| $\gamma = 1.0798$ | calibrated to match mean wage |
| $\beta = 4.2377$ | calibrated to match share above tax cap |
| **Social Security:** | |
| $\tau = 11.99\%$ | balances the Social Security budget |
| $b(w)$ piecewise continuous | current law SS benefit earning formula |
| **Demographics and misc:** | |
| $T = 82$ | maximum economic lifespan (age 18 to 100) |
| $t_R = 49$ | length of career (age 18 to 67) |
| $S(t) = 1 - (t/T)^{2.58}$ | survival function yielding 3.3 workers:retirees |
| $\rho = 0$ | discount rate (free parameter) |
| $\sigma = 3$ | midpoint CRRA value from literature |
| $r = 0$ or $r = 0.029$ | fully-funded parametrization or real interest rate from Trustee's Report |

Table 1: Summary of Baseline Calibration of Parameters

$$\int_0^1 \int_0^T g(w)e^{-\rho t}S(t)u(c(t|w))dtdw = \int_0^1 \int_0^T g(w)e^{-\rho t}S(t)u(c^{ss}(t|w)(1-\Delta))dtdw.$$

Here $c(t|w)$ is consumption without Social Security, with the bequests that correspond to an economy without Social Security, and $c^{ss}(t|w)$ is consumption with Social Security, with the corresponding bequests.

The compensating variation $\Delta$ is the fraction of lifetime consumption an individual with Social Security would be willing to give up such that their ex-ante expected lifetime utility is equal to their ex-ante expected utility without Social Security. If $\Delta$ is positive, Social Security increases the well-being of the individual.

Table 2 reports the ex-ante compensating variation $\Delta$ for different parameterizations. The ex-ante compensating variation is large and negative in the Perfect Mobility Economy (Economy I), suggesting that households are ex-ante worse off with Social Security. This is due to the crowding-out of private saving which reduces the size of accidental bequests. Everyone in Economy I receives the same bequest. Low wage individuals rely on the bequest as a large source of lifetime income. Thus, the reduction in bequest income is particularly harmful for those at the bottom of the wage distribution. From an ex-ante perspective, before wage-type is known, individuals are worse off in a world with lower bequests and Social Security. Social Security is harmful across several different parameterizations.

In the No Mobility Economy with dynastic bequests (Economy II), Social Security is welfare enhancing in the baseline parameterization with $r = 0$. The ex-ante compensating variation in Economy II is equal to 17% of consumption. This result is driven by the progressive benefit earning rule of Social Security. The program transfers wealth from high wage individuals to low wage individuals. From an ex-ante perspective, before wage-type is revealed, this is welfare enhancing. Given our assumption of constant population and wages, an interest rates below zero would be dynamically inefficient. Our baseline $r = 0$ is the lowest interest rate that is efficient. If we consider a higher interest rate, the economy is even more dynamically efficient, and Social Security is relatively less attractive compared to private savings. As the interest rate increases, an individual's ex-ante willingness to pay for Social Security falls because the crowding out of private savings is more costly when the rate of return on savings exceeds the rate of return on Social Security. The compensating variation is negative, for example, if the interest rate is 0.029 as in the second and third columns of Table 2. The redistribution of the program outweighs the crowding out when the interest rate is less than 0.016 (given our other parameters).

Bequests are passed down within wage type in Economy II. Income inequality is persistent and there is no private wealth redistribution across different ancestral lines. In this setting, the longevity insurance provided by Social Security is completely

| compensating variation $\Delta$ | | | | | | |
|---|---|---|---|---|---|---|
| | baseline model | | | | linear benefit | progressive income tax |
| | $\rho = 0$ $r = 0$ | $\rho = 0$ $r = 0.029$ | $\rho = 0.029$ $r = 0.029$ | $\rho = 0.029$ $r = 0$ | $\rho = 0$ $r = 0$ | $\rho = 0$ $r = 0$ |
| Economy I: Perfect Mobility | -0.69 | -0.41 | -0.86 | $-1$ | -0.89 | -0.21 |
| Economy II: No Mobility | 0.17 | -0.10 | -0.09 | 0.17 | 0 | 0.05 |
| Economy III: Thrown in Ocean | 0.33 | 0.05 | 0.05 | 0.33 | 0.13 | 0.14 |

Table 2: Welfare effects of Social Security. Ex-ante compensating variation $\Delta$ for different parameterizations of the interest rate $r$ and discount rate $\rho$.

offset by the crowding out of bequests. For any given wage type, the gain from annuitization provided through Social Security is exactly offset by a reduction in bequests. The crowding out of bequests perfectly offsets the longevity insurance. Our analysis therefore extends the results of Caliendo et al. (2014) who analytically show that the benefits of longevity insurance are unwound by reduced accidental bequests in a model with homogeneous wages. We explore this idea further in Section 4.3.

In Economy III, Social Security is always welfare enhancing from an ex-ante perspective. In this economy, private saving is inefficient in the sense that all the accidental bequests are wasted and do not contribute to the consumption of any generation. In contrast, Social Security does not waste resources. Thus, the welfare gains of annuitization through Social Security are quite large. In our baseline $r = 0$ parameterization, the welfare gain is equal to approximately one third of lifetime consumption. This is consistent with earlier papers that show large welfare gains from Social Security (Hubbard and Judd (1987), İmrohoroğlu et al. (1995), among others), and that show the value of annuitization in general is very large, in the range of 30% to 50%, when bequest income is not accounted for (Kotlikoff and Spivak (1981), Mitchell et al. (1999), Brown (2001), and Davidoff et al. (2005)).

## 4.2 Compensating Variation by Wage-type

The ex-ante welfare gains and losses of the previous section are driven by the ways Social Security changes the lifetime consumption of low wage individuals. To strengthen our intuition, in this section we calculate the compensating variation by wage-type. The compensating variation $\Delta(w)$ is defined as the percentage of lifetime consumption that an individual of wage type $w$ is willing to pay for Social Security. A positive number indicates that Social Security improves welfare.

$$\int_0^T e^{-\rho t} S(t) u(c(t|w)) dt = \int_0^T e^{-\rho t} S(t) u(c^{ss}(t|w)(1 - \Delta(w))) dt.$$

The compensating variation by wage type is illustrated in Figure 1 (for the baseline case of $r = \rho = 0$). In the Perfect Mobility Economy with uniform bequests (Economy I), Social Security is harmful for low wage individuals and beneficial for high wage individual when the interest rate is zero. In other words, Social Security is actually regressive in this economy, and intergenerational economic mobility is entirely the source of this result. The negative effect for low wage individuals is very painful, since the reduction in bequest income represents a large decline in lifetime income. The equilibrium uniform bequest falls from 0.024 to 0.006 with the introduction of Social Security ($6,082 to $1,629 in 2018 dollars given our wage distribution). The bequest is larger than wage income for the bottom 7.8% of the population without Social Security, and only 1.9% with Social Security. The compensating variation (CV) is largest for wage earners right at the second point of the benefit earning rule (wage equal to 1.24 times the mean wage). More than anyone else, these individuals benefit from the progressive Social Security benefit more than they are harmed by the reduction in bequest income. For individuals earning higher wages, the CV is still positive, but decreasing in wage.

The pattern is reversed for the No Mobility Economy with dynastic bequests (Economy II). When the interest rate is zero, the CV is positive only for low wage individuals who benefit from the progressivity of Social Security benefits. The CV is only positive for individuals who receive more income from Social Security than they pay in taxes. Individuals who pay more in taxes than they receive in benefits are harmed by the program. The CV is lowest for individuals earning wages right at the tax and benefit cap (2.47 times the mean wage). Individuals above the tax cap are harmed less since they pay a smaller portion of their total income in Social Security taxes. All of the welfare gains and losses in Economy II are due to the within-cohort redistribution of Social Security. This is visible in Figure 2 which plots the implied transfers of Social Security by wage type along with the compensating variation $\Delta(w)$. The welfare gain is only positive where the implied transfers are also positive. We calculate the implied transfers of Social Security as the expected benefits minus the expected taxes of a given wage-type. If we model a counter-factual economy with linear Social Security benefits (where there is no income redistribution), the CV is zero for all wage-types. Although Social Security provides longevity insurance, it does so by crowding out private savings. In net, individuals are no better or worse with a linear Social Security program in Economy II.

In Economy III where bequests are thrown in the ocean, Social Security is welfare enhancing for all wage-types for the baseline parameterization. This is because the crowding out of private savings and bequests does not reduce consumption. Rather, Social Security reduces the amount of resources that are wasted by redistributing resources from those who die early to those who live well into old age. The welfare gain is largest for the lowest wage earners due to the progressivity of Social Security benefits.

Figure 3 illustrates the welfare effects for a higher interest rate, $r = 0.029$. With this interest rate, the compensating variation $\Delta(w)$ is negative for all wage types in Economies I and II. The CV is most negative for the lowest wage earners in the Perfect Mobility Economy with uniform bequests (Economy I). Bequests are larger in this parameterization with a positive interest rate and so the crowding out of bequests is even more harmful to individuals than when $r = 0$. The introduction of Social Security reduces bequests from 0.036 to 0.019 ($9,134 to $4,825) which reduces the share of the population for whom bequests are larger than wage income from 11.9% to 6.1%. In the No Mobility Economy with wage-specific dynastic bequests (Economy II), Social Security is likewise welfare reducing for all wage types and most harmful for wage earners in the middle of the distribution (below the tax cap). In Economy III (where bequests are thrown in the ocean), the CV is only positive for very low wage types. Although the crowding out of bequests does not harm households, mandatory saving through Social Security is harmful for middle and high wage types since Social Security pays a lower rate of return than private savings in this parameterization.

## 4.3 Linear Benefit Earning Rule: Disentangling Longevity Insurance from Lifetime Income Insurance

In the US, Social Security benefits are calculated according to a progressive (concave) benefit earning rule. This redistributes lifetime income from high wage earners to low wage earners and provides partial ex-ante insurance against having a low wage. In this section, we consider an alternative non-redistributive Social Security benefit earning rule. In this counter-factual example, Social Security benefits replace a constant fraction $\phi$ of wages up to the tax and benefit cap. That is, $b(w) = \phi w$ for $w < w_c$ and $b(w) = \phi w_c$ for $w \geq w_c$. We solve for the replacement rate $\phi = 0.39$ that balances the Social Security budget.

This counter-factual Social Security system only provides longevity insurance; it no longer provides wage-type insurance. The value of this longevity insurance varies across our three economies. The ex-ante compensating variation $\Delta$ for all three economies is presented in Table 2. We also calculate the wage-specific compensating variation $\Delta(w)$ that shows the percentage of lifetime consumption that an individual of wage type $w$ is willing to pay for Social Security. This is presented in Figure 4 for

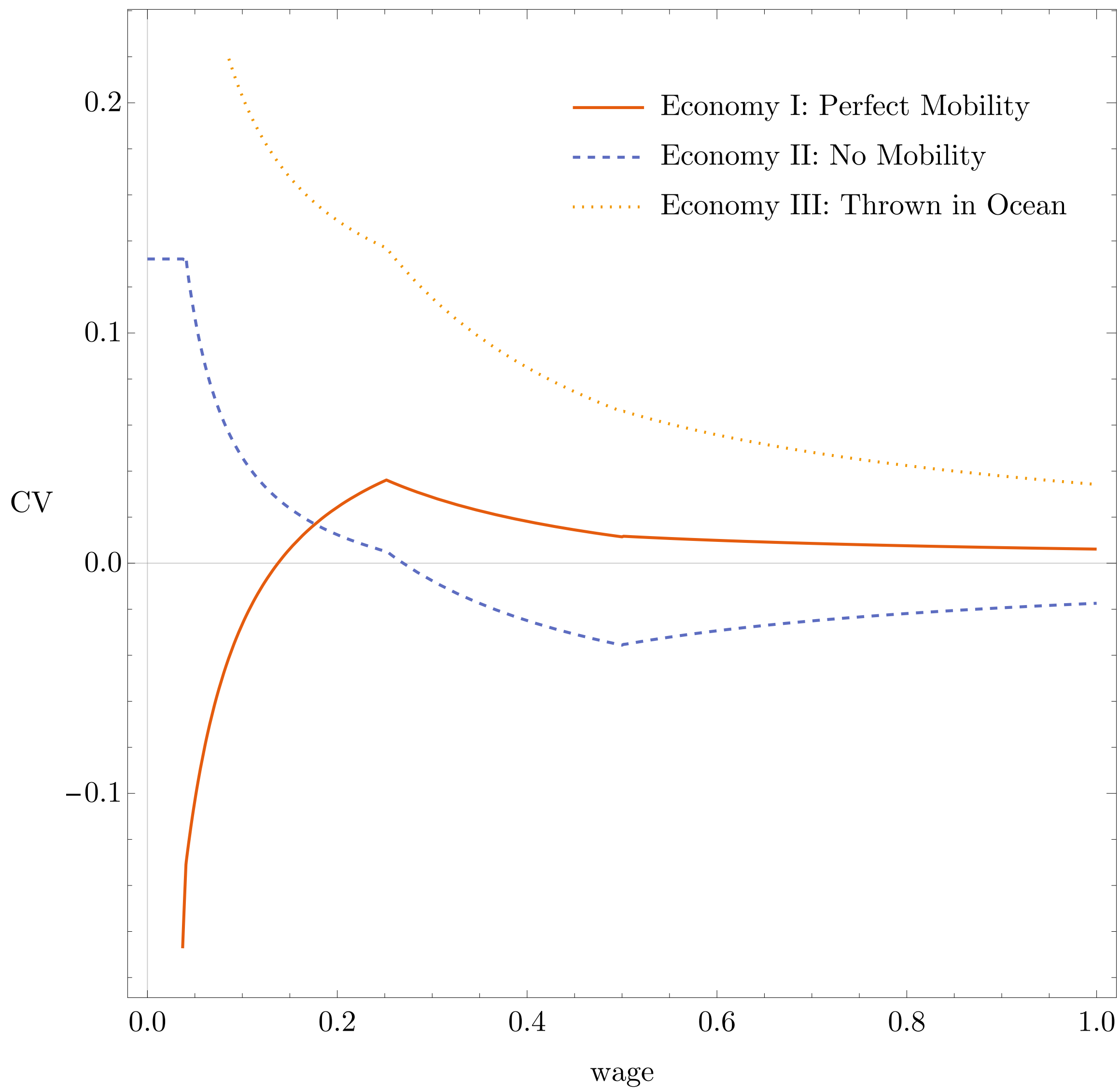


Figure 1: Compensating variation $\Delta(w)$ for $r = 0$, $\rho = 0$.

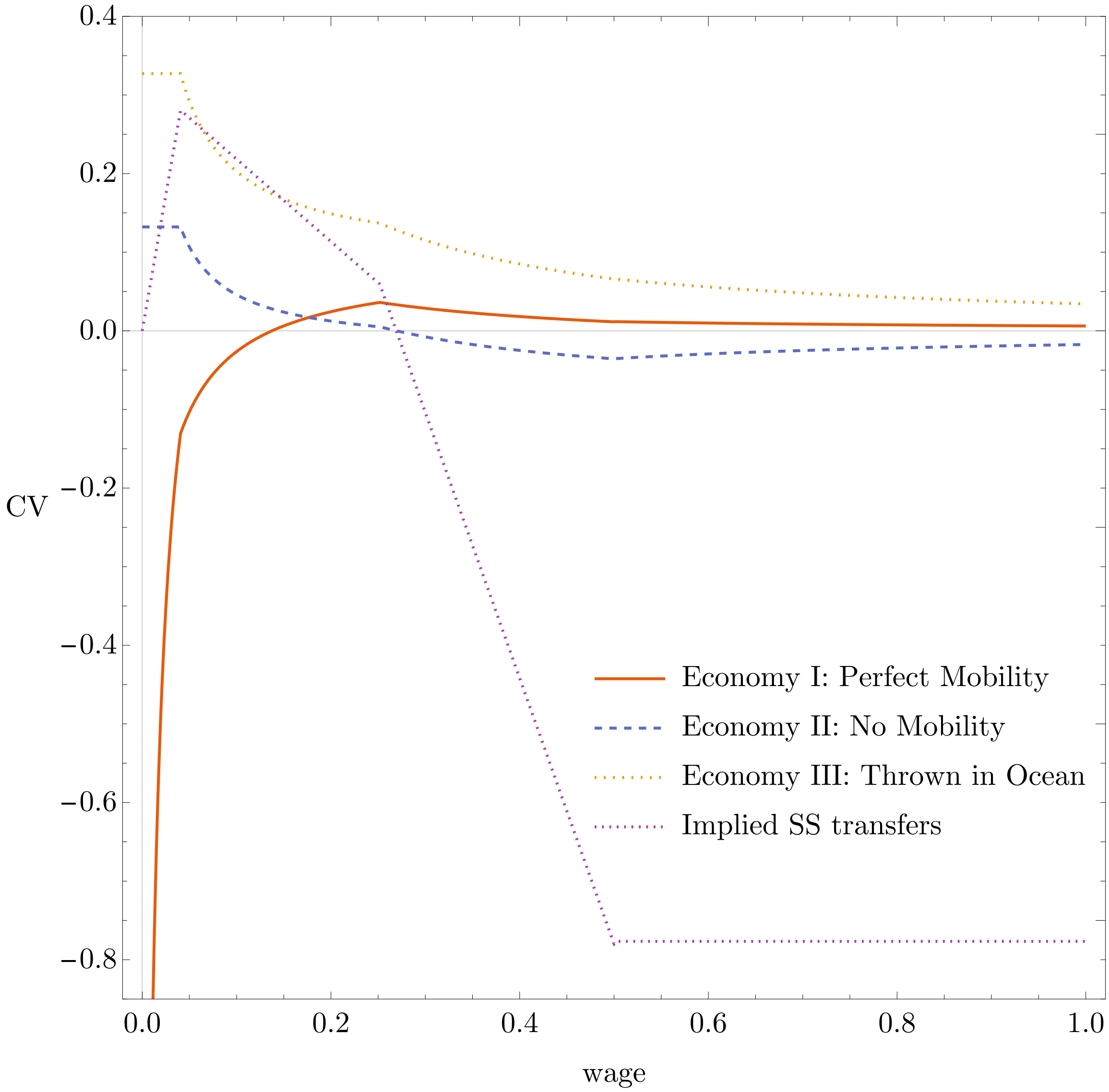


Figure 2: Implied Social Security transfers and compensating variation $\Delta(w)$ for $r = 0$, $\rho = 0$.

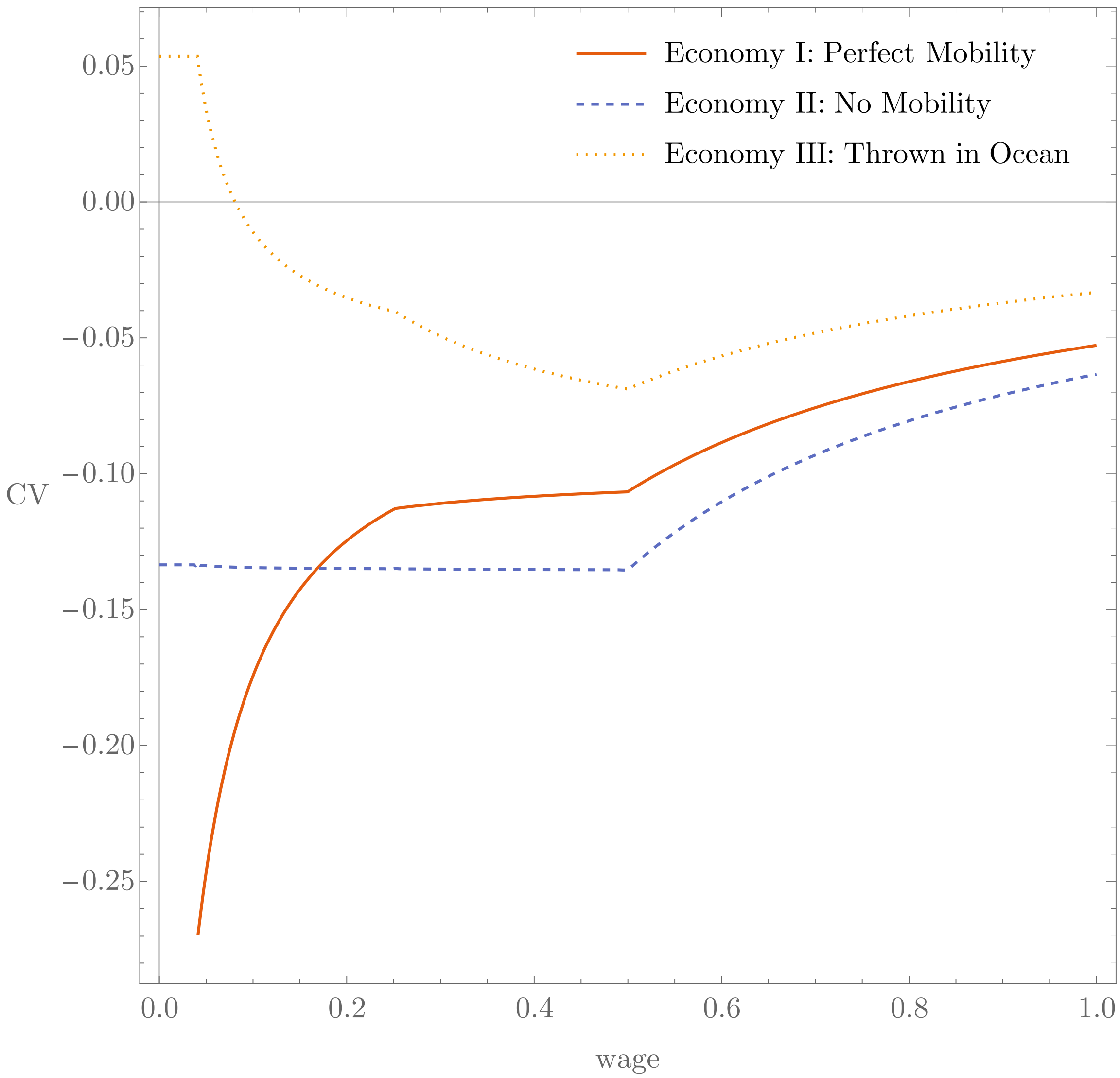


Figure 3: Compensating variation $\Delta(w)$ for $r = 0.029$, $\rho = 0$.

all three economies.

In the Perfect Mobility Economy with uniform bequests (Economy I), the ex-ante compensating variation $\Delta$ is -0.89 compared to the baseline $\Delta$ of -0.69 with progressive benefits. From an ex-ante perspective, an individual would be willing to give up 89% of her consumption to avoid Social Security in this economy. Social Security provides longevity insurance, but at the expense of crowding out private savings and accidental bequests. From an ex-ante perspective, the individual is much worse off with Social Security and lower bequests. This effect is driven by the welfare cost to low wage workers. The wage-specific compensating $\Delta(w)$ is negative for all wages below the mean ($w = 0.2031$), and positive for all wages above.

In the No Mobility Economy with dynastic bequests (Economy II), the ex-ante compensating variation $\Delta$ is zero, compared to 0.17 in the baseline redistributive system. With linear Social Security benefits, the welfare benefit of longevity insurance provided through Social Security is exactly offset by the reduction in wage-specific accidental bequests as in Caliendo et al. (2014). The wage-specific compensating $\Delta(w)$ is also zero for all $w$.

In Economy III where bequests are thrown in the ocean, the ex-ante compensating variation $\Delta$ is 0.13 compared to 0.33 in the baseline (progressive) Social Security system. The wage-specific compensating variation $\Delta(w)$ is always positive. It is constant for $w$ below the Social Security tax and benefit cap, and then declines for wages above the cap.

## 4.4 Adding a Progressive Income Tax

Our analysis thus far has assumed that the only avenue for the government to redistribute wealth is through a progressive a pay-as-you-go Social Security system. Of course, in reality, the government redistributes through a variety of tax and expenditure programs, and we expect that a progressive Social Security system is potentially less valuable in a world with other redistribution programs. In this section, we add a progressive income tax calibrated to US income taxes and a lump-sum transfer payment to proxy for the additional redistributive programs of the government.

We intentionally make the income tax and transfer program perfectly efficient and perfectly redistributive in the following sense. It is efficient because there are no administrative costs or other wasteful aspects to running the program. It is redistributive because individuals of all income levels enjoy the same cash transfer from the government. These specific assumptions allow us to create the greatest possible contrast with our baseline model in which there is no redistribution outside of Social Security.

The individual pays income tax on wage earnings $\tau_y(w)$ for all $t \in [0, t_R]$. The

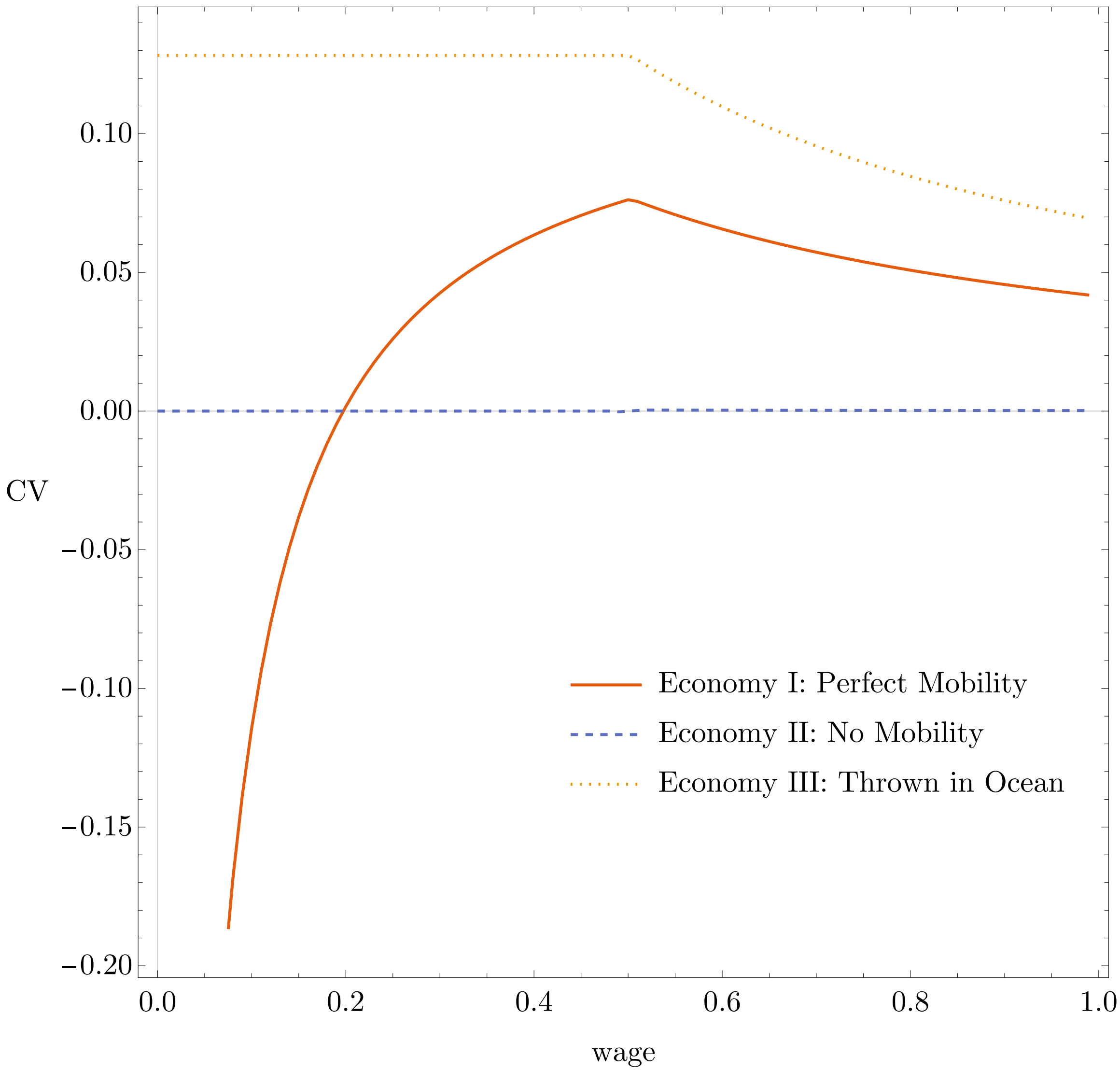


Figure 4: Compensating variation $\Delta(w)$ for $r = 0$, $\rho = 0$, with linear (non-redistributive) Social Security benefits.

government in turn provides a cash payment $G$ for all $w$ and all $t$. The government's non-Social Security budget must balance separately, hence:

$$G = \frac{\int_0^1 \int_0^{t_R} g(w)S(t)\tau_y(w)dtdw}{\int_0^T S(t)dt}.$$

The income tax function $\tau_y(w)$ is a piecewise linear function calibrated to match the 2018 federal income tax schedule for a single individual claiming the standard deduction.[9] Given this calibration, $G = 0.0197$, which corresponds to 9.7% of the mean wage.

We conduct the same welfare analysis as in the previous sections in all three model economies including the progressive income tax $\tau_y(w)$ and cash payment $G$. The far right column of Table 2 presents the ex-ante compensating variation calculations $\Delta$ for all three economies with progressive income taxes and cash payments assuming $r = \rho = 0$. Figure 5 present the corresponding wage-specific compensating variation $\Delta(w)$.

In the Perfect Mobility Economy with uniform bequests (Economy I), the ex-ante welfare cost of Social Security $\Delta$ is -0.21 compared to -0.69 in the baseline model without income taxes and transfers. Social Security still reduces ex-ante expected utility by crowding out accidental bequests; however, the harmful effect to low wage workers is mitigated by the progressive income tax and uniform cash transfer. The wage-specific compensating $\Delta(w)$ that shows the percentage of lifetime consumption that an individual of wage type $w$ is willing to pay for Social Security is qualitatively similar with a progressive income tax and in the baseline model (with Social Security only). In both cases, the wage-specific welfare effect is lowest for the lowest wage workers.

In the No Mobility Economy with dynastic bequests (Economy II), the ex-ante welfare effect of Social Security $\Delta$ is 0.05 in the model with progressive income taxes and cash transfers. This compares to our baseline model without income taxes where we found the welfare effect to be 0.17. The redistributive welfare gains from Social Security are dampened if the economy already includes other income redistribution. The lowest wage earners do not benefit as much from Social Security in a world with progressive income taxes because the Social Security benefit is proportional to wages and thus small compared to the cash transfer $G$. This is visible in Figure 5; the compensating variation approaches zero as the wage approaches zero. By modeling

[9] The marginal tax rates are 10%, 12%, 22%, 24%, and 32%. The standard deduction is \$12,000 (normalized to 0.04673 for our wage distribution). The thresholds at which the marginal tax rate increases are \$9,525, \$38,700, \$82,500, \$157,500, and \$200,000 (normalized to 0.037, 0.151, 0.321, 0.613, and 0.779 for our wage distribution). The sixth marginal income tax bin (37% for wages over \$500,000) is excluded from our analysis, since our maximum calibrated wage $w = 1$ corresponds to a wage income of \$256,800.

government spending $G$ as a cash transfer, we have stacked the model against Social Security. If government spending $G$ were modeled as an additively separable good instead, it would not compete directly with Social Security and the welfare gain associated with Social Security would be larger.

Finally, in Economy III where bequests are thrown in the ocean, the ex-ante welfare effect of Social Security $\Delta$ is 0.14 compared to 0.33 in the baseline model. The wage-specific compensating $\Delta(w)$ is still positive for all wage earners, but is smaller than in the model without income taxes. In the model with income taxes, the cash transfer $G$ represents a large portion of lifetime earnings for low wage individuals ($G > w$ for the bottom 6.5% of the wage distribution). These individuals benefit from Social Security, but the welfare effect is relatively small since Social Security benefits make up a small portion of their lifetime resources.

Adding a progressive income tax to the model dampens the welfare effects of Social Security, but does not change the qualitative interpretation. In the Perfect Mobility Economy, Social Security reduces welfare by crowding out private intergenerational wealth transfers. In the No Mobility Economy, Social Security improves welfare by paying progressive benefits and providing partial insurance against low lifetime earnings (in addition to the progressivity of the income tax and government transfer $G$). Similarly, in a partial equilibrium framework, Social Security improves ex-ante welfare by providing longevity insurance and lifetime earning insurance.

# 5 Extensions

In our baseline analysis we assume bequests are accidental and factor prices are fixed. These assumptions allow us to focus in a clean way on economic mobility. In this section we study the robustness of our baseline conclusions by relaxing these assumption in sequence.

## 5.1 Intentional Bequests

In our baseline model, we consider an equilibrium economy in which individuals both give and receive accidental bequests. In this section we extend the analysis to consider intentional bequests. To keep the model tractable, there are two generations. Generation 1 is the first generation ever to exist and Generation 2 is the last generation. Generation 1 starts with no assets but accumulates wealth over the life cycle, and it does so knowing that upon death these assets will be transmitted one-for-one to specific individuals (progeny) in Generation 2. Further, Generation 1 values this transfer.

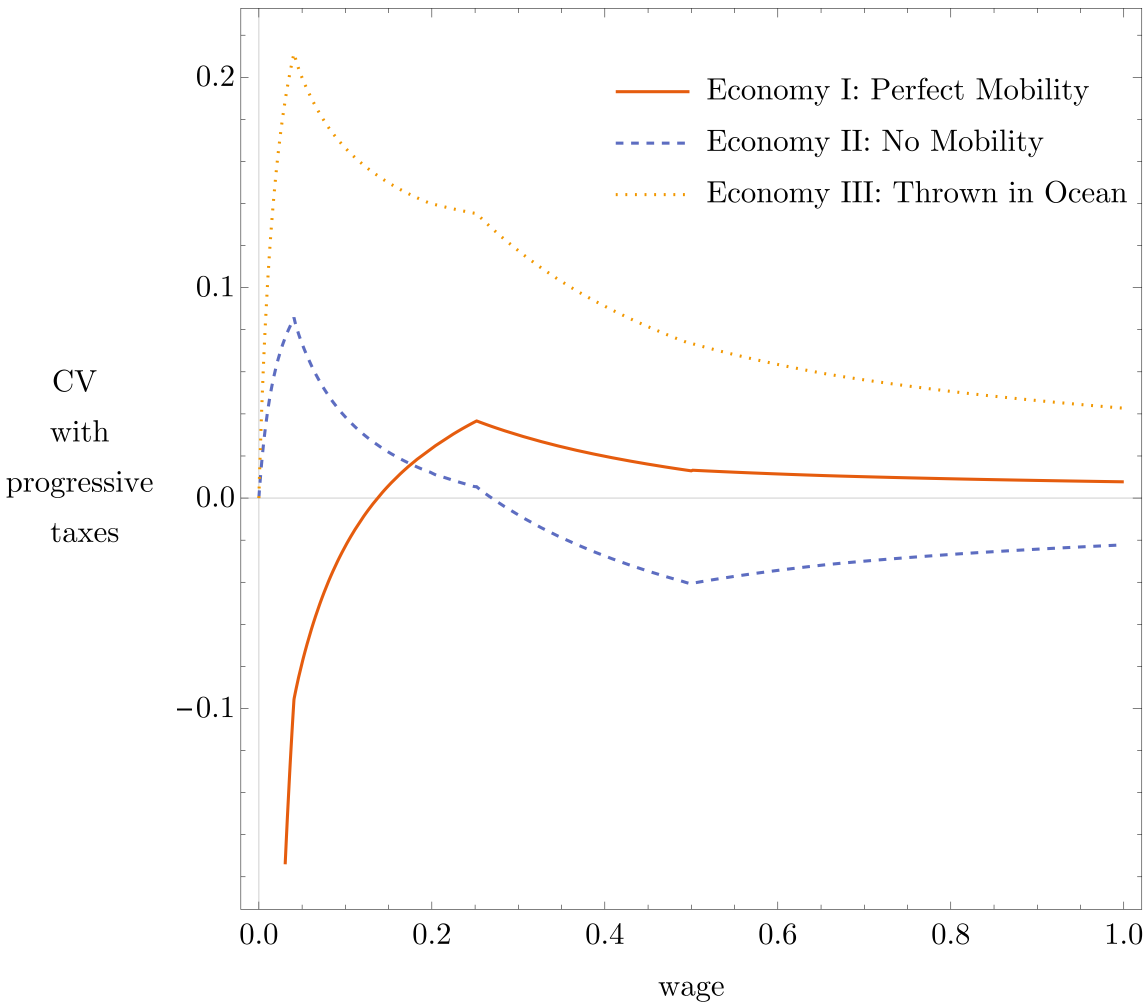


Figure 5: Compensating variation $\Delta(w)$ for $r = 0$, $\rho = 0$, in a world with progressive income taxes.

We assume the individual in Generation 1 is altruistic and recursively solves a dynamic stochastic problem whereby Generation 2's lifetime utility is the continuation value of Generation 1's asset holdings. The optimization problem is stochastic not only because both generations face longevity risk, but more specifically because the longevity risk of Generation 1 creates uncertainty about the level of bequest income that will be transmitted to Generation 2.[10]

We study welfare from the perspective of Generation 1 to understand how the presence of a bequest motive might change our conclusions about Social Security's welfare effect. Not only does Social Security insure Generation 1's own longevity risk and lifetime income risk, but it also provides the same insurance to the next generation. However, Social Security distorts asset holdings and in turn bequest inheritances, and Generation 1 cares about the inheritance of Generation 2. To keep the model tractable and to keep our focus on economic mobility, we assume Generation 2 receives inheritance income when they start working.[11] Both generations have CRRA utility,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

An individual in Generation 1 earns wages $w_1$. An individual in Generation 2 earns wages $w_2$, which depends on the nature of economic mobility.

We focus on the case of No Mobility to check to the sensitivity of Social Security's welfare gains, as this was the baseline assumption that produced welfare gains and we would like to know if these gains can survive in a model that has been augmented to include an intentional bequest motive. Hence, $w_1$ is randomly drawn from the probability density $g(w_1)$ with unit support $[0, 1]$, and $w_2 = w_1$.

We assume a pay-as-you-go Social Security system whose budget balances generation by generation in a longitudinal sense. That is, ex ante the Law of Large Numbers ensures that taxes collected from a given generation equals benefits collected by that same generation. Hence, this implies that we can simply reuse the baseline $\tau$ and

[10] Our baseline model follows the convention of a "timeless equilibrium" concept without a beginning or end of time. While this is a convenient modeling device, the timeless feature of the equilibrium makes the model intractable if one wants to go a step further and model specific bequest linkages between parents and children. The intractability arises because the size of the bequest from a given parent to child would depend on the entire history of bequests in that parent's ancestral line. Nevertheless, we find it potentially useful to build a model with specific parent-child linkages when studying intentional bequests, because in doing so we are able to connect the the bequest utility of Generation 1 to the utility that Generation 2 receives from the bequest through a tractable, dynamic-stochastic recursive problem.

[11] Otherwise the timing and magnitude of inheritance income received by Generation 2 are stochastic, and this is a rather complex problem that is studied elsewhere in the literature (Cottle Hunt and Caliendo (2021)). Here we wish to side step this complexity to focus our attention on the mobility issue.

baseline $b(w)$ rule from Table 1, because those values can be thought to satisfy an instantaneous aggregate budget constraint or can be interpreted in a longitudinal sense as we are doing here.

The individual in Generation 1 with wage $w$ solves a recursive problem in which the utility of the next generation is embedded into Generation 1's own utility. Generation 1 knows that if it dies with assets $B(t)$ based on death at age $t$ and leaves these assets to Generation 2, then the latter will solve the following optimization problem

$$\max_{c(z),k(z)} : U_2 = \int_0^T S(z)u(c(z))dz,$$

subject to

$$\dot{k}(z) = rk(z) + y(z) - c(z),$$

$$k(0) = B(t),$$

$$k(T) = 0,$$

$$y(z) = \begin{cases} w - \tau \min(w, w_c), \text{ for } z \in [0, t_R], \\ b(w), \text{ for } z \in [t_R, T]. \end{cases}$$

The solution is

$$c_2^*(z) = \frac{B(t) + Y}{A} e^{rz/\sigma} S(z)^{1/\sigma},$$

where

$$Y \equiv \int_0^T e^{-rv} y(v)dv, \text{ and } A \equiv \int_0^T e^{-rv+rv/\sigma} S(v)^{1/\sigma} dv,$$

and solution utility is

$$U_2^*(B(t)) = \int_0^T S(z)u(c_2^*(z))dz.$$

Now, working backwards, with $\alpha$ measuring the strength of altruism, Generation 1 optimizes according to

$$\max_{c(t),B(t)} : U_1 = \int_0^T \Big( S(t)u(c(t)) + (-\dot{S}(t))\alpha U_2^*(B(t)) \Big) dt,$$

subject to

$$\dot{B}(t) = rB(t) + y(t) - c(t),$$

$$y(t) = \begin{cases} w - \tau \min(w, w_c), \text{ for } t \in [0, t_R], \\ b(w), \text{ for } t \in [t_R, T], \end{cases}$$

$$B(0) = 0,$$

$$B(T) = \text{free}.$$

The Hamiltonian is

$$\mathcal{H} = S(t)u(c(t)) + (-\dot{S}(t))\alpha U_2^*(B(t)) + \lambda(t)[rB(t) + y(t) - c(t)]$$

with first-order conditions

$$\frac{\partial \mathcal{H}}{\partial c(t)} = S(t)c(t)^{-\sigma} - \lambda(t) = 0$$

$$\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial B(t)} = \dot{S}(t)\alpha \frac{\partial U_2^*(B(t))}{\partial B(t)} - \lambda(t)r$$

$$\lambda(T) = 0.$$

Differentiate the Maximum Condition with respect to $t$

$$\dot{S}(t)c(t)^{-\sigma} - \sigma S(t)c(t)^{-\sigma-1}\dot{c}(t) - \dot{\lambda}(t) = 0$$

and combine with the Multiplier Equation

$$\dot{S}(t)c(t)^{-\sigma} - \sigma S(t)c(t)^{-\sigma-1}\dot{c}(t) = \dot{S}(t)\alpha \frac{\partial U_2^*(B(t))}{\partial B(t)} - S(t)c(t)^{-\sigma}r.$$

Rearrange terms to obtain the Euler equation

$$\dot{c}(t) = \frac{1}{\sigma}\left(r + \frac{\dot{S}(t)}{S(t)}\right)c(t) - \frac{1}{\sigma}\left(\frac{\dot{S}(t)}{S(t)}\right)\alpha \frac{\partial U_2^*(B(t))}{\partial B(t)} c(t)^{\sigma+1}.$$

Finally, noting that

$$\begin{aligned} U_2^*(B(t)) &= \frac{1}{1-\sigma}\int_0^T S(z)\left(\frac{B(t)+Y}{A}e^{rz/\sigma}S(z)^{1/\sigma}\right)^{1-\sigma} dz \\ \frac{\partial U_2^*(B(t))}{\partial B(t)} &= \left(\frac{B(t)+Y}{A}\right)^{-\sigma}, \end{aligned}$$

we can write the final form Euler equation as

$$\dot{c}(t) = \frac{1}{\sigma}\left(r + \frac{\dot{S}(t)}{S(t)}\right)c(t) - \frac{1}{\sigma}\left(\frac{\dot{S}(t)}{S(t)}\right)\alpha\left(\frac{B(t)+Y}{A}\right)^{-\sigma} c(t)^{\sigma+1}.$$

The Transverality Condition provides the needed endpoint to use this Euler equation to simulate consumption. Note the Maximum Condition has the indeterminate form at $t = T$,

$$c(T)^{-\sigma} = \frac{\lambda(T)}{S(T)} = \frac{0}{0},$$

hence,

$$\begin{aligned}
\lim_{t\to T} c(t)^{-\sigma} &= \lim_{t\to T} \frac{\dot{\lambda}(t)}{\dot{S}(t)} \\
&= \lim_{t\to T} \frac{\dot{S}(t)\alpha\frac{\partial U_2^*(B(t))}{\partial B(t)} - S(t)c(t)^{-\sigma}r}{\dot{S}(t)} \\
&= \lim_{t\to T} \alpha\frac{\partial U_2^*(B(t))}{\partial B(t)} - \lim_{t\to T}\frac{S(t)}{\dot{S}(t)}c(t)^{-\sigma}r \\
&= \lim_{t\to T} \alpha\frac{\partial U_2^*(B(t))}{\partial B(t)} \\
&= \alpha\left(\frac{B(T)+Y}{A}\right)^{-\sigma}.
\end{aligned}$$

Hence, terminal consumption satisfies

$$c(T)^{-\sigma} = \alpha\left(\frac{B(T)+Y}{A}\right)^{-\sigma}.$$

We denote the solution consumption and saving paths for Generation 1 as $(c_1^*(t), B_1^*(t))$.

The ex ante welfare of an individual in Generation 1, before realizing their wage type $w$, is

$$\mathbb{E}(U_1) = \int_0^1 g(w)\left(\int_0^T \left(S(t)u(c_1^*(t)) + (-\dot{S}(t))\alpha U_2^*(B_1^*(t))\right)dt\right)dw.$$

**Method for Computing $E(U_1)$:**

Step 1: For a given $w$, guess initial consumption $c(0)$.

Step 2: Using this guess, simulate the timepath $(c(t), B(t))$ using the following system of equations

$$\begin{gathered}
\dot{c}(t) = \frac{1}{\sigma}\left(r + \frac{\dot{S}(t)}{S(t)}\right)c(t) - \frac{1}{\sigma}\left(\frac{\dot{S}(t)}{S(t)}\right)\alpha\left(\frac{B(t)+Y}{A}\right)^{-\sigma}c(t)^{\sigma+1}. \\
\dot{B}(t) = rB(t) + y(t) - c(t), \\
B(0) = 0.
\end{gathered}$$

Step 3: Check to see if the following terminal condition is satisfied

$$c(T)^{-\sigma} = \alpha\left(\frac{B(T)+Y}{A}\right)^{-\sigma}.$$

Step 4: If yes, then the timepath $(c_1^*(t), B_1^*(t))$ for an individual in Generation 1 with wages $w$ has been identified. In not, then go back to Step 1 and repeat.

Step 5: Repeat Steps 1-4 for each value of $w \in [0, 1]$ and store the associated solution timepaths $(c_1^*(t), B_1^*(t))$ for each $w$.

Step 6: Compute $\mathbb{E}(U_1)$ using the solution timepaths $(c_1^*(t), B_1^*(t))$ for each $w$.

We compare $\mathbb{E}(U_1)$ with and without Social Security, for various values of $\alpha$ (strength of the bequest motive). We plot the percentage difference in expected utility in Figure 6. A positive number indicates expected utility is higher with Social Security than without. We use our baseline parameterization in Table 1, and we set the interest rate to 2.9% rather than 0%. If the return on saving $r$ is low enough and/or $\alpha$ is low enough, the individual in Generation 1 is discouraged from saving and instead endows the individual in Generation 2 with debt. As we have not constrained assets to be positive in the optimization problem itself, we wish to avoid such model outcomes by selecting $r$ and $\alpha$ so that the asset holdings of Generation 1 are positive. Also, we assume altruism is "perfect" if $\alpha = 1$, and hence we do not consider values higher than this. When $\alpha = 1$, Generation 1 cares as much about the welfare of Generation 2 as Generation 2 cares about themselves.

When altruism is perfect, Social Security does not improve the welfare of Generation 1. There are competing forces at work behind this result. On the one hand, Social Security insures individuals (and their children) against lifetime earnings risk—low earners enjoy a higher benefit replacement rate. Also, Social Security pays benefits as a life annuity. On the other hand, Social Security crowds out private saving and this is costly for the usual reason that individuals are harmed when the return to forced saving is low relative to the return to private saving. But the crowding out of private saving is now additionally costly because Generation 1 attaches bequest valuation to their own asset holdings. Overall, when $\alpha = 1$ Social Security is not welfare improving in this model, but it can be welfare improving when $\alpha < 1$ (i.e., when altruism is less than perfect). We find Social Security is welfare improving for $\alpha = 0.5$, for example. Because the ultimate welfare effect of Social Security depends on the strength of intergenerational altruism—which is a utility-side, unobservable parameter—we have to be careful to note that without deeper empirical evidence on this parameter, the potential welfare gains of Social Security can be positive or negative.[12]

Our results fit into the larger literature on bequest motives (see for example, Modigliani (1988), Hurd (1990), Bernheim (1991), Laitner (2002), or Horioka (2014) for a discussion and empirical evidence). The recursive way in which we modeled

[12]There is some empirical support for less than perfect altruism. Laitner (2001) models altruism in a similar way to this section of our paper. In Laitner's model, parents place a weight of $\xi$ on their children's utility (and $\xi^2$ on their grandchildren's utility and so on recursively). He calibrates the model and finds $\xi = 0.83$ best matches US data.

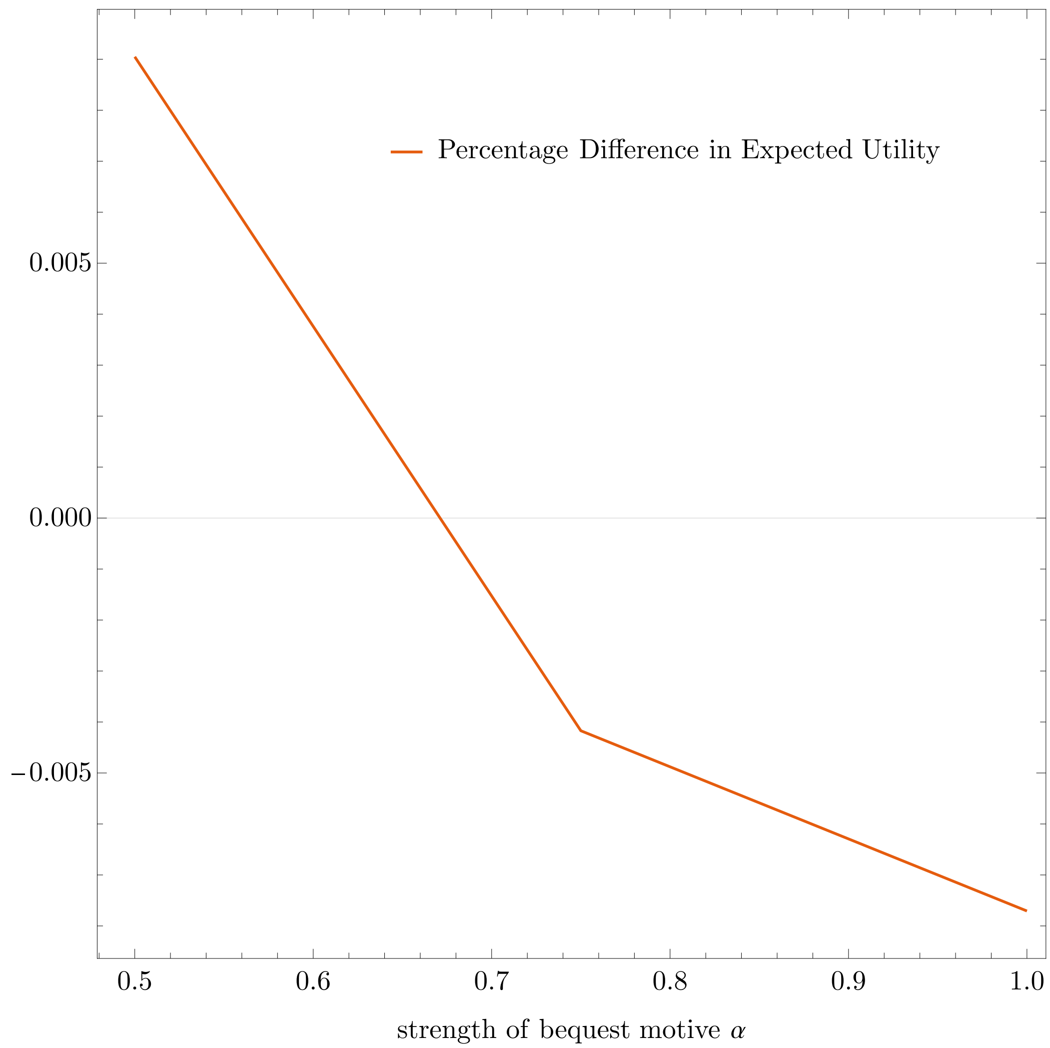


Figure 6: Percentage difference in expected utility of Generation 1 for different values of $\alpha$, with and without Social Security.

an altruistic bequest motive is similar in spirit to models such as Tomes (1981), or Laitner (2001), among many others.[13] We model survival uncertainty for both the parent and child, which means our bequests combine an intentional motive and an accidental bequest. We add to the literature by considering Social Security's welfare role in insuring both longevity risk and life-time earnings risk. Our welfare analysis captures the competing forces of an unfunded pension on the ex-ante wellbeing of a household that faces a risk of a low wage, longevity risk, and also cares about the consumption of the next generation, all in the context of an economy that lacks economic mobility across generations.

## 5.2 Production Economy

Factor prices are fixed in our baseline analysis. This is a convenient way to isolate the mechanisms of interest. However, because Social Security is a large program that directly affects the incentive to save for retirement, which in turn affects aggregate capital accumulation, the program's ultimate effect on the economy and welfare will depend on how it affects total output and factor prices. In this section we extend our model from an endowment economy to a production economy with endogenous factor prices.

In our production economy, individuals are endowed with innate labor productivity $l$, which is distributed according to $g(l)$ on the unit support $[0, 1]$. The wage rate per unit of productivity is $w$. Therefore, a given individual's labor earnings at a moment in time are $wl$. As in our baseline analysis, we place a cap on taxable earnings, but here we denote the cap in terms of labor productivity, $l_c$. That is, individuals with productivity below the cap ($l < l_c$) pay Social Security taxes $\tau wl$, while individuals with productivity at or above the cap ($l \geq l_c$) pay taxes $\tau w l_c$.

The Social Security benefit earning rule is $b(wl)$. The individual receives bequest income $B(l)$. In Economy I with Perfect Mobility $B(l) = B$ for all productivity types. In Economy II, the case of No Mobility, the individual receives bequest income from an ancestral line that shares the same labor productivity.

Based on drawing labor productivity $l$, the individual chooses $(c(t), k(t))_{t\in[0,T]}$ to

[13]The literature has suggested several other motivations for bequests aside from altruism. Bequest can be the result of survival risk and incomplete insurance markets, as in our baseline model and in Hurd (1987, 1989), or Gokhale et al. (2001). Bequests can result from self-interested exchange with one's heirs (Bernheim et al. (1985)), as inter-generational risk-sharing (Kotlikoff and Spivak (1981)), or parents might experience a "joy of giving" or "warm glow" (Altig et al. (2001) or De Nardi (2004)).

maximize lifetime utility, which gives solution paths for $c(t)$ and $k(t)$

$$c(t) = S(t)^{\frac{1}{\sigma}} e^{\frac{(r-\rho)t}{\sigma}} \left( \frac{\int_0^{t_R} e^{-rt}(wl - \tau \min(wl, wl_c) + B(l))dt + \int_{t_R}^T e^{-rt}(b(wl) + B(l))dt}{\int_0^T e^{(\frac{(r-\rho)}{\sigma} - r)t} S(t)^{\frac{1}{\sigma}} dt} \right),$$

$$k(t) = \int_0^t e^{r(t-v)}(wl - \tau \min(wl, wl_c) + B(l) - c(v))dv, \text{ for } t \leq t_R,$$

$$k(t) = e^{r(t-t_R)} k(t_R) + \int_{t_R}^t e^{r(t-v)}(b(wl) + B(l) - c(v))dv, \text{ for } t > t_R.$$

At the macro level, aggregate capital $K$ at a moment in time is the total asset holdings summed across age and productivity type

$$K = \int_0^1 \int_0^T g(l) S(t) k(t|l) dt dl,$$

and aggregate labor $L$ is likewise defined as

$$L = \int_0^1 \int_0^{t_R} g(l) S(t) l dt dl.$$

Total production takes the Cobb Douglas form

$$Y = K^\alpha L^{1-\alpha}$$

where $\alpha$ is capital's share. Factors are priced according to their marginal products

$$r = \alpha \frac{Y}{K} - \delta$$

$$w = (1-\alpha) \frac{Y}{L},$$

where $\delta$ is the depreciation rate. Finally, in Economy I, the Perfect Mobility Economy, bequest income $B$ satisfies the aggregate bequest constraint

$$\int_0^T S(t) B dt = \int_0^T \left( -\dot{S}(t) \right) k(t|l, B) dt.$$

In Economy II, the No Mobility Economy, bequest income $B(l)$ for each productivity type satisfies

$$\int_0^T S(t) B(l) dt = \int_0^T \left( -\dot{S}(t) \right) k(t|l, B(l)) dt \text{ for all } l \in [0, 1]$$

and the Social Security budget is in balance

$$\tau = \frac{\int_0^1 \int_{t_R}^T g(l)S(t)b(wl)dtdl}{\int_0^{l_c} \int_0^{t_R} g(l)S(t)wldtdl + \left(\int_{l_c}^1 g(l)dl\right) \int_0^{t_R} S(t)wl_c dt}.$$

A **Stationary Equilibrium** is comprised of household allocations $(c(t|l), k(t|l))_{t\in[0,T]}$ for all $l$ that solve each individual's full information optimization problem given a productivity density $g(l)$ on $[0, 1]$, survival probabilities $S(t)$ on $[0, T]$, factor prices $r$ and $w$, a distribution of individual asset holdings and labor that satisfy aggregate resource constraints, a distribution of bequest income $B$ or $B(l)$ that satisfies the bequest constraint (or a continuum of resource constraints for each productivity type), and Social Security tax rate $\tau$, taxable wage cap $wl_c$, and benefit earning rule $b(wl)$ that jointly balance the Social Security budget.

To calibrate the model we closely follow our baseline parameterization in Table 1 where possible. The distribution $g$ follows the same beta distribution as in Table 1

$$g(l) = \frac{l^{\gamma-1}(1-l)^{\beta-1}}{\int_0^1 l^{\gamma-1}(1-l)^{\beta-1}dl}$$

with $\gamma$ and $\beta$ selected as before. With $l_c = 0.5$, this parameterization ensures the mean and the tax cap are in the appropriate relationship relative to one another, and that the share of individuals above the tax cap is also empirically accurate. The Social Security benefit earning rule $b(wl)$ continues to follow the US rule, and the Social Security tax rate is given by the balanced budget condition. We set capital's share to $\alpha = 0.35$ and the depreciation rate to $\delta = 8\%$ following convention in the general equilibrium literature.

**Method for Computing the Stationary Equilibrium**

Step 1: Guess a value for aggregate capital $K$.

Step 2: From this guess, compute factor prices $r$ and $w$.

Step 3: Using the wage rate from the previous step, compute the Social Security benefit and the tax rate $\tau$ that balances the budget.

Step 4: Compute bequest income $B$ for Economy I that satisfies the aggregate bequest condition. For Economy II compute bequest income $B(l)$ for all productivity types as the solution to a continuum of aggregate bequest conditions, one for each type.

Step 5: Compute the optimal consumption and saving profiles for all productivity types, $(c(t|l), k(t|l))_{t\in[0,T]}$, given $r$, $w$, $b(wl)$, $\tau$, $B$ or $B(l)$ from previous steps.

Step 6: Compute aggregate capital $K$ as the sum of the asset holdings of living individuals of all ages and productivity types.

Step 7: If the calculated $K$ from Step 6 matches the guess from Step 1, then stop. Otherwise go back to Step 1 and repeat.

We conduct the same welfare analysis as in the baseline model to measure the effectiveness of Social Security in providing insurance against longevity risk and earnings risk (i.e., productivity type risk). Specifically we calculate a compensating variation that shows the percentage of lifetime consumption that an individual is willing to pay to live in a world with Social Security, before their productivity-type is revealed. The addition of general equilibrium factor prices is quantitatively important, but does not change the underlying insurance mechanisms.

In Economy I with Perfect Mobility Social Security continues to be welfare reducing, in part because it crowds out uniform bequests which are redistributive. When $\rho = 0$, we find the compensation variation to be -0.31, and when $\rho = 0.029$ we find it to be -0.43.

In Economy II, our results are still sensitive to the discount rate $\rho$ and the interest rate. With the discount rate set to $\rho = 0$, we find the compensating variation is 0.23. Under this parameterization, the endogenous interest rate in the economy is relatively low and the benefit of longevity risk and earnings risk insurance are enough to offset the cost of crowding out private savings. However, when we consider a higher discount rate $\rho = 0.029$, the interest rate is higher which leads to a compensating variation of -0.13.[14] Although quantitatively different, this pattern of welfare results follow our findings from Table 2 where factor prices were fixed. Overall, this confirms the main conclusion that the welfare gains from Social Security are higher when the economy lacks economic mobility.

# 6 Conclusion

Social Security is designed, at least in part, to pool risk and provide wage-type and longevity insurance. We evaluate the effectiveness of the program in providing social insurance along those two margins. We show that the ability of the program to provide this insurance depends critically on the nature of intergenerational economic mobility. We model three economies to illustrate this point.

In the Perfect Mobility Economy (Economy I), accidental bequests are distributed evenly to all survivors. Individuals of all wage-types and ages receive the same trans-

[14]When $\rho = 0$, the interest rate is $r = -0.016$ without Social Security and $r = 0.016$ with Social Security. When $\rho = 0.029$ the interest rate is $r = 0.01$ without Social Security and 0.044 with Social Security.

fer. In this setting, Social Security is welfare reducing, because it crowds out private savings and reduces bequests. Although the program provided longevity insurance and pays progressive benefits, individuals are ex-ante better off uninsured.

In the No Mobility Economy (Economy II), accidental bequests are distributed within wage type. Economic mobility is limited in this setting. Social security is welfare enhancing in this economy by providing partial wage-type insurance. The welfare gain is due entirely to the progressivity of Social Security benefits. The welfare gains of annuitization are completely off-set by the reduction in private bequests.

Finally, in Economy III, accidental bequests are thrown in the ocean. In this partial equilibrium setting, Social Security is welfare enhancing both because it annuities some of the individuals wealth and also because it redistributes income.

A growing body of evidence suggests economic mobility is limited in the US, at least for some. In this setting, the redistribution provided through Social Security improves ex-ante wellbeing. This improvement comes through collective risk sharing along the lifetime earning dimension. This is an important insight for policymakers to consider when evaluating the future of Social Security.

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