Social Security and Longevity Risk: An Analysis of Couples

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December 10, 2021

Abstract

To help manage longevity risk, Social Security pays three types of benefits to retirees: retirement benefits as a life annuity, spousal benefits until the death of the primary earner, and survivor benefits after the death of primary earner. How effective is Social Security at insuring couples against the joint longevity risks that they face? We take a public finance approach to this important question by comparing a Laissez Faire economy to a variety of different public insurance structures including the First Best, the Second Best, and US Social Security. We find that the welfare gains to couples from participating in the US Social Security system are large, and the survivor benefit feature is the key to this result while the spousal benefit provides very little efficiency gains. In fact, the optimal mixture of spousal and survivor benefits (i.e., the Second Best) improves ex ante efficiency by providing a larger payment to widows and a smaller spousal benefit than the current US system. We obtain these results using a theoretical model of couples who have full information about gender-specific longevity risks and solve a dynamic stochastic (regime-switching) problem to optimally hedge these risks.

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1 Introduction

Social Security pays benefits to retirees as a life annuity, and in the absence of competitive annuity markets, this feature is commonly understood to provide welfare gains by allowing individuals to pool their longevity risk. Such risk sharing is referred to as Social Security's longevity insurance role.

When evaluating Social Security's longevity insurance role, the typical starting point is a life-cycle model of single individuals. Of course, modeling singles rather than couples keeps the analysis clean and tractable, and many important economic lessons can be learned at this level of abstraction. However, the US Social Security system has two important features relating to couples: spousal benefits and survivor benefits. First, if both individuals are alive, then a spouse can collect benefits equal to the larger of their own benefits or half of their spouse's benefits. For example, if a husband's average earnings exceeds that of the wife's, then he collects benefits based on his own earning history while she collects benefits equal to the larger of her own benefits based on her earning history or half of her husband's benefits. Second, Social Security provides benefits to the surviving spouse. For example, if a husband passes away first, then the widow will collect the larger of his benefit or her benefit for the remainder of her life.

In this paper we evaluate Social Security's longevity insurance role from the perspective of a couple. We develop a life-cycle consumption/saving model of a husband and wife, and we solve the couple's optimization problem recursively. We first consider the contingent problems of the widow and widower upon the death of their spouse, and we solve these problems to obtain the continuation value of asset holdings at each possible date of death of the spouse. Moving back to the beginning of the life cycle, we then embed these continuation values into the initial problem of a couple who must form a consumption and saving plan that optimally hedges the joint longevity risks that they face. We work in continuous time and we derive an analytical solution to this dynamic stochastic control problem.[1]

[1] We take into consideration gender-specific longevity risk, since women in the US have a longer life expectancy than men. As such, we focus our analysis on a male-female couple. The modeling framework we develop can also accommodate male-male

We take a public finance approach to the problem by characterizing and comparing the couple's consumption, saving, and welfare across a variety of scenarios, including a Laissez Faire economy with no government, an economy with ex ante efficient (First Best) insurance, an economy with US Social Security, and an economy with various permutations to US Social Security including removing some of its features as well as reconfiguring the program to reach the Second Best welfare level.

Our baseline analysis focuses on a married, single-earner household consisting of a working male with a shorter life expectancy than the female spouse who does not participate in the labor force. Of course, the prevalence of dual-earner households in America has changed dramatically over the last half century, rising from a small minority to a significant majority (Fisher and Johnson (2019)), and we also consider a dual-earner couple with wage parity between husband and wife as an alternative bookend assumption. Survivor benefits and spousal benefits are redundant features for dual-earner couples who earn the same wage. That is, a dual-earner couple would not use the spousal or survivor benefit if both spouses earn the same wage—instead both spouses would claim identical retirement benefits based on their identical earnings history. In contrast, these marriage-related features of Social Security are especially relevant (and would always be claimed) for single-earner couples and so we select this particular family structure for our baseline analysis.

We evaluate the effectiveness of Social Security in providing longevity insurance in a setting that is designed to allow Social Security the greatest possible chance to improve welfare. The couple in our model is unable to insure their longevity risk (no annuity markets) and does not adjust their joint labor supply. Their only avenue of self-insurance is through precautionary savings. These assumptions create room for a public, Second Best option to improve ex ante welfare.

Yet, even in this setting, we find that the spousal benefit is not very useful to single-earner couples. This is an important result because it is an expensive feature to finance. In contrast, eliminating the

or female-female couples.

survivor benefit makes single-earner couples much worse off. In fact, the couple would be better off fending for themselves in Laissez Faire without Social Security than participating in a program with only retirement and spousal benefits and no survivor benefits. If the goal is to help single-earner couples insure their joint longevity risk, then the current Social Security system could improve the ex ante wellbeing of single-earner couples even more if it were revised to pay larger survivor benefits and smaller spousal benefits, holding taxes and retirement benefits fixed. To the extent that policy makers wish to better protect widows who did not work, increasing the survivor benefit by shrinking the spousal benefit is a budget-neutral way to achieve that goal.[2] Finally, while dual-earner couples do benefit from the annuitization feature of Social Security, their welfare gains are significantly smaller than those accruing to single-earner couples.

A few authors have expanded the seminal longevity risk model from Yaari (1965) to include an analysis of couples. Hurd (1999) studies the joint optimization problem facing couples and our theoretical analysis is similar, though we focus specifically on the welfare effects of Social Security. Likewise, Kotlikoff and Spivak (1981) examine the degree to which couples may naturally hedge their longevity risk through cooperative optimization of their joint resources and by naming the other as the sole beneficiary of their joint wealth. Our analysis incorporates these features, and it goes a step further in seeking to understand the importance of Social Security in providing additional longevity insurance beyond what is naturally found through self-insurance within the family.

Brown and Poterba (2000) calculate the welfare gains to couples who purchase annuities from competitive private markets and find that the welfare gains accruing to couples tend to be smaller than for singles. Although our technical treatment of couples is similar, they calculate the welfare gains from

[2] To focus on Social Security's longevity insurance role, we shut down other potential roles: households do not vary by earning type so there is no redistributive role for Social Security, and households optimize their resources with full information about the risks that they face, so there is no role for Social Security to rescue irrational savers. Also, the public and private rates of interest on the storage of wealth are equal (set to zero for simplicity), so there are no dynamic (in)efficiencies associated with our welfare results. All welfare gains are therefore strictly the result of households sharing their longevity risk through Social Security.

access to competitive annuities on top of Social Security, while we study the welfare effects of Social Security when competitive annuity markets are missing.[3]

Like our paper, Li (2018) estimates the welfare effects of Social Security survivor benefits. She explicitly includes a life insurance market in a setting with a representative male decision-maker who has a calibrated bequest motive toward his children and spouse. On the other hand, we explicitly model the joint optimization problem of a couple that takes into account the behavior of surviving spouses and nests the continuation value of asset holdings into the couple's joint optimization problem with stochastic timing of spousal death. Also, while she focuses on the income-based longevity gap, we focus on the gender-based longevity gap, and we contrast the welfare effects of various aspects of Social Security (survivor benefits versus spousal benefits).[4]

Our paper complements several papers that investigate the effects of Social Security's spousal and survivor benefits on labor market decisions. Borella et al. (2019) find that eliminating the marriage-related Social Security provisions would drastically increase the labor market participation of married women over the life cycle and increase the savings of couples. Groneck and Wallenius (2020) also find that Social Security depresses married female labor supply. Similarly, Nishiyama (2019) shows that removing spousal and survivor benefits would increase the female labor participation rate. Kaygusuz (2015) calculates the labor supply and welfare effects of eliminating Social Security's redistributive features—including spousal and survivor benefits—and finds that female labor supply would increase as would the wellbeing of high-

[3]One limitation in our study is the absence of life insurance markets. In reality a couple can purchase life insurance against the untimely death of a spouse, and holding at least some life insurance is common, especially among married households (Chambers et al. (2003)). However, most of the poverty experienced by surviving women tends to be the result of inadequate life insurance coverage according to Bernheim et al. (2003), and notwithstanding relatively high participation rates in life insurance markets, the aggregate welfare gains associated with such participation appear to be quite small, even if insurance is actuarially fair, according to Chambers et al. (2003). Given that a lack of sufficient coverage appears to be a key determinate of financial insecurity, and given Social Security's objective to ease poverty in old age, we focus on studying Social Security's welfare properties in a setting where the couple does not participate in insurance markets but does self-insure with full information about the distribution of mortality risks to males and females. Also, Chambers et al. (2011) find that equilibrium life insurance holdings are virtually unaffected by the absence or presence of Social Security survivor benefits.

[4]In Li (2018), reducing survivor benefits creates welfare gains in general equilibrium but creates welfare losses when prices are held fixed.

skill dual earner households at the expense of single-earner households. The welfare analysis in Kaygusuz (2015) is conceptually similar to ours. In both cases the desirability of a particular Social Security feature is examined by turning that feature off in the model. Our analysis complements Kaygusuz's by considering survivor and spousal benefits separately, and by searching for the optimal mix of survivor and spousal benefits for a given level of taxes and retirement benefits.

Finally, our analysis relates to many papers that explore the welfare gains of Social Security's annuity benefit for singles (Hubbard and Judd (1987), İmrohoroğlu et al. (1995), Brown (2000), Caliendo et al. (2014), Cottle Hunt and Caliendo (2021)), and the vast literature that computes the welfare gains of annuitization in general (Kotlikoff and Spivak (1981), Mitchell et al. (1999), Brown (2001), Davidoff et al. (2005), and Lockwood (2012)).[5]

Additionally, there is a large literature that goes beyond longevity insurance to address Social Security's role in insuring lifetime earnings risk and disability risk, as well as its role in helping those who do not save optimally. This literature typically weighs the potential advantages of Social Security against the distortions that it imposes to aggregate labor supply, aggregate capital accumulation, and the transmission of wealth across generations. We do not tackle these broader issues but instead focus narrowly on a few new insights relating specifically to its longevity insurance role for couples.

2 The Setup

A household is comprised of a couple—a husband and wife. Children are not the focus of this paper and are not included in the model. Time is continuous and indexed by t. At $t = 0$ a husband and wife enter the workforce at the same time. The male earns wage income w_M and the female earns wage income w_F up until the retirement age, t_R, which is the same for both individuals. Our focus is on the

[5]For a discussion the potential interplay between Social Security and the problem of adverse selection in private annuity markets see Hosseini (2015).

insurance provided to widows through Social Security, and we are particularly interested in evaluating the effectiveness of such insurance for the historically-relevant case in which $w_M \geq w_F$. Upon retirement, each individual collects Social Security benefits b_M and b_F respectively, which are paid as life annuities for as long as the individuals live. Because $w_M \geq w_F$, the male's Social Security benefit will be the same whether or not the female is alive. However, the female's benefit will depend on whether the male is alive. If so, the female collects a benefit denoted b_F^1, and if the male is deceased then she will collect a benefit b_F^2. Social Security is financed through taxation of wage income at rate τ.

The maximum possible survival date of both individuals is $t = T$. The male survival probability is $S_M(t)$ and the female survival probability is $S_F(t)$, with $S_M(0) = S_F(0) = 1$ and $S_M(T) = S_F(T) = 0$. The female will be calibrated to have a higher life expectancy than the male.

The couple is of one mind when it comes to economic decision making, and they split consumption evenly when both are alive. Let $c(t)$ denote *total* household consumption and $k(t)$ denote the couple's joint asset holdings. All saving is done in a risk-free storage account. With this assumption, the welfare effect of Social Security is purely associated with the sharing of longevity risk (which is our focus), rather than for other well-known issues related to the dynamic (in)efficiency properties of the economy.

Period utility is of the CRRA variety. If an individual's consumption is x, then that individual experiences period utility

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma}.$$

Household utility is the sum of the individual utilities of the male and female.

Let $y(t)$ be the household's total disposable income, which can take on three different forms. If both individual's are alive at time t, we denote disposable income as

$$y(t) = \begin{cases} (1-\tau)w_M + (1-\tau)w_F, \text{ for } t \in [0, t_R], \\ b_M + b_F^1, \text{ for } t \in [t_R, T]. \end{cases}$$

If only the male is alive at time t, then we denote disposable income as

$$y_M(t) = \begin{cases} (1-\tau)w_M, \text{ for } t \in [0, t_R], \\ b_M, \text{ for } t \in [t_R, T]. \end{cases}$$

And if only the female is alive at time t, then we denote disposable income as

$$y_F(t) = \begin{cases} (1-\tau)w_F, \text{ for } t \in [0, t_R], \\ b_F^2, \text{ for } t \in [t_R, T]. \end{cases}$$

Social Security's budget is balanced and the system is efficiently financed, which means that expected taxes paid must equal expected benefits received over the life cycle of the household,

$$\int_0^{t_R} (S_M(t)\tau w_M + S_F(t)\tau w_F)dt = \int_{t_R}^{T} S_M(t) b_M dt + \int_{t_R}^{T} S_F(t) \left(S_M(t) b_F^1 + (1 - S_M(t)) b_F^2 \right) dt.$$

3 Couple's Dynamic Stochastic Problem: Optimal Hedging of Longevity Risk

We solve the problem recursively. We first consider the problems facing the widow and widower upon the loss of their spouse, and then we work backwards to the initial problem when both are alive.[6]

[6]Our theoretical methods relate to the regime-switching optimal control literature. Households in our model experience a regime switch at the death of the first spouse, which creates its own modeling complexities but is conceptually similar to the regime switching methods dating back to the pioneering work of Kamien and Schwartz (1971), Dasgupta and Heal (1974), Kemp and Long (1977), Hoel (1978), Tomiyama (1985), Amit (1986), Clarke and Reed (1994), Tahvonen and Withagen

Step 1. The Widow and Widower Problems

Let us first consider the widow's problem at the moment the husband passes away at time t with household asset holdings $k(t)$. Her optimal consumption plan for the remainder of her uncertain lifespan is the solution to

$$\max : \int_t^T S_F(z)u(c(z))dz,$$

subject to

$$\dot{k}(z) = y_F(z) - c(z),$$

$$k(t) \text{ given},$$

$$k(T) = 0.$$

The solution is

$$c_F^2(z|t, k(t)) = C_F(t, k(t))S_F(z)^{1/\sigma},$$

where

$$C_F(t, k(t)) \equiv \frac{k(t) + \int_t^T y_F(z)dz}{\int_t^T S_F(z)^{1/\sigma}dz}.$$

We likewise define the female continuation value of assets $k(t)$ at the death of her husband at time t as

$$U_F^2(t, k(t)) \equiv \int_t^T S_F(z)u(c_F^2(z|t, k(t)))dz.$$

On the other hand, if the wife dies first at time t with household asset holdings $k(t)$, the husband's

(1996). Numerous additional applications of regime-switch models include Makris (2001), Boucekkine et al. (2004), Dogan et al. (2011), Saglam (2011), Abel and Eberly (2012), Boucekkine et al. (2013a), Boucekkine et al. (2013b), Boucekkine et al. (2013c), Bi et al. (2013), Davig and Foerster (2014), and Caliendo et al. (2019).

remaining consumption is the solution to

$$\max : \int_t^T S_M(z)u(c(z))dz,$$

subject to

$$\dot{k}(z) = y_M(z) - c(z),$$

$$k(t) \text{ given},$$

$$k(T) = 0.$$

The solution is

$$c_M^2(z|t, k(t)) = C_M(t, k(t))S_M(z)^{1/\sigma},$$

where

$$C_M(t, k(t)) \equiv \frac{k(t) + \int_t^T y_M(z)dz}{\int_t^T S_M(z)^{1/\sigma}dz}.$$

And we define the male continuation value of assets $k(t)$ at the death of his wife at time t as

$$U_M^2(t, k(t)) \equiv \int_t^T S_M(z)u(c_M^2(z|t, k(t)))dz.$$

Step 2. The Couple's Initial Dynamic Stochastic Problem

Working back to time $t = 0$, the couple's ex ante problem involves selecting household consumption

$c(t)$ and asset holdings $k(t)$ to maximize expected utility[7]

$$\mathbb{E}(U) = \int_0^T \left(S_M(t)S_F(t)2u\left(\frac{c(t)}{2}\right) + \left(-\dot{S}_M(t)\right) U_F^2(t,k(t)) + \left(-\dot{S}_F(t)\right) U_M^2(t,k(t)) \right) dt.$$

subject to the following constraints

$$\dot{k}(t) = y(t) - c(t), \text{ for } t \in [0,T],$$

$$k(0) = k(T) = 0.$$

The Hamiltonian $\mathcal{H}$ with multiplier $\lambda(t)$ is

$$\mathcal{H} = S_M(t)S_F(t)2u\left(\frac{c(t)}{2}\right) + \left(-\dot{S}_M(t)\right) U_F^2(t,k(t)) + \left(-\dot{S}_F(t)\right) U_M^2(t,k(t)) + \lambda(t)[y(t) - c(t)],$$

and the necessary conditions include the Maximum Condition

$$\frac{\partial \mathcal{H}}{\partial c(t)} = S_M(t)S_F(t)\left(\frac{c(t)}{2}\right)^{-\sigma} - \lambda(t) = 0,$$

and the multiplier equation

$$\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial k(t)} = \dot{S}_M(t)\frac{\partial}{\partial k(t)}U_F^2(t,k(t)) + \dot{S}_F(t)\frac{\partial}{\partial k(t)}U_M^2(t,k(t)).$$

Note that with a bit of algebra we can show

$$\frac{\partial}{\partial k(t)}U_i^2(t,k(t)) = C_i(t,k(t))^{-\sigma} \text{ for } i \in \{M,F\}.$$

[7]Expected utility has been written here in a recursive form that allows us to use Pontryagin's Maximum Principle. The derivation of this expression itself is rather involved, and in the appendix we provide a comprehensive derivation.

Therefore the multiplier equation can be rewritten as

$$\dot{\lambda}(t) = \dot{S}_M(t) C_F(t, k(t))^{-\sigma} + \dot{S}_F(t) C_M(t, k(t))^{-\sigma}.$$

Differentiate the Maximum Condition with respect to t

$$\left(\dot{S}_M(t)S_F(t) + S_M(t)\dot{S}_F(t)\right)\left(\frac{c(t)}{2}\right)^{-\sigma} - \sigma S_M(t)S_F(t)\left(\frac{c(t)}{2}\right)^{-\sigma-1}\frac{\dot{c}(t)}{2} = \dot{\lambda}(t).$$

Combining this with the mutiplier equation gives the Euler equation

$$\dot{c}(t) = \frac{1}{\sigma}\left(\frac{\dot{S}_M(t)}{S_M(t)} + \frac{\dot{S}_F(t)}{S_F(t)}\right)c(t) - \frac{\dot{S}_M(t)C_F(t,k(t))^{-\sigma} + \dot{S}_F(t)C_M(t,k(t))^{-\sigma}}{\sigma S_M(t)S_F(t)} 2\left(\frac{c(t)}{2}\right)^{\sigma+1} \quad \text{for } t \in [0,T].$$

The first term in the Euler equation captures the familiar effect of longevity risk on consumption. Other things equal, consumption starts high and falls in proportion to the rate of change in the survival function. Of course, in our model there are two individuals whose survival rates factor into the Euler equation, but otherwise the first term conveys just the information that we would expect from a standard Euler equation: the household wants consumption to be relatively high during the early years when the probabilities of survival are highest.

However, the second term in the Euler equation works in the opposite direction. Notice that the second term is a function of the marginal continuation values of capital for each spouse upon the death of the other. A high marginal continuation value means the Euler equation will be upward sloping, other things equal, as the household finds it optimal to accumulate capital. While the marginal continuation value is always positive, it is especially high when capital is low and for the case where the primary earner dies first. For instance, when capital is low early in the life cycle, there is an especially high continuation value attached to an extra unit of capital to a surviving, non-working spouse upon the death of the other, and this can be a powerful motive to save.

The Euler equation, together with the law of motion $\dot{k}(t) = y(t) - c(t)$ and the boundary conditions $k(0) = k(T) = 0$, pin down the constant $c(0)$. This completes the analytical derivation of the solution. We denote the solution consumption and saving paths while both individuals are alive as $\{c_1^*(t), k_1^*(t)\}_{t\in[0,T]}$.

4 Ex Ante Efficient Allocation: The First Best

Before proceeding to our policy experiments, it is helpful to establish a benchmark for comparison. Here we derive the Ex Ante Efficient allocation, which is unattainable in our decentralized equilibrium without competitive annuity insurance markets. Establishing this benchmark allows us to assess the performance of Social Security as a Second Best program for hedging longevity risk.

Through the Law of Large Numbers, the Ex Ante Efficient allocation provides a perfect hedge against the couple's joint longevity risk according to the following optimization problem. Notationally, $c(t)$ continues to stand for total household consumption at time t, which is the sum of the consumption of the female and male $c(t) = c_F(t) + c_M(t)$.

$$\max_{\{c_M(t), c_F(t)\}} : \int_0^T \left(S_M(t)u(c_M(t)) + S_F(t)u(c_F(t))\right) dt$$

subject to

$$\int_0^{t_R} \left(S_M(t)w_M + S_F(t)w_F\right) dt = \int_0^T \left(S_M(t)c_M(t) + S_F(t)c_F(t)\right) dt.$$

With Lagrange multiplier λ, the first-order conditions are

$$S_M(t)u'(c_M(t)) - \lambda S_M(t) = 0$$

$$S_F(t)u'(c_F(t)) - \lambda S_F(t) = 0.$$

These conditions imply that male and female consumption must be constant across the life cycle and equal to one another. From the problem's constraint,

$$c_F = c_M = \frac{\int_0^{t_R} (S_M(t)w_M + S_F(t)w_F)\, dt}{\int_0^T (S_M(t) + S_F(t))\, dt}.$$

Hence, Ex Ante Efficient (First-Best) *household* consumption is

$$c_{FB} = c_F + c_M = 2 \times \frac{\int_0^{t_R} (S_M(t)w_M + S_F(t)w_F)\, dt}{\int_0^T (S_M(t) + S_F(t))\, dt},$$

with First-Best household expected utility

$$\mathbb{E}(U)_{FB} = \int_0^T (S_M(t) + S_F(t))\, u(c_{FB}/2)dt.$$

This allocation is not feasible to the couple because it is based on an expected budget constraint, whereas in reality the couple must instead plan for every possible contingency regarding the joint longevity risk that they face. In other words, the couple does not have the luxury of making plans based on the Law of Large Numbers as is done in the derivation of the Ex Ante Efficient allocation. In what follows, we examine how well various Social Security structures fare relative to the First Best.

5 Welfare

The optimized value of expected utility for a couple that follows the optimal dynamic strategy to hedge their joint longevity risk is

$$\mathbb{E}(U) = \int_0^T \left(S_M(t)S_F(t)2u\left(\frac{c_1^*(t)}{2}\right) + \left(-\dot{S}_M(t)\right)U_F^2(t,k_1^*(t)) + \left(-\dot{S}_F(t)\right)U_M^2(t,k_1^*(t)) \right) dt.$$

In our welfare calculations, we compare various permutations of the US Social Security to a Laissez Faire world with no Social Security. We compute the fraction of lifetime consumption the couple would commit to giving up, ex ante before mortality risks are realized, to live in a world with Social Security rather than Laissez Faire,

$$\Delta_{SS} = 1 - \left(\frac{\mathbb{E}(U)_{LF}}{\mathbb{E}(U)_{SS}}\right)^{\frac{1}{1-\sigma}},$$

where $\mathbb{E}(U)_{LF}$ is expected utility in a Laissez Faire world with no Social Security and $\mathbb{E}(U)_{SS}$ is expected utility with a Social Security program of some type in place.

6 Quantitative Analysis

6.1 Calibration

Individuals enter the model as they begin their working life at age 18. The maximum possible age for both men and women is 100 years and the retirement date is age 67. On average, women live longer than men. We calibrate the male and female survival functions to match the life-expectancy at birth from the Social Security life tables.[8] We set the CRRA parameter $\sigma = 3$ which is a common mid-point of estimated values. The parameters that are common to all policy experiments are listed in Table 1.

[8]See https://www.ssa.gov/oact/STATS/table4c6.html

parameter		target	value
σ	CRRA coefficient	mid-point in literature	3
T	maximum age	age 100	82
t_R	retirement age	age 67	49
$S_M(t)$	male survival function	male life-expectancy 75.97	$1-(t/T)^{2.4124}$
$S_F(t)$	female survival function	female life-expectancy 80.96	$1-(t/T)^{3.3067}$

Table 1: Baseline parameters common to all policy experiments

To create contrast with a typical model in which everyone works and earns the same wage, we focus on a single-earner household and therefore set $w_F = 0$. In this case, as long as the male is still alive, the female receives half of the male's Social Security benefit $b_F^1 = b_M/2$ during the retirement years. However, if the male passes away first, then the female receives the full male benefit for the remainder of her life. Hence, $b_F^2 = b_M$. Hence, for a given b_M, the Social Security tax rate τ is determined by the Social Security balanced budget.

$$\tau = \frac{\int_{t_R}^{T} S_M(t) b_M dt + \int_{t_R}^{T} S_F(t)\left(S_M(t) b_M/2 + (1 - S_M(t)) b_M\right) dt}{\int_0^{t_R} S_M(t) w_M dt}.$$

We set the baseline male Social Security benefit to 0.35 to match the average replacement rate. The tax rate is chosen to balance the Social Security budget and is $\tau = 0.1964$. This tax rate is somewhat higher than the 10.6% OASI tax because in this calibration only half of the population work while all receive benefits. We normalize the male wage to unity.[9] The baseline policy parameters are listed in Table 2.

Figure 1 plots the path of planned household consumption when both individuals are alive, $c_1^*(t)$, with the baseline Social Security parameterization, without Social Security, as well as the path of First Best consumption $c_{FB}(t)$. The planned consumption path $c_1^*(t)$ is followed until one of the individuals in the

[9] We abstract from wage heterogeneity in our analysis to focus on spousal and survivor benefits. This simplification means we miss the redistribution that occurs because of the progressivity of Social Security benefits. Social Security also redistributes towards those who enjoy a longer lifespan and the wealthy tend to live longer than the poor (see Liebman (2002), Coronado et al. (1999, 2002, 2011), Bishnu et al. (2019), and Goda et al. (2011)).

couple dies, at which point the surviving spouse consumes $c_i^2(t)$ for $i \in \{M, F\}$. The path of planned household consumption $c_1^*(t)$ is hump shaped over the life cycle due to precautionary saving to partially self-insure against the joint survival risk. In contrast, the path of First Best consumption $c_{FB}(t)$ is a flat line.

With the simple calibration that we are using, a standard model of a single individual would have ever-decreasing consumption across the life cycle. The rate of decrease in consumption would be in proportion to the rate of decrease in the individual's survival function. In our model with couples, this force is likewise in effect but there is a counter-force at play as well: the couple knows that premature death of a wage earner can damage the household's wage income flow and this gives the household the incentive to build up precautionary savings against this risk. This incentive is particularly strong for a single-earner couple without Social Security, because the household would need to rely on its own precautionary savings in the event of the death of the wage earner. This gives consumption without Social Security its pronounced hump shape. On the other hand, when Social Security is factored into the model, the need to quickly accumulate precautionary savings when the household is young is mitigated because Social Security pays survivor benefits. Hence, when Social Security is present, the household feels much less urgency to save when young and as a result consumption is only slightly hump shaped.

Figure 2 plots the path of realized household consumption given a variety of dates of death of either spouse. The top panel of the graph plots consumption assuming the female survives and the bottom plots consumption assuming the male survives. Note that both graphs plot *household* consumption. For the first part of the life cycle both spouses are alive and household consumption is given by $c_1^*(t)$. Following the death of either spouse, household consumption falls (as the number of household members decreases), and consumption is given by either $c_F^2(t)$ or $c_M^2(t)$ for the surviving spouse. Of course, mortality risk is a continuous random variable in our model, but for the ease of illustration, we present just a few of the many possible shocks that can occur.

For a single earner household, the loss of the earner can be devastating to the household budget. While the surviving spouse does pick up a significant survivor benefit through Social Security that equals the lost Social Security benefit of the deceased earner, these benefits do not replace lost household wages and the survivor essentially surrenders their own Social Security benefits to obtain the larger benefits of the deceased. These lost sources of revenue cause the household budget to contract and household consumption must be scaled back. The earlier the death of the sole earner, the larger the contraction. On the other hand, very early death is a relatively low-probability event and the household has rational expectation and hedges this risk optimally through precautionary saving. While the timing and ordering of the deaths of the spouses are unknown, the household has full information about the distribution of these risks and selects an optimal consumption-saving plan that accounts for all possible contingencies. In other words, the downward corrections are the household's optimal responses to each possible death shock, and consumption before the shock takes all of these possibilities into account. Finally, note that each of the post-shock contingent plans involve the consumption of the surviving spouse eventually converging to zero by the end of the life cycle, which is a standard result in life-cycle models with survival risk and no competitive annuities. The surviving individual consumes progressively less with time because they naturally discount future consumption according to their own survival probabilities.

6.2 Policy Experiments

We perform three sets of policy experiments to assess the welfare implications of various adjustments to the current Social Security system. These experiments help to shed light on which aspects of the current system are the most and least valuable in terms of longevity insurance within couples. We pay particular attention to the opportunity cost associated with various aspects of Social Security.

In the first set of experiments, we study the welfare effect of turning off Social Security survivor

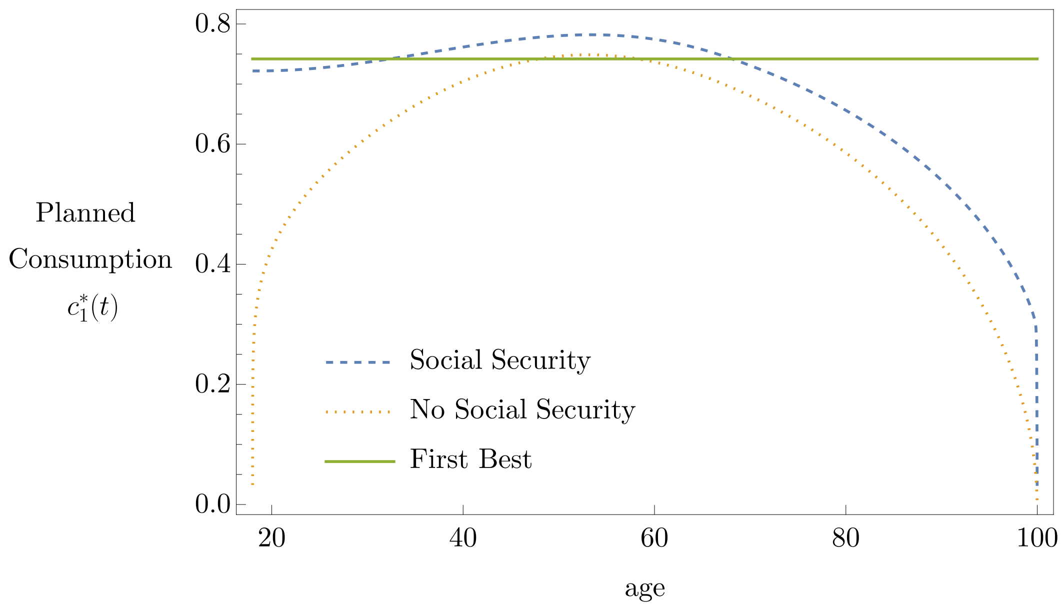

Figure 1: Path of planned household consumption when both individuals are alive, $c_1^*(t)$, with the baseline Social Security parameterization (blue dashed line), without Social Security (dotted yellow line), as well as First Best consumption $c_{FB}(t)$ (solid green line).

benefits. Within this first experiment, we consider two different cases. In one case, the budgetary savings associated with eliminating survivor benefits accrues directly to the household in the form of a lower Social Security tax rate. In this case the household keeps a greater portion of its wage earnings, but it must self-insure against the loss of Social Security benefits to the household at the death of the first spouse. In the other case, the budgetary savings associated with the elimination of survivor benefits is rolled into larger Social Security retirement and spousal benefits while holding the Social Security tax rate fixed at its baseline level. Thus, we are careful to distinguish between the differing welfare effects of eliminating survivor benefits and rebating the savings directly to households on the one hand, and eliminating survivor benefits and rolling the savings into other Social Security benefits on the other hand.

In the second set of experiments, we leave survivor benefits in place but we remove the spousal benefit provision. Within this second experiment, we again consider two different cases. In one case, the budgetary savings associated with the elimination of spousal benefits accrues directly to the household in the form of a lower Social Security tax rate. In the other case, the budgetary savings is used to fund

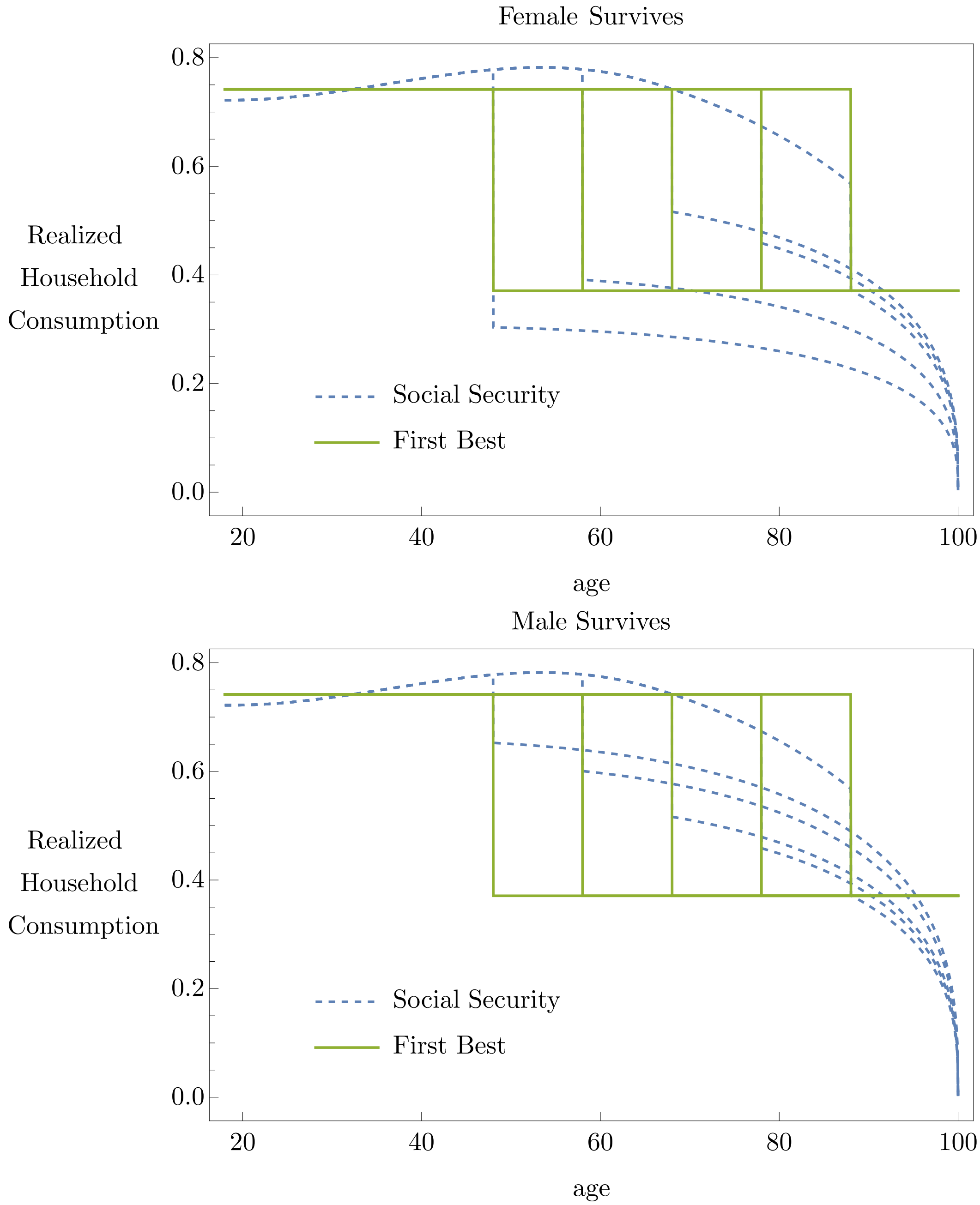

Figure 2: Path of realized household consumption for various lengths of spousal life with Social Security (blue dashed lines) as well as First Best consumption $c_{FB}(t)$ (solid green line). The top figure plots consumption if the female spouse survives for various possible dates of death of her spouse. The bottom figure plots consumption if the male spouse survives.

larger Social Security retirement and survivor benefits, while holding the Social Security tax rate fixed at its baseline level.

In the third and final set of experiments, we eliminate survival benefits and spousal benefits altogether. As with the other two sets of experiments, we study the case in which the budgetary savings associated with the canceled components of Social Security accrue to the household through a reduction in Social Security taxes, as well as the case in which the budgetary savings is rolled into larger Social Security retirement benefits while holding the tax rate fixed.

The details of each experiment are enumerated below and the parameters for each are summarized in Table 2.

- *Experiment 1: No Survivor Benefits.* In this experiment, we compare current US Social Security to a hypothetical Social Security system that has spousal benefits but no survivor benefits. For this case we have $b_F^1 = b_M/2$ and $b_F^2 = 0$. The budget of the hypothetical Social Security system is balanced if

$$\tau \int_0^{t_R} S_M(t) w_M dt = \int_{t_R}^T S_M(t) b_M dt + \int_{t_R}^T S_F(t) S_M(t) b_M/2 dt.$$

  We run this experiment in two ways. In Experiment 1A, we hold b_M fixed at its baseline value $b_M = 0.35$ and we let the tax rate τ adjust to a (lower) level that balances the budget. This experiment captures the welfare effect of removing survivor benefits and allowing the household to keep the associated tax savings. In Experiment 1B, we also remove survivor benefits but we load the associated tax savings into larger retirement and spousal benefits. That is, we hold the tax rate τ fixed at its baseline value and we let b_M adjust to a (higher) level that balances the budget.

- *Experiment 2: No Spousal Benefits.* In this experiment, we compare current US Social Security to a hypothetical Social Security system that has survivor benefits but no spousal benefits. For this case we have $b_F^1 = 0$ and $b_F^2 = b_M$. The budget of the hypothetical Social Security system is balanced if

$$\tau \int_0^{t_R} S_M(t) w_M dt = \int_{t_R}^{T} \left(S_M(t) + S_F(t) - S_F(t) S_M(t) \right) b_M dt.$$

We run this experiment in two ways. In Experiment 2A, we hold b_M fixed at its baseline value $b_M = 0.35$ and we let the tax rate τ adjust to a (lower) level that balances the budget. This experiment captures the welfare effect of removing spousal benefits and allowing the household to keep the associated tax savings. In Experiment 2B, we also remove spousal benefits but we load the associated tax savings into larger retirement and survivor benefits. That is, we hold the tax rate τ fixed at its baseline value and we let b_M adjust to a (higher) level that balances the budget.

- *Experiment 3: No Spousal Benefits and No Survivor Benefits.* In this experiment, we compare current US Social Security to a hypothetical Social Security system that has no spousal benefits and no survivor benefits. For this case we have $b_F^1 = b_F^2 = 0$. The budget of the hypothetical Social Security system is balanced if

$$\tau \int_0^{t_R} S_M(t) w_M dt = \int_{t_R}^{T} S_M(t) b_M dt.$$

We run this experiment in two ways. In Experiment 3A, we hold b_M fixed at its baseline value $b_M = 0.35$ and we let the tax rate τ adjust to a (lower) level that balances the budget. This experiment captures the welfare effect of removing spousal and survivor benefits and allowing the household to keep the associated tax savings. In Experiment 3B, we also remove spousal and survivor benefits but we load the associated tax savings into a larger retirement benefit. That is, we hold the tax rate fixed at its baseline value and we let b_M adjust to a (higher) level that balances the budget.

Experiment 1: No Survivor Benefits. In this experiment we compare the current US Social

		Baseline Social Sec.	No Survivor Benefits		No Spousal Benefits		No Spousal or Surviv. Benefits	
parameter			**1A**	**1B**	**2A**	**2B**	**3A**	**3B**
w_M	male wage	1	1	1	1	1	1	1
w_F	female wage	0	0	0	0	0	0	0
b_M	male benefit	0.35	0.35	0.5150	0.35	0.4159	0.35	0.6715
b_F^1	spousal benefit	$b_M/2$	$b_M/2$	$b_M/2$	0	0	0	0
b_F^2	survivor benefit	0.35	0	0	0.35	0.4159	0	0
τ	balanced-budget tax	0.1964	0.1334	0.1964	0.1653	0.1964	0.1024	0.1964

Table 2: Single-earner policy experiment parameterizations

Security system to a hypothetical system that has spousal benefits, but no survivor benefits. Given our single-earner parameterization, this means that the non-working spouse collects a Social Security benefit equal to half of her spouse's during retirement if her spouse is alive (the spousal benefit) and nothing if her spouse dies (no survivor benefit).

Figure 3 plots the path of planned household consumption $c_1^*(t)$ for policy experiment 1A (no survivor benefit, taxes reduced, other benefits unchanged) and 1B (no survivor benefit, taxes constant, other benefits increased), as well as the path for the current Social Security system. The lack of survivor benefits means that the household faces the possibility of the surviving spouse relying solely on private savings to finance consumption. Given our parameterization, if the husband dies before the wife, then the wife will need to rely on their savings to finance her consumption after he dies. This is true regardless of the husband's date of death. By assumption, the female wage is zero. The couple engages in aggressive precautionary saving early in life to insure against the risk that the husband dies young (or at least before the wife), leaving the wife without an income stream. As the couple ages, they are able to consume more, as they have built up sufficient assets to prevent the wife from being destitute in old age, should she survive longer than the husband. Planned consumption in the first half of life is higher with reform 1A (lower taxes) compared to reform 1B (higher benefits).

In our baseline model of a single earner couple, the death of the earner (male) creates enormous risk for the female. In the model, the female does not remarry and does not work. In actuality both of these adjustments are possible, and both would tend to mitigate the financial risk to surviving females. Therefore our baseline calibration accentuates the insurance role of Social Security survivor benefits. We choose this calibration intentionally because it helps to set a boundary on how important Social Security can be to a financially vulnerable subset of the population—women who are not able to return to work (perhaps for health or other reasons) and who do not remarry. Later in the paper we swing to the other extreme and consider the welfare effect of Social Security for dual earner couples where male and female earnings are equal.

Figure 4 plots the path of realized household consumption for policy experiments 1A and 1B given various dates of spouse death. The left graphs plot household consumption if the wife lives longer than the husband, for a few possible dates of death of the husband. Household consumption is smoother—has a smaller drop at the date of the husband's death—with the current Social Security system compared to the hypothetical system without survivor benefits. This is because the survivor benefit provides partial insurance against the risk that the wage-earning spouse dies. Without the survivor benefit, consumption early in life is much lower due to precautionary savings. Following the death of the husband, household consumption falls by a greater amount since the household stops receiving Social Security income. The right graphs illustrate the path of household consumption if the husband survives, for a few possible dates of death of the wife. Consumption is flatter over the life cycle with the current Social Security system because the household does not need to save as aggressively when they are very young. Notice that the death of the wife triggers a smaller reduction in consumption because the negative income shock is less severe than when the husband dies first; indeed, consumption may even jump up with the death of the wife as the household realizes that the more adverse shock of the male dying first has been avoided.

Experiment 2: No Spousal Benefits. In this experiment we compare current US Social Security to a hypothetical system that has survivor benefits, but no spousal benefits. Given our single-earner

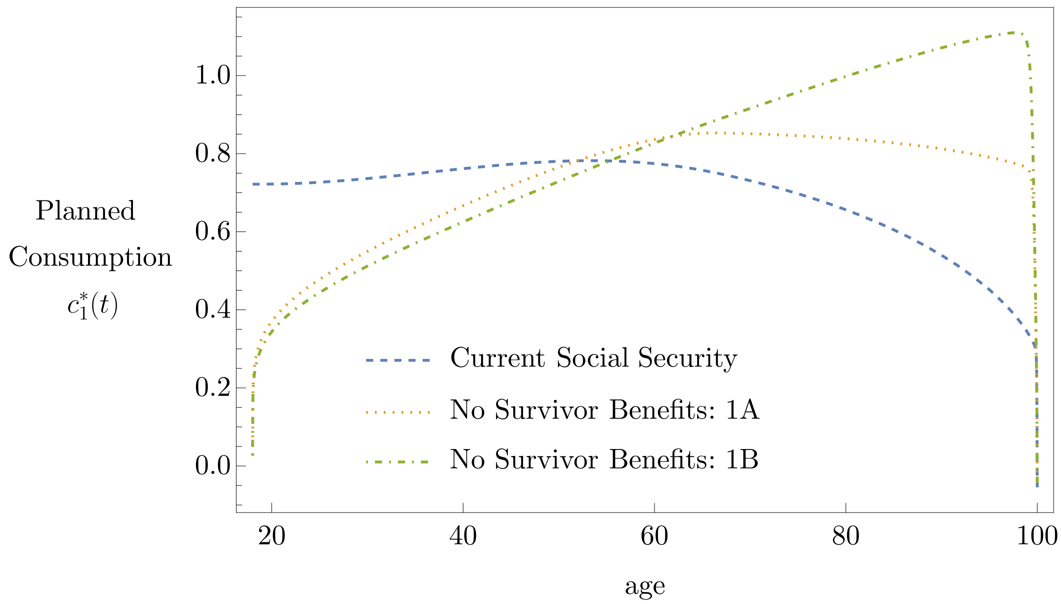

Figure 3: Policy Experiment 1: No Survivor Benefits. Path of planned household consumption $c_1^*(t)$.

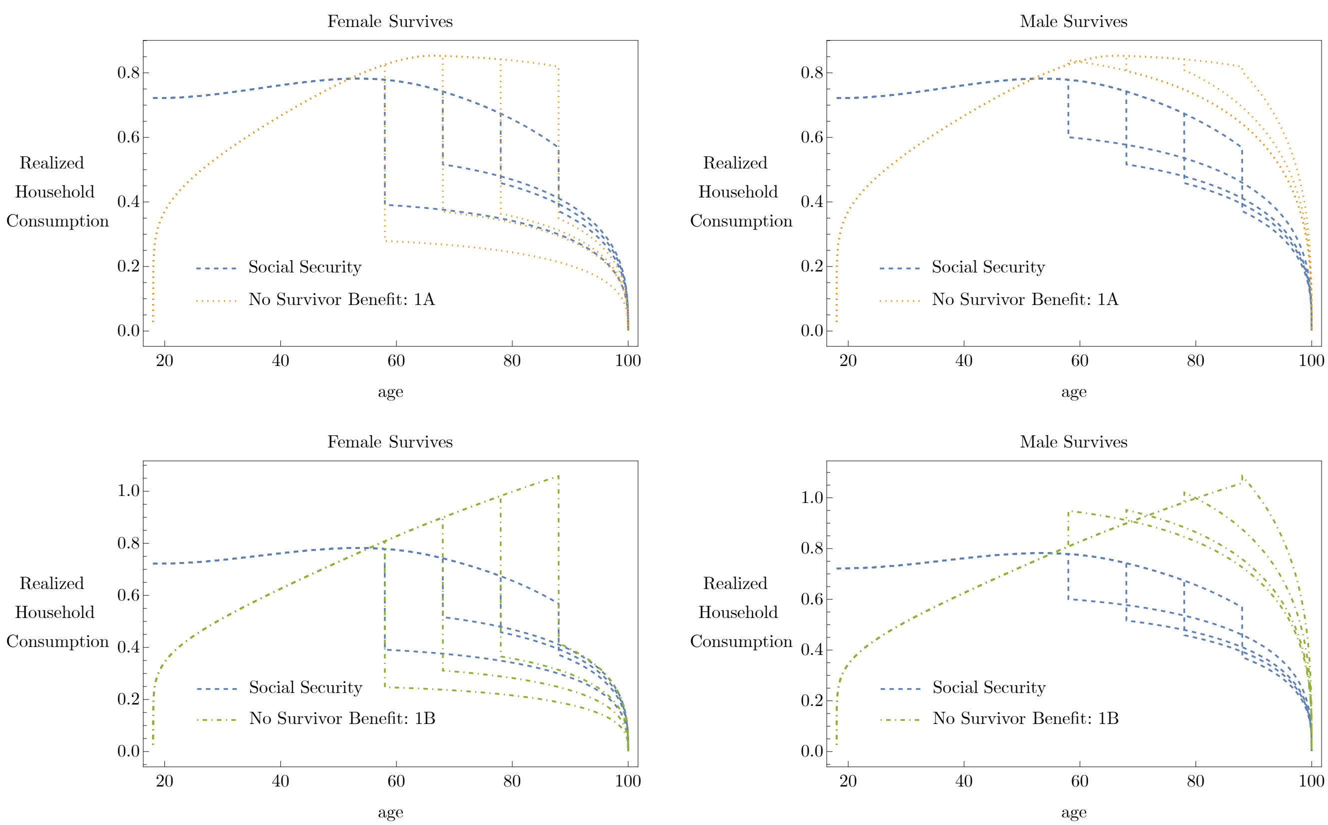

Figure 4: Policy Experiment 1: No Survivor Benefits. Realized household consumption given various spousal lifespans. The top panels show experiment 1A and the bottom panels show 1B. The left graphs show household consumption when the female survives and the right show consumption when the male survives. In all four graphs consumption given the current Social Security system is plotted as a dashed blue line and consumption without survivor benefits is the dashed yellow line (1A) or dot dashed green line (1B).

parameterization, this means that the non-working spouse collects a Social Security benefit if her spouse dies (survivor benefit) but nothing during the years that he is alive (no spousal benefit). Retirement income is constant regardless of which spouse(s) are alive.

Figure 5 plots the path of planned household consumption $c_1^*(t)$ for policy experiment 2A (no spousal benefit, taxes reduced, other benefits unchanged) and 2B (no spousal benefit, taxes constant, other benefits increased), as well as the path for the current Social Security. Early in life, planned household consumption is higher under experiment 2B than either experiment 2A or the current Social Security system. This is because benefits are large and certain in experiment 2B, which decreases the need for precautionary savings. Later in life, planned consumption is nearly identical for experiments 2A and 2B, although both are lower than the current Social Security system. This is because retirement income is lower when the spousal benefit is removed.

Figure 6 plots a few example paths of realized household consumption for policy experiment 2A and 2B, given different dates of death for the first spouse to pass away. The left panels show consumption assuming the female survives while the right shows consumption if the male survives. Experiment 2A is in the top figures and 2B is in the bottom figures. Across all four figures a similar pattern is visible: realized household consumption is relatively smoother over the life cycle when spousal benefits are removed compared to the current Social Security system. Consumption early in life is flatter and the drop in consumption at the death of the first spouse is smaller when there isn't a spousal benefit. Retirement income is constant regardless of which spouse dies first, so there is no need for households to save to offset a reduction in retirement income that occurs under the current system if the male spouse dies.

Experiment 3: No Spousal or Survivor Benefits. In this experiment we compare the current Social Security system to a system without spousal or survivor benefits. Given our single-earner parameterization, this means that the non-working spouse does not receive benefits. The household only receives

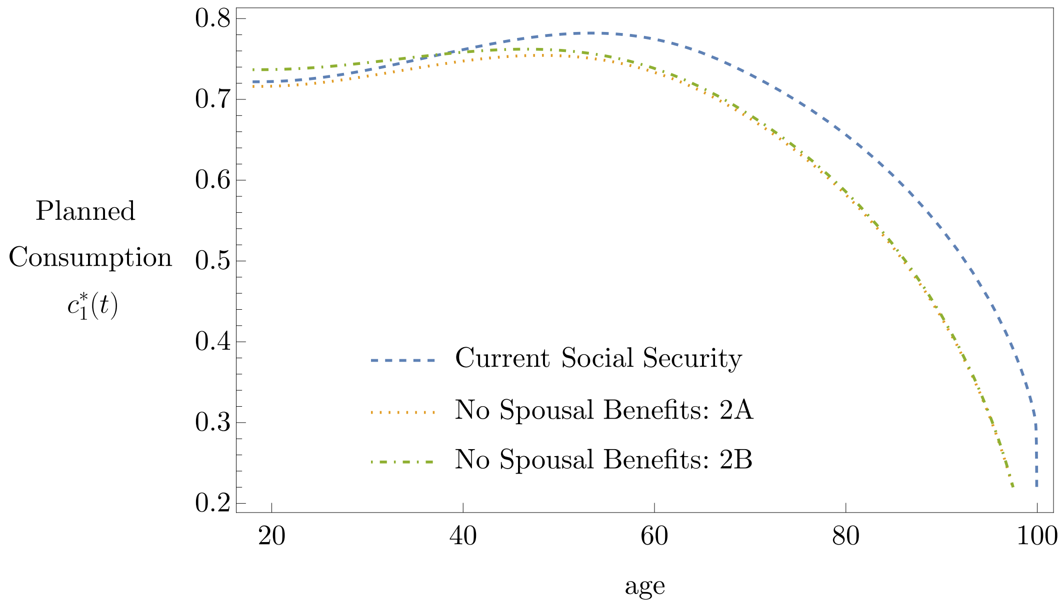

Figure 5: Policy Experiment 2: No Spousal Benefits. Path of planned household consumption $c_1^*(t)$.

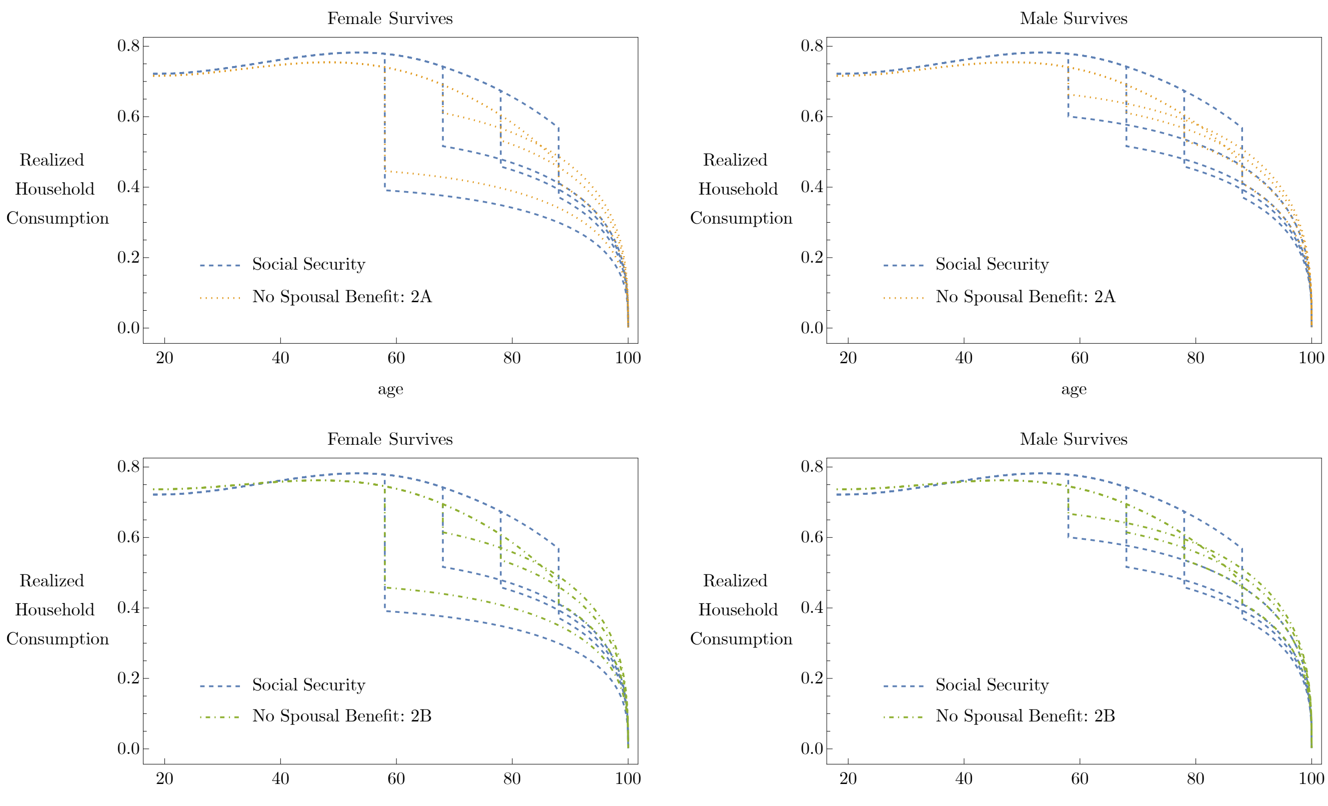

Figure 6: Policy Experiment 2: No Spousal Benefits. Realized household consumption given various spousal lifespans. The top panels show experiment 2A and the bottom panels show 2B. The left graphs show household consumption when the female survives and the right show consumption when the male survives. In all four graphs consumption given the current Social Security system is plotted as a dashed blue line and consumption without spousal benefits is the dashed yellow line (2A) or dot dashed green line (2B).

Social Security benefits while the working spouse is alive.

Figure 7 plots the path of planned household consumption $c_1^*(t)$ for policy experiment 3A (no spousal or survivor benefits, taxes reduced, retirement benefit unchanged) and 3B (no spousal or survivor benefits, taxes constant, retirement benefit increased), as well as the path for the current Social Security. In the absence of spousal and survivor benefits, the household has a strong precautionary saving motive. Early in life the household saves a large fraction of their income to insure against the risk that the wife outlives her husband. The wife neither receives wage income or Social Security benefits. Thus, if she outlives her spouse, she is only able to consume their savings. If both spouses live into old age, household consumption is quite high, given the accumulation of assets earlier in life. This is particularly true in experiment 3B, where the cost-savings of removing the spousal and survivor benefits are loaded into larger Social Security benefits for the working spouse.

Figure 8 plots a set of realized household consumption paths given different dates of spousal death for experiment 3A (top panels) and 3B (bottom panels). The left side plots household consumption assuming the wife survives. Household consumption falls by more at the date of death of the male spouse under either experiment than under the current system. This is because the household loses all retirement benefits when the male spouse dies (as well as any wage income if he dies before retirement). In contrast, if the male spouse survives longer than his partner, household consumption increases at the date of death. The only risk the surviving husband faces is his own longevity, which is partially insured through his annuity benefit and partially insured by the assets he accumulated earlier in life.

6.3 Welfare Results

The expected utility for the couple for each of the policy experiments are shown in the middle column of Table 3. Ex-ante expected utility for the couple is highest with the current US Social Security system.

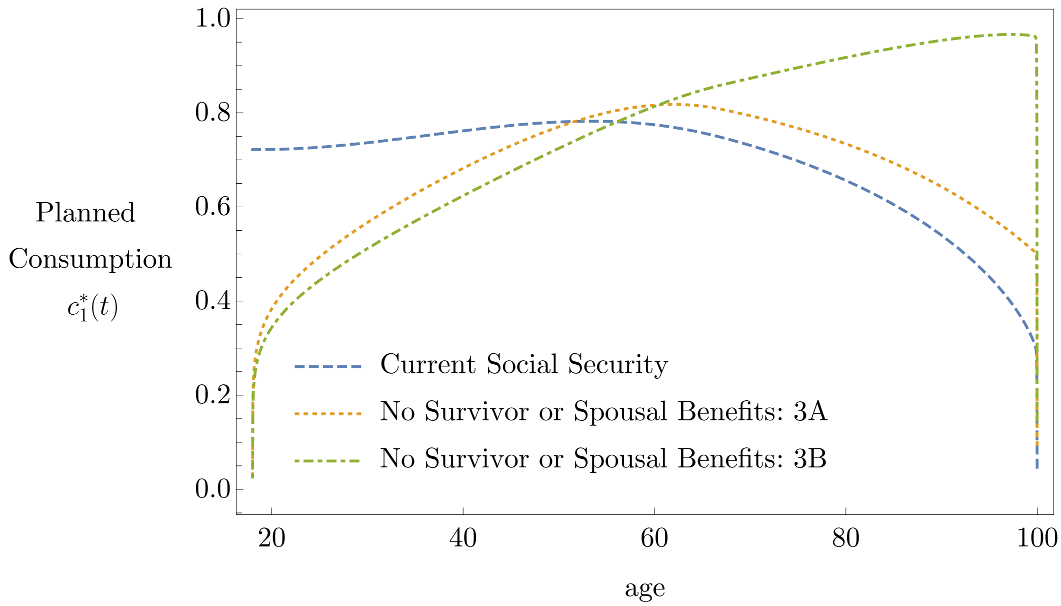

Figure 7: Policy Experiment 3: No Spousal or Survivor Benefits. Path of planned household consumption $c_1^*(t)$.

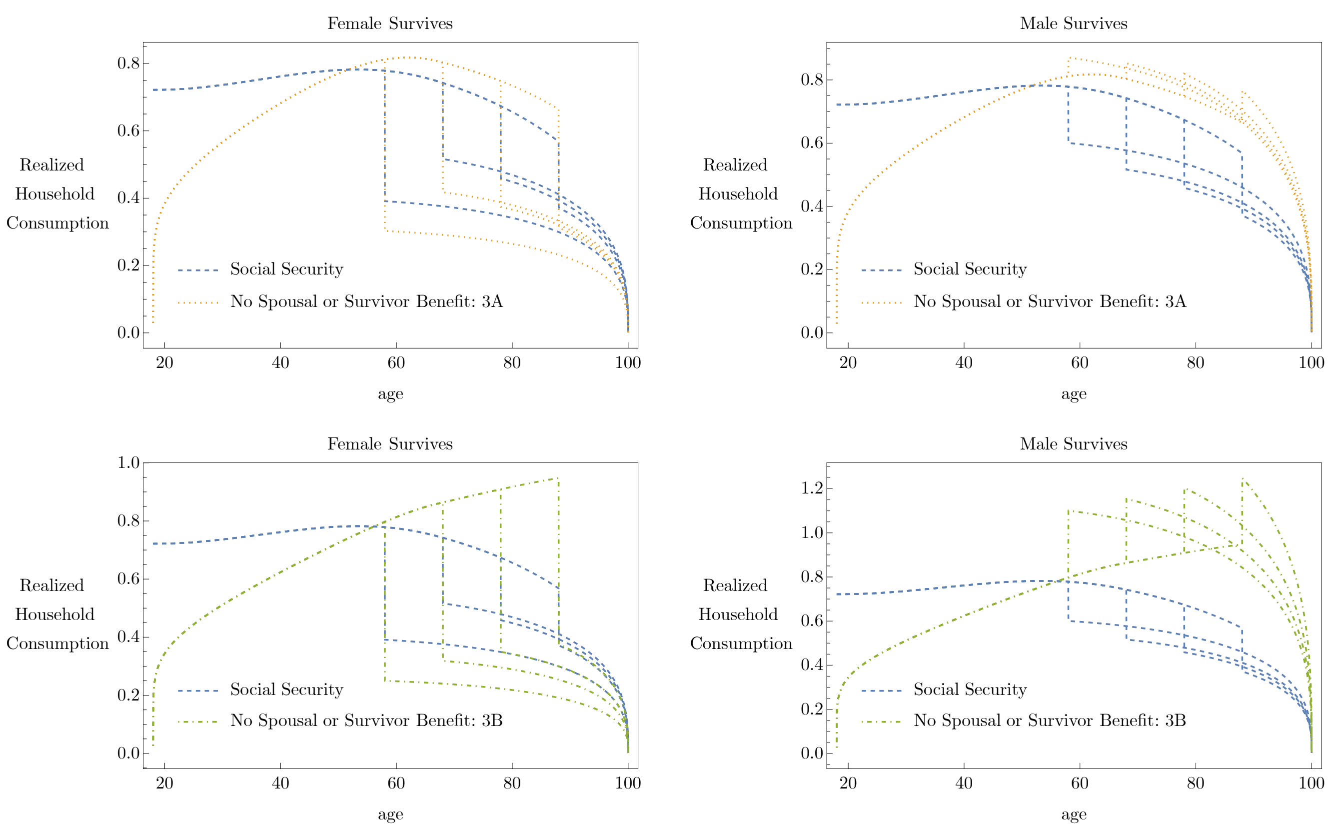

Figure 8: Policy Experiment 3: No Spousal or Survivor Benefits. Realized household consumption given various spousal survival. The top panels show experiment 3A and the bottom panels show 3B. The left graphs show household consumption when the female survives and the right show consumption when the male survives. In all four graphs consumption given the current Social Security system is plotted as a dashed blue line and consumption without spousal or survivor benefits is the dashed yellow line (3A) or dot dashed green line (3B).

Experiments 2A and 2B, which remove the spousal benefit, provide the next highest expected utility. Expected utility is lowest in experiment 3B where spousal and survivor benefits are removed and Social Security budget is balanced by raising the retirement benefit.

The welfare gains of a particular Social Security system relative to Laissez Faire are shown in the right column of Table 3. This welfare metric shows the fraction of lifetime consumption the couple would commit to giving up, ex ante before mortality risks are realized, to live in a world with a particular Social Security arrangement rather than Laissez Faire. For example, the couple would be willing to give up 24.9% of their consumption in a world without Social Security in order to live in a world with the current US system. The welfare gains of policy experiment 2 (No Spousal Benefit) are nearly as large. A couple would be willing to give up 23.2% or 24.7% of their consumption in a world without Social Security in order to live in a world with a Social Security system that paid retirement and survivor benefits only but did not pay spousal benefits. In contrast, the welfare gains of policy experiment 1 (No Survivor Benefit) are negative. A couple would be have to be compensated 7.5% or 14.2% of their consumption to be willing to live in a world with a Social Security system that paid retirement and spousal benefits but did not offer survivor benefits. These comparisons lead us to conclude that the survivor benefit that accrues to widows is a desirable component of a Social Security system for couple with a single earner, while the spousal benefit is much less important.[10]

We summarize our policy experiments thus far as follows:

1. Couples in our model experience very large welfare gains from the US Social Security system. Ex ante (before mortality risks are realized) the couple would be willing to give up 24.9% of their

[10] Our analysis measures the desirability of a particular Social Security benefit type by turning that benefit off and comparing the couple's ex ante expected utility to Laissez Faire. An alternative approach to measure the welfare gains of a particular Social Security feature is to compute the couple's ex ante willingness to live in a world with a single Social Security feature turned on compared to Laissez Faire. We find a couple would be willing to give up 4.5% or 15.7% of consumption to live in a world with Social Security that paid only survivor benefits (with the cost-saving of eliminating other benefits loaded into a larger survivor benefit or lower taxes). In contrast the welfare gain associated with a Social Security system that pays only spousal benefits is negative: -0.3% ot -14.5% depending on how the cost-savings of eliminating other benefits are financed.

Experiment		$\mathbb{E}(U)$	Δ_{SS}
No Social Security		-832.63	
Baseline US Social Security		-469.22	24.9%
No Survivor Benefit	1A	-961.52	-7.5%
	1B	-1086.23	-14.2%
No Spousal Benefit	2A	-491.18	23.2%
	2B	-472.41	24.7%
No Survivor or Spousal	3A	-920.03	-5.1%
	3B	-1092.68	-14.6%

Table 3: Welfare Results of Policy Experiments. The middle column shows the value of expected utility for the couple. The right column shows the fraction of lifetime consumption the couple would commit to giving up, ex ante before mortality risks are realized, to live in a world with a Social Security program rather Laissez Faire. A positive number indicates the couple prefers the Social Security system to Laissez Faire.

lifetime consumption in order to live in a world with the current Social Security system of retirement benefits, spousal benefits, and survivor benefits, compared to a separate Laissez Faire world without Social Security.

2. We decompose this welfare gain into the portions that can be attributed to Social Security's various features, and we find that eliminating the survivor benefit that accrues to widows (while retaining the retirement benefit life annuity feature to the sole earner and retaining the spousal benefit feature) results in a very undesirable Social Security program. In this case, the couple would be much better off fending for themselves in a Laissez Faire world without Social Security. While a Social Security program without survivor benefits would still have the advantage of paying retirement benefits as a life annuity up to the death of the sole earner, the program isn't free because it requires the couple to pay taxes to finance those benefits and the program acts to create a problematic amount of income risk for the household: now the widow would experience a large, negative income shock at the death of her spouse–regardless of if he dies before or after reaching retirement. A rational, forward-looking couple in our model would rather self-insure their combined longevity risks rather than pay taxes to participate in a program with incomplete annuitization that only lasts until the death of the sole earner. This result holds true whether the budgetary savings associated with the

elimination of survival benefits is used to fund a larger level of other benefits or to reduce taxes.

3. Removing the spousal benefit feature (while retaining the retirement benefit life annuity feature and the survivor benefit feature) results in a modified Social Security program that confers almost the same level of welfare gains as the current Social Security program. This is true whether the budgetary savings associated with the elimination of spousal benefits is used to fund a larger level of other benefits or to reduce taxes. Hence, the spousal benefit feature appears to offer very little in terms of welfare gains. This is an important results because it is an expensive feature to finance: in our model, spousal benefits cost about 3% of wages to fund.

Finally, it is interesting to compare the performance of Social Security to the First Best. We've shown above that the US Social Security program provides significant welfare gains to the couple who faces joint survival risk, and we've shown that the survivor benefits are the key to this result. Naturally, it is also interesting to know how well Social Security compares to the First Best. Of course, by definition, Social Security is inferior to the First Best, but by how much? Does Social Security close most of the welfare gap between Laissez Faire and the First Best? The answer is yes. The welfare gain associated with the First Best relative to Laissez Faire is 27.3%, which is just a few percentage points higher than the welfare gain of Social Security. So on the one hand, it is rather remarkable that the US Social Security program comes so close to the utility conferred but the First Best. Yet on the other hand, there is room for potential improvement by adjusting the structure of Social Security somewhat, and this is the topic of the next section.

7 The Second Best

Up to this point in the paper, we have considered the welfare effects of Social Security by turning on and off Social Security and its various components. This exercise allows us to understand how much value the program adds to the welfare of the couple. We now take our analysis a step further by searching for the optimal mixture of spousal and survivor benefits within a balanced-budget Social Security system, while holding fixed the Social Security tax and retirement benefits at their baseline levels. Because we are imposing some constraints in our optimization procedure—we are searching for an optimal parameterization of Social Security within a given system structure—we do not intend to identify the global optimum program but rather are aiming to establish the optimal blend of spousal and survivor benefits within a system that is otherwise identical to the current US system. And, importantly, because we are holding taxes fixed at the baseline level, we are seeking to understand whether there is a resource-neutral way to reconfigure the program to improve its ability to help couples cope with longevity risk. We refer to the solution to this optimization problem as the Second Best.[11]

Figure 9 plots the locus of possible spousal and survivor benefits that jointly balance the Social Security budget holding the tax rate and retirement level at their baseline levels. On one extreme, the budget can be balanced with survivor benefits of 0.52 (larger than the retirement benefit of 0.35) and no spousal benefit. On the other extreme, the budget can be balanced with a spousal benefit of 0.53 and no survivor benefit. The Second Best solution is somewhat close to the current Social Security system. Both points are indicated on the graph. The Second Best solution is to set spousal benefits to 0.125 and survivor benefits to 0.399; this is compared to the current system with spousal benefits of 0.175 and survivor benefits of 0.35. Holding retirement benefits and the Social Security tax rate fixed, the optimal blend of spousal and survivor benefits would pay higher survivor benefits and lower spousal benefits compared to the current US system.

Planned household consumption $c_1^*(t)$ (which corresponds to realized household consumption while both spouses are alive) is flatter under the Second Best Social Security system compared to the current

[11] Optimal Social Security reform is not our focus but is the subject of a large literature including Huang et al. (1997), Conesa and Garriga (2008), Kitao (2014), and McGrattan and Prescott (2017) among many other examples.

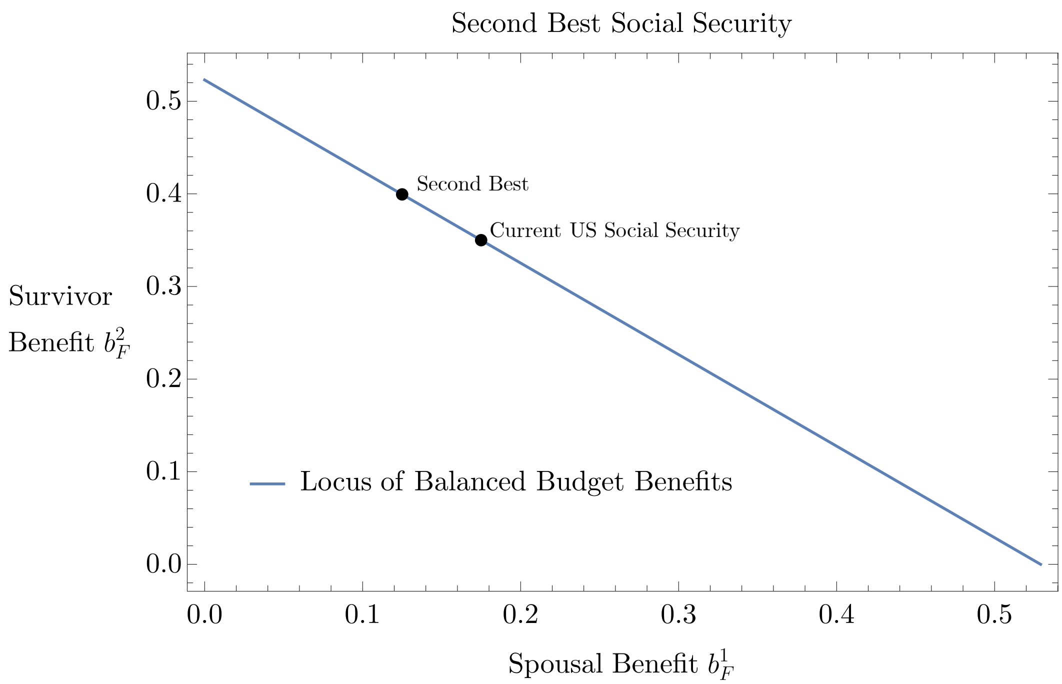

Figure 9: Locus of possible balanced budget benefits. The horizontal axis show the spousal benefit b_F^1 and the vertical axis shows the survivor benefit b_F^2 that jointly balance the Social Security budget holding the tax rate and retirement benefit fixed.

US Social Security system. This is visible in Figure 10. A flatter planned consumption profile is preferable to the household and closer to the First Best path of consumption. Figure 11 plots realized household consumption for a small number of spousal dates of death for the current US Social Security system, the First Best solution, as well as the Second Best Social Security system. Realized consumption under the Second Best Social Security system is closer to the First Best than is the current Social Security system. That is because the Second Best system does a better job of insuring the household's joint survival risk than current Social Security. Note in particular that realized consumption is smoother over the life cycle if the female spouse survivors with Second Best Social Security compared to current Social Security.

Couples have higher ex ante expected utility under the Second Best than under current Social Security. The expected utility associated with various points along the locus of possible benefits are listed in Table 4. The table also shows the fraction of lifetime consumption the couple would commit to giving up, ex ante before mortality risks are realized, to live in a world with a Social Security program rather Laissez

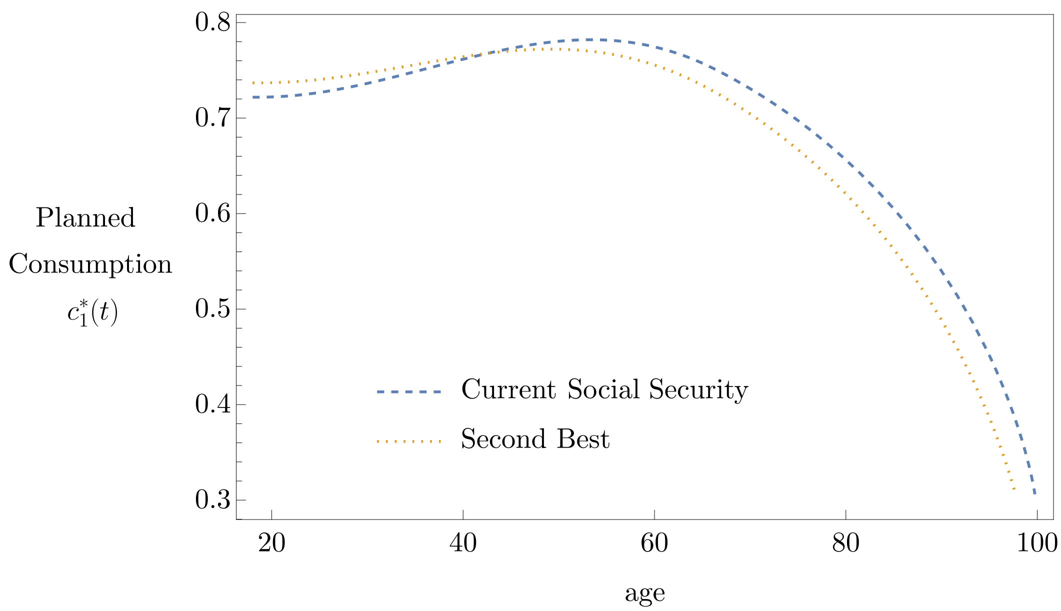

Figure 10: Second Best Social Security. Path of planned household consumption $c_1^*(t)$.

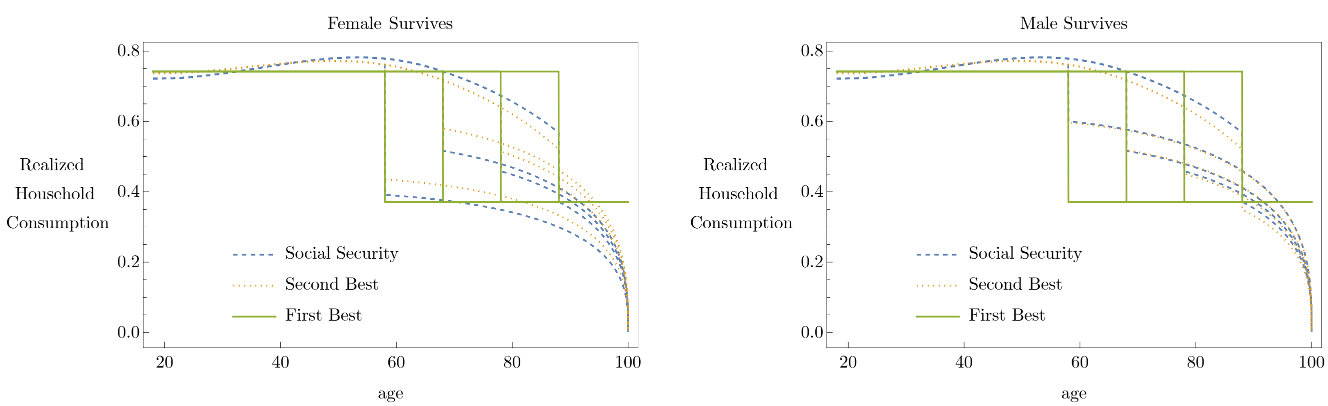

Figure 11: Second Best Social Security. Realized household consumption given various spousal lifespans. The left graphs show household consumption when the female survives and the right show consumption when the male survives. In both graphs consumption given the current Social Security system is plotted as a dashed blue line, Second Best Social Security is the dashed yellow line, and First Best is the solid green line.

Locus Point	b_F^1	b_F^2	$\mathbb{E}(U)$	Δ_{SS}
	0	0.523	-485.01	23.7%
Second Best	0.125	0.399	-465.19	25.3%
US Social Security	0.175	0.350	-469.22	24.9%
	0.35	0.177	-555.43	18.3%
	0.45	0.078	-692.83	8.8%
	0.50	0.029	-834.42	-0.1%
	0.53	0	-1190.33	-19.6%

Table 4: Welfare Results of Second Best Social Security. The first and second numerical columns show the spousal and survivor benefits that jointly balance the Social Security budget. The third column shows the value of expected utility for the couple. The right column shows the fraction of lifetime consumption the couple would commit to giving up, ex ante before mortality risks are realized, to live in a world with a Social Security program rather Laissez Faire. A positive number indicates the couple prefers the Social Security system to Laissez Faire.

Faire. Unsurprisingly, the welfare gain of the Second Best system is higher than that of current Social Security. The welfare gain of the Second Best system is 25.3% compared to 24.9% for the current system. The welfare gain in either case is driven mainly by having a large survivor benefit. At one extreme of the locus of possible benefits, a system that pays survivor benefits but no spousal benefits (holding retirement benefits and taxes constant) has a welfare gain of 23.7% compared to Laissez Faire. This is a little lower than the current system and lower than the Second Best, but still much higher than many points along the locus. At the other extreme, the welfare gain of a Social Security system with spousal benefits but no survivor benefits (again holding retirement benefits and taxes at the same level as the current US system) is negative, suggesting couples would prefer to self-insure their risk rather than participate in a Social Security system without survivor benefits.

8 Dual Earners

We focus our main analysis on a couple with a single earner to provide the largest contrast to a model with an individual earner. A couple with a single earner has the most to gain from a spousal benefit and a survivor benefit, since one half of the couple never earns wage income (by assumption). As we show, single-earner couples would be willing to sacrifice a large share of their consumption in a world without

Social Security to live in a world with the current US Social Security system or a hypothetical system with retirement and survivor benefits but no spousal benefits.

Of course, in reality many households are dual-earner couples. If both partners earn the same wage, then neither the spousal or survivor benefit are relevant, since each partner receives the same retirement benefit based on their own wages. For example, if the female wage is set to unity $w_F = w_M = 1$, then both spouses would collect the same retirement benefit $b_M = b_F^1 = b_F^2 = 0.35$. Under this parameterization, the tax rate that balances the Social Security budget is 11.11%. The couple would be willing to give up 6.9% of their consumption, ex ante before mortality risks are realized, to live in a world with Social Security rather Laissez Faire. The couple's expected utility would be the same under any of our policy experiments (with or without spousal or survivor benefits), since they would never use the spousal or survivor benefit. The welfare gain of Social Security is much smaller for the dual-earner couple with wage parity than for the single-earner couple because the dual-earner couple does not face the risk of the wife becoming a widow without wage income and with a retirement annuity.

In a similar vein, if the lower-earning spouse's wage is close enough to the higher-earning spouse's wage, the spousal benefit is irrelevant. Recall, the spousal benefit allows the lower-earning spouse to collect a benefit based on either their own earning history, or to collect half of their partner's benefit, whichever is higher. Thus if the lower-earning spouse's wage is close to the higher-earning spouse, the lower-earning spouse will choose their own benefit rather than half of their spouse's. The survivor benefit is relevant for a dual-earner couple who earn different wages, but the welfare gain will be smaller than for a single-earner couple. The dual-earner couple uses the survivor benefit if the higher-earning spouse passes away first. In that case, the surviving spouse would see their Social Security benefit increase from their own amount (either based on their wages, or the spousal benefit, whichever is higher) to the amount their spouse received. This partially offsets the loss of household income that occurs with the death of the higher-earning spouse, but is (potentially) less valuable than in the case of a couple with a single earner.

Spousal and survivor benefits as provided in the current US Social Security system redistribute from dual-earner households to single-earner households, as well as from couples with wage parity to couples with a large wage gap. This is because both partners in the dual-earner couple pay Social Security taxes on their wages, but the survivor benefit is only claimed by couples with a wage-gap or a single earner, and the spousal benefit is only claimed by couples with a large-enough wage gap or a single earner. Whether or not this redistribution is desirable is a question of social preferences or political economy. Couples who earn the same wage (or similar wages) would prefer a system without spousal and survivor benefit because either their taxes could be reduced or the cost savings could be passed into larger retirement benefits. However, single-earner couples would prefer the current system.

It is worth emphasizing that the single-earner couples in our model are willing to pay high taxes to finance Social Security entirely on their own. The welfare gains experienced by single-earner couples in our model do not come at the expense of any other group (since all couples in baseline the model are single earners). Thus, the single-earner couples in our model are not subsidized by dual-earner couples—they entirely self-finance the Social Security system. In that sense, our analysis understates the positive welfare gains to single-earner households. If marriage related benefits were instead subsidized by dual-earner couples, the single-earner households would experience an even larger welfare gain from Social Security.

9 Leaving Rationality

Not everyone seems to save adequately for retirement and this fact has stimulated a large literature on the causes and consequences of such behavior as well as potential public policy solutions. In this section we check the robustness of our results to our underlying assumption that individuals save optimally for retirement in the face of longevity risk. Here we swing to the other extreme and assume the couple does not save at all and instead relies exclusively on Social Security benefits during retirement. Surveys indicate that such behavior is prevalent in the US and accounts for as much as 1/3 households.

Total household consumption $c(t)$ is equal to total household disposable income $y(t)$ when both individuals are alive, and is equal to $y_M(t)$ when only the male is alive and $y_F(t)$ when only the female is alive. While the household itself does not optimize, we can compute the level of ex ante expected utility associated with the household's suboptimal behavior,

$$\mathbb{E}(U) = \int_0^T \left(S_M(t) S_F(t) 2u\left(\frac{y(t)}{2}\right) + (1 - S_M(t)) S_F(t) u\left(y_F(t)\right) + (1 - S_F(t)) S_M(t) u\left(y_M(t)\right)\right) dt.$$

We utilize the same parameter values as in our baseline model with one exception. The coefficient of relative risk aversion σ must be less than 1 or expected utility is undefined in a Laissez Faire world (because consumption goes to zero after retirement since the household does not save). We set $\sigma = 0.5$, and we re-run all of the same Social Security policy experiments for comparison. Our results are in Table 5.

To be brief, we can summarizing our findings with irrational behavior as follows. First, the US Social Security program continues to confer large welfare gains, and the magnitude of these gains is similar to our baseline model with full rationality. Second, eliminating Social Security survivor benefits significantly reduces the welfare gains associated with Social Security, consistent with our baseline model with rational agents. But unlike our baseline model, here Social Security without survivor benefits continues to be a welfare improving program. Hence, our previous finding that survivor benefits are the key to Social Security's welfare gains is sensitive to our assumption that agents are rational, which is intuitive because the attraction of Laissez Faire for a rational couple is that they can use the tax savings from eliminating Social Security to build up a larger precautionary savings balance. This large balance is transmitted

Experiment		$\mathbb{E}(U)$	Δ_{SS}
No Social Security		125.56	
Baseline US Social Security		144.44	24.4%
No Survivor Benefit	1A	139.24	18.7%
	1B	139.68	19.2%
No Spousal Benefit	2A	143.61	23.6%
	2B	125.56	24.0%
No Survivor or Spousal	3A	138.33	17.6%
	3B	139.39	18.9%

Table 5: Welfare Results of Policy Experiments: Non-optimizing Households. The middle column shows the value of expected utility for the couple who does not save at all. The right column shows the fraction of lifetime consumption the couple would commit to giving up, ex ante before mortality risks are realized, to live in a world with a Social Security program rather Laissez Faire. A positive number indicates the couple prefers the Social Security system to Laissez Faire.

to the survivor while Social Security benefits are not (when there is no survivor provision). Clearly, this mechanism is based on the couple taking the initiative to save on their own. Third, as with rational agents, eliminating spousal benefits only slightly reduces the welfare gains associated with Social Security, leaving us with the same conclusion as before: spousal benefits do not add much to the couple's welfare.

10 Conclusion

In this paper we study a life-cycle problem of a couple who makes optimal consumption and saving decisions in the face of mortality risk. The individuals in the couple face different mortality risks based on gender, and they have full information about these risks. We solve the dynamic stochastic problem of the couple recursively by deriving continuation functions for the widow and widower, and then we embed these functions into an initial problem with uncertainty about the order and timing of death of the individuals. We work in continuous time and we solve the problem analytically.

While the couple makes rational decisions with full information, competitive insurance markets are closed in the model and so there is room for a public, Second Best option to improve ex ante welfare. Our focus is on understanding whether Social Security spousal and survivor benefits improve the ex ante welfare of the couple, and our setting with closed insurance markets gives Social Security the greatest chance to succeed in raising the couple's welfare.

We take a public finance approach to the problem by characterizing and comparing the couple's consumption, saving, and welfare across a variety of scenarios, including a Laissez Faire economy with no government, an economy with perfect (First Best) insurance, an economy with US Social Security, and an economy with various permutations to US Social Security including removing some of its features as well as reconfiguring the program to reach the Second Best welfare level.

Even in this environment that is biased in favor of Social Security, we do not find much of a welfare role for spousal benefits. On the other hand, survivor benefits are incredibly important: removing survivor benefits while leaving retirement benefits and spousal benefits in place would make the couple worse off than in a Laissez Faire economy where they had to self insure their joint survival risks. Likewise, the Social Security program could be reconfigured in a way that improves welfare without an increase in taxes or a reduction in retirement benefits: reduce spousal benefits and use the cost savings to increase survivor benefits. In this way, the program would help couples to better hedge their longevity risk.

11 Appendix

Here we derive expected utility from scratch. To keep track of notation, let v be a random variable denoting the timing of female death and let t be a random variable denoting the timing of male death. Ultimately, these are dummies of integration and can be adjusted for convenience. Expected utility is the sum of two terms

$$\mathbb{E}(U) = \mathbb{I} + \mathbb{III},$$

where

$$\mathbb{I} \equiv \int_0^T \left(-\dot{S}_F(v)\right) \int_0^v \left(-\dot{S}_M(t)\right) \left(\int_0^t 2u\left(\frac{c(z)}{2}\right) dz + \int_t^v u(c_F^2(z|t, k(t)))dz\right) dtdv$$

$$\mathbb{III} \equiv \int_0^T \left(-\dot{S}_M(t)\right) \int_0^t \left(-\dot{S}_F(v)\right) \left(\int_0^v 2u\left(\frac{c(z)}{2}\right) dz + \int_v^t u(c_M^2(z|v, k(v)))dz\right) dvdt.$$

The first term corresponds to all possible joint dates of death for which $v \geq t$ and the second term corresponds to all $v \leq t$. Notice the order of integration of the iterated integrals differs in these terms. The purpose of this appendix is to demonstrate how to transform expected utility—from this almost impenetrable form—into a form for which we can utilize Pontryagin's Maximum Principle.

With a change in the order of integration of the outer integrals in the first term, $\int_0^T \int_0^v (\cdot)\, dtdv = \int_0^T \int_t^T (\cdot)\, dvdt$, we have

$$\mathbb{I} \equiv \int_0^T \int_t^T \left(-\dot{S}_M(t)\right) \left(-\dot{S}_F(v)\right) \left(\int_0^t 2u\left(\frac{c(z)}{2}\right) dz + \int_t^v u(c_F^2(z|t, k(t)))dz\right) dvdt.$$

To compress notation,

$$A_F(t, v) \equiv \int_0^t 2u\left(\frac{c(z)}{2}\right) dz + \int_t^v u(c_F^2(z|t, k(t)))dz.$$

Using integration by parts we have

$$\begin{aligned}
\mathbb{I} &= \int_0^T \left(-\dot{S}_M(t)\right)\left(\int_t^T \left(-\dot{S}_F(v)\right) A_F(t,v)dv\right)dt \\
&= \int_0^T \left(-\dot{S}_M(t)\right)\left(-S_F(T)A_F(t,T) + S_F(t)A_F(t,t) + \int_t^T S_F(v)u(c_F^2(v|t,k(t)))dv\right)dt \\
&= \int_0^T \left(-\dot{S}_M(t)\right)\left(S_F(t)\int_0^t 2u\left(\frac{c(z)}{2}\right)dz + \int_t^T S_F(v)u(c_F^2(v|t,k(t)))dv\right)dt \\
&= \int_0^T \left(-\dot{S}_M(t)\right)\left(S_F(t)\int_0^t 2u\left(\frac{c(z)}{2}\right)dz + U_F^2(t,k(t))\right)dt \\
&= \int_0^T \left(-\dot{S}_M(t)\right)S_F(t)\int_0^t 2u\left(\frac{c(z)}{2}\right)dzdt + \int_0^T \left(-\dot{S}_M(t)\right)U_F^2(t,k(t))dt.
\end{aligned}$$

Likewise, turning to term $\mathbb{III}$, we denote

$$A_M(t,v) \equiv \int_0^v 2u\left(\frac{c(z)}{2}\right)dz + \int_v^t u(c_M^2(z|v,k(v)))dz,$$

and we can rewrite $\mathbb{III}$

$$\mathbb{III} = \int_0^T \left(-\dot{S}_M(t)\right)\left(\int_0^t \left(-\dot{S}_F(v)\right) A_M(t,v)dv\right)dt.$$

With a change in the order of integration , $\int_0^T \int_0^t (\cdot)\, dvdt = \int_0^T \int_v^T (\cdot)\, dtdv$, we have

$$\mathbb{III} = \int_0^T \left(-\dot{S}_F(v)\right)\left(\int_v^T \left(-\dot{S}_M(t)\right) A_M(t,v)dt\right)dv$$

which can be rewritten further using integration by parts

$$\begin{aligned}
\mathbb{III} &= \int_0^T \left(-\dot{S}_F(v)\right)\left(\left(-S_M(T)\right) A_M(T,v) + S_M(v)A_M(v,v) + \int_v^T S_M(t)u(c_M^2(t|v,k(v)))dt\right) dv \\
&= \int_0^T \left(-\dot{S}_F(v)\right)\left(S_M(v)\int_0^v 2u\left(\frac{c(z)}{2}\right) dz + \int_v^T S_M(t)u(c_M^2(t|v,k(v)))dt\right) dv \\
&= \int_0^T \left(-\dot{S}_F(v)\right)\left(S_M(v)\int_0^v 2u\left(\frac{c(z)}{2}\right) dz + U_M^2(v,k(v))\right) dv \\
&= \int_0^T \left(-\dot{S}_F(v)\right) S_M(v)\int_0^v 2u\left(\frac{c(z)}{2}\right) dzdv + \int_0^T \left(-\dot{S}_F(v)\right) U_M^2(v,k(v))dv \\
&= \int_0^T \left(-\dot{S}_F(t)\right) S_M(t)\int_0^t 2u\left(\frac{c(z)}{2}\right) dzdt + \int_0^T \left(-\dot{S}_F(t)\right) U_M^2(t,k(t))dt.
\end{aligned}$$

Now, recall that $\mathbb{E}(U) = \mathbb{I} + \mathbb{III}$, so

$$\begin{aligned}
\mathbb{E}(U) &= \int_0^T \left(-\dot{S}_M(t)\right) S_F(t)\int_0^t 2u\left(\frac{c(z)}{2}\right) dzdt + \int_0^T \left(-\dot{S}_M(t)\right) U_F^2(t,k(t))dt \\
&\quad + \int_0^T \left(-\dot{S}_F(t)\right) S_M(t)\int_0^t 2u\left(\frac{c(z)}{2}\right) dzdt + \int_0^T \left(-\dot{S}_F(t)\right) U_M^2(t,k(t))dt \\
&= \int_0^T \int_0^t \left(\left(-\dot{S}_M(t)\right) S_F(t) + \left(-\dot{S}_F(t)\right) S_M(t)\right) 2u\left(\frac{c(z)}{2}\right) dzdt \\
&\quad + \int_0^T \left(-\dot{S}_M(t)\right) U_F^2(t,k(t))dt + \int_0^T \left(-\dot{S}_F(t)\right) U_M^2(t,k(t))dt.
\end{aligned}$$

And with one final change in the order of integration, $\int_0^T \int_0^t (\cdot)\, dzdt = \int_0^T \int_z^T (\cdot)\, dtdz$, we have

$$\begin{aligned}
\mathbb{E}(U) &= \int_0^T \left(\int_z^T \left(\left(-\dot{S}_M(t)\right) S_F(t) + \left(-\dot{S}_F(t)\right) S_M(t)\right) dt\right) 2u\left(\frac{c(z)}{2}\right) dz \\
&\quad + \int_0^T \left(-\dot{S}_M(t)\right) U_F^2(t,k(t))dt + \int_0^T \left(-\dot{S}_F(t)\right) U_M^2(t,k(t))dt \\
&= \int_0^T \left(-S_M(T)S_F(T) + S_M(z)S_F(z)\right) 2u\left(\frac{c(z)}{2}\right) dz \\
&\quad + \int_0^T \left(-\dot{S}_M(t)\right) U_F^2(t,k(t))dt + \int_0^T \left(-\dot{S}_F(t)\right) U_M^2(t,k(t))dt \\
&= \int_0^T \left(S_M(t)S_F(t)2u\left(\frac{c(t)}{2}\right) + \left(-\dot{S}_M(t)\right) U_F^2(t,k(t)) + \left(-\dot{S}_F(t)\right) U_M^2(t,k(t))\right) dt,
\end{aligned}$$

which completes the derivation of expected utility.

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