Short Planning Horizons and the *Save More Tomorrow* Program*

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Abstract

Designed by Thaler and Benartzi (2004), the *Save More Tomorrow* (SMarT) program enables employees to pre-authorize an automatic escalation of their savings contribution rate into employer-sponsored retirement saving plans, wherein the escalation of the contribution rate typically occurs with the receipt of pay raises. We offer theoretical support for the SMarT program in a model setting with well-functioning credit markets and shortsighted individuals who plan only a fixed number of years into the future. We find that the SMarT saving rate that maximizes lifetime utility is positive, suggesting that the SMarT program increases the welfare of shortsighted individuals. We compute consumption equivalent variations and find that shortsighted individuals with a five-year planing horizon (i) would be better off by an equivalent of 13 percent of lifetime consumption as a result of participating in the SMarT program if they do not have access to a social security program; (ii) would be better off by 9 percent of lifetime consumption as a result of participating in the SMarT program in place of participating in social security; and, (iii) would be better off by 3 percent of lifetime consumption as a result of participating in the SMarT program in tandem with participating in a social security program. The consumption equivalent variations are smaller, although generally still positive, for individuals with longer planning horizons, and the consumption equivalent variations are larger for individuals with shorter planning horizons. Our results contrast with an existing, documented finding in which the SMarT program does not affect the consumption allocations (and therefore does not improve the welfare) of individuals with hyperbolic discount functions, at least in the absence of sizable credit market imperfections.

Keywords: *Save More Tomorrow Program, shortsightedness, short planning horizon, life-cycle consumption and saving, time-inconsistent dynamic optimization, social security*

JEL Classification: C61, D15, D91, H55

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1 Introduction

In this paper we examine theoretically whether or not the Save More Tomorrow (or SMarT) savings program can improve the welfare of program participants who are shortsighted. This is motivated by evidence that some individuals might save too little during their working years, and as such, they can end up in the position of not having adequate resources to finance spending needs during retirement.[1] In order to assist individuals at saving adequately for retirement, Richard Thaler and Shlomo Benartzi (2004) introduced the Save More Tomorrow program. The key feature of the program is that employees commit in the present to save more in the future when they receive a pay raise.[2] The SMarT program is engineered to leverage several behavioral biases in order to better prepare individuals for retirement: employees procrastinate increased saving to the future when raises occur, employees never experience the pain from a drop in take-home pay given that increased saving is synchronized to raises, and lastly, psychological inertia is an impediment to program attrition. Since its inception, the concept of the SMarT program has become popular with potential participants (Helman, Copeland and VanDerhei (2006)), and the program itself has been implemented widely as a feature of employer-sponsored retirement plans.[3] For example, features of the SMarT program were encouraged legislatively as part of the *U.S. Pension Protection Act* which was signed into law in 2006, and 51 percent of employers offered SMarT programs as part of their 401(k) plans as of 2011 (Benartzi and Thaler (2013)).

Empirical evidence suggests that the SMarT program is effective at increasing employee contributions into employer-sponsored savings plans (Benartzi and Thaler (2013)). It has also been projected that the program will provide significant help to households at becoming prepared financially for retirement, especially among low-income earners (VanDerhei (2007)). However, it is still an open question as to whether or not participation in the SMarT program actually increases the *overall* savings of program participants. Indeed, the possibility exists that participants increase their contributions into employer-sponsored retirement savings plans while simultaneously decreasing their holdings of other savings assets and/or while simultaneously incurring added debt. From a theoretical perspective, the

[1] For example, see Benartzi and Thaler (2013), Hanna, Kim and Chen (2016), and the Federal Reserve Report on Economic Well-Being of U.S. Households in 2017 (report number 201805) for the United States. See Burnett, Davis, Murawski, Wilkins and Wilkinson (2018) for Australia, and Knoff, Been, Alessie, Caminada, Goudswaard and Kalwij (2016) for the Netherlands.

[2] The synchronization of increases in the contribution rate with pay raises reflects what Benartzi (2012) describes as "*Save More Tomorrow 2.0*".

[3] Within the financial services industry, the Save More Tomorrow program is usually referred to as an "automatic escalation" feature of the employee contribution rate into an employer-sponsored retirement saving plan.

welfare benefit or cost of participation in a SMarT program is also an open question. We contribute to the effort of answering this question by providing theoretical evidence that the SMarT program can improve the welfare of shortsighted, boundedly-rational individuals.

Thaler and Benartzi (2004) emphasize the phenomenon of hyperbolic discounting as the primary motivation for why the SMarT program might help individuals to save successfully for retirement. Indeed, they state that "Hyperbolic discounting implies that opportunities to save more in the future will be considered more attractive than those in the present" (p.169). Despite this clear emphasis on hyperbolic discounting, there is a recent area of research which documents that individuals do not benefit much, if at all, from such a program in model settings with hyperbolic discounting and well-functioning credit markets. Specifically, if credit markets are well-functioning, then the SMarT program does little or nothing to alter the desired consumption allocations because individuals simply offset savings contributions via the SMarT program by reducing their holdings of other savings assets and/or by going into debt (Findley and Caliendo (2020); Findley and Cottle Hunt (2020)). Moreover, in a life-cycle model setting with shortsightedness compounded with hyperbolic discounting, Findley and Caliendo (2014) report that the presence of hyperbolic discounting does not affect consumption allocations. This suggests that shortsightedness, rather than hyperbolic discounting, could be a promising dimension along which to explore the potential role of the SMarT program at improving welfare by helping individuals to become better prepared for retirement.

We construct a life-cycle model of consumption and saving that is representative of shortsighted (myopic) individuals who plan only a fixed number of periods into the future.[4] The presence of short planning horizons triggers time-inconsistent decision making (time-inconsistent dynamic optimization) by individuals. This is due to the fact that shortsighted individuals fail to account for that part of the future which is beyond the endpoint of their planning horizons; yet as time progresses, the planning horizon slides forward and incorporates new information, which can lead to frequent re-optimization as an individual ages. The short planning horizon mechanism has been documented to match several key empirical observations from the life-cycle experience, including a hump-shaped age-consumption profile and a drop in consumption at retirement (see Park and Feigenbaum (2018) for an overview).

The idea that households have a short planning horizon when formulating consumption and saving

[4] Our model representation of the SMarT program follows Findley and Caliendo (2007, 2010), nested within the short planning horizon life-cycle framework of Caliendo and Aadland (2007) and Findley and Caliendo (2009).

plans was supported at least as early as Strotz (1956) and Friedman (1963). Indeed, Carroll and Summers (1991; p.307,335) report,

> We suggest that both our data and the available time-series evidence are consistent with Milton Friedman's view that people save to smooth consumption over several years but, because of liquidity constraints, caution, or shortsightedness do not seek to smooth consumption over longer horizons... Indeed, Milton Friedman explicitly rejected the idea that consumers had horizons as long as a lifetime in discussing the permanent income hypothesis.

Moreover, Carroll (2001) reports on Milton Friedman's claim that the marginal propensity to consume from transitory income is one-third, which implies that households have a 3-year planning horizon in the context of the permanent income hypothesis. There is also some empirical evidence that is consistent with the idea that people have short planning horizons when making key financial decisions. For example, the 1992 wave of the Health and Retirement Study reports the stated financial planning horizons of respondents age fifty and over in the United States: 61 percent of respondents report that "the next few months", "next year", or "next few years" is the "time period that is most important" when it comes to "planning your family's saving and spending". The remaining 30 percent of respondents reported "the next five to ten years", while only 9 percent reported a financial planning horizon of "more than ten years". But of particular interest to us, including a short planning horizon is an intuitive way to modify the life-cycle consumption and saving framework such that it can easily generate predictions of insufficient saving for retirement.

Using our life-cycle model with short planning horizons, we document that shortsighted individuals can easily benefit from participation in the SMarT program given the range of short planning horizons that we consider (from 1-year up to 15-year planning horizons). The SMarT program increases the retirement savings of these individuals in addition to raising their consumption possibilities during retirement. The welfare benefits from participating in the SMarT program can be quite large: a shortsighted individual who plans only one year into the future would be better off by an equivalent of 50 percent of lifetime consumption as a result of participating in the SMarT program with a 20 percent contribution rate from pay raises. The welfare gains are smaller in magnitude for individuals with longer planning horizons, but still significantly positive: a shortsighted individual who plans 15 years into the future would be better off by 1 percent of lifetime consumption from participating in the program with a 10 percent contribution rate from pay raises. Moreover, the welfare gains from participation in the SMarT program generally increase as the contribution rate from pay raises increases.

We find that shortsighted individuals in our model can also benefit from government-operated pensions, such as a pay-as-you-go social security program. Indeed, we explore the relationship between pay-as-you-go social security and the SMarT program with two types of exercises. First, we find that shortsighted individuals are better off from participating in the SMarT savings program as opposed to a pay-as-you-go social security program that has a replacement rate that is parameterized to that of the United States. This finding is robust for the entire range of short planning horizons and SMarT contribution rates that we consider (1-year to 15-year planning horizons; SMarT contribution rates all the way up to 100 percent of pay raises). Second, if the planning horizon is less than five years, then we always find an improvement in welfare as a result of added participation in the SMarT program (for the entire range of SMarT contribution rates) when the individual already participates in a pay-as-you-go social security program. We also document an improvement in welfare across most of the range of SMarT contribution rates when the length of the planning horizon is between five and fifteen years. These exercises suggest that the SMarT program is an excellent substitute for a pay-as-you-go social security program, and the SMarT program is also an excellent complement to pay-as-you-go social security.

The mechanism that underlies these findings is that shortsighted individuals do not anticipate their needed retirement resources (along with the associated loss of wage income) until just a few years before retirement occurs. For example, if retirement occurs at age sixty-five and if an individual has a 5-year planning horizon, then he or she does not anticipate retirement until age sixty. Absent participation in the SMarT program, the individual will have accumulated little savings by age sixty, and as a result, the individual will respond by reducing consumption dramatically in a desperate attempt to accumulate enough assets for retirement. But in a world with participation in the SMarT program, the accumulated saving assets from participation are valued highly by the individual at age sixty and beyond. The shortsighted individual is aware of his contributions into the SMarT savings asset throughout the working phase of the life cycle. But since the accumulated savings from SMarT participation are not accessed until after retirement has occurred, the accumulated savings from the SMarT program do not become salient until the individual gets close to retirement. As such, shortsighted individuals do not unwind very much, if any, of their SMarT contributions via reducing their holdings of other savings assets. This theoretical feature of our model is conceptually consistent with recent empirical evidence from Denmark in which households are “passive savers” because they do not materially adjust their household balance sheet as a result of changes in the tax treatment of saving contributions to retirement accounts (Chetty, Friedman, Leth-Petersen, Nielsen and Olsen (2014)).

Mechanically, our findings hinge on the modeling assumption that shortsighted individuals intend to spend down their liquid savings (or to repay any accumulated debt) by that future point in time that corresponds to the endpoint on each of their respective planning horizons. This modeling assumption is the embodiment of myopia or shortsightedness. This is due to the fact that if individuals with short planning horizons intend instead to hold assets at future dates beyond the endpoint of their planning horizons, then this would imply that they are planning for future consumption beyond the endpoint or that they have a bequest motive. Either way, this would indicate that they are not truly shortsighted.[5]

2 Model

Age is continuous and indexed by t. The representative individual starts work at age $t = 0$, retires at age $t = T$, and passes away with certainty at age $t = \bar{T}$. The individual receives the wage-income flow $w(t) = wq(t)$ for $t \in [0, T]$ where w is the market wage and $q(t)$ is a longitudinal age-efficiency profile. We model $q(t)$ as a fourth-order polynomial to capture the increasing hump-shaped pattern of wage-income over the life cycle, although our results are qualitatively similar when we use other specifications. Individuals pay social security taxes at rate θ all throughout the working phase of the life cycle, and they receive social security benefits that are illiquid, $b = \int_0^T \theta w(t)dt/(\bar{T} - T)$ for $t \in [T, \bar{T}]$, which embodies pay-as-you-go financing of the social security program. Consumption at each instant in time, $c(t)$, is the control variable.

Individuals have two separate types of private savings accounts. The first asset account balance, $S(t)$, is illiquid and it is funded via an employer-sponsored retirement saving plan (e.g., a 401(k) plan) with a Save More Tomorrow program feature, wherein a higher contribution rate is applied to pay raises. The asset balance is annuitized at retirement by the employer-sponsored retirement plan

[5]See Park and Feigenbaum (2018) for an example of a short planning horizon model in which individuals are not shortsighted: individuals have full information about what will happen beyond the endpoint of their planning horizons, yet they simply do not care about that part of their future when it comes to their active decision making in the present.

provider. The other account, $k(t)$, is liquid and it is directly managed by individuals themselves. Both types of accounts accrue the real market rate of interest, r. With boundary conditions $S(0) = 0$ and $S(\bar{T}) = 0$, the SMarT savings asset balance evolves according to the laws of motion

$$\frac{dS(t)}{dt} = rS(t) + \gamma[w(t) - w(0)], \text{ for } t \in [0, T], \tag{1}$$

$$\frac{dS(t)}{dt} = rS(t) - A, \text{ for } t \in [T, \bar{T}], \tag{2}$$

where γ is the SMarT savings contribution rate and A is the constant real annuity that exhausts the SMarT savings asset balance by the date of death. Given these laws of motion and boundary conditions, the constant real annuity received during retirement is identified as

$$A = \frac{\int_0^T \gamma[w(t) - w(0)] \exp[r(T-t)]dt}{\int_T^{\bar{T}} \exp[r(T-t)]dt}, \text{ for } t \in [T, \bar{T}]. \tag{3}$$

In this paper we abstract from the higher-order problem of whether or not shortsighted individuals choose to participate in the SMarT program. We assume that individuals are enrolled in the program via their employer-sponsored retirement saving plan as soon as they start working, which is consistent with the recent literature on the power of default settings, like automatic enrollment into 401(k) plans by plan sponsors (e.g., Choi, Laibson, Madrian, and Metrick (2004)). Moreover, we assume participation in the program occurs during the working years, which is motivated by Thaler and Benartzi's observation that, "It seems as though inertia plays the most dominant role in the program, where defaulting employees into the program results in nearly universal participation" (Benartzi, Peleg, and Thaler (2012, p.252)). Thaler and Benartzi (2004) report that 98 percent of participants remained in the SMarT program after two wage increases in the initial experiments. We assume that any disposal income (net of payroll taxation and employer-sponsored SMarT program contributions) that is not consumed will flow into the liquid savings account, $k(t)$, which has the boundary conditions $k(0) = 0$ and $k(\bar{T}) = 0$.

We assume that the length of a given individual's planning horizon, x, is less than or equal to the length of the retirement period (i.e., the restriction $x \leq \bar{T} - T$ is imposed for mathematical convenience). This restriction is employed to improve the tractability of the model. The life span of a given individual is therefore partitioned into four distinct phases:

- Phase 1: the individual is working and is not aware of retirement, $t \in [0, T - x]$
- Phase 2: the individual is working and is aware of retirement, $t \in [T - x, T]$
- Phase 3: the individual is retired and is not aware of death, $t \in [T, \bar{T} - x]$
- Phase 4: the individual is retired and is aware of death, $t \in [\bar{T} - x, \bar{T}]$

A given individual's saving and consumption problem can be solved for each phase, using the appropriate boundary conditions and laws of motion for the liquid savings account, $k(t)$. The time-inconsistent dynamic optimization problem for each phase is characterized below.

2.1 Phase 1

A given individual solves the following interior optimal control problem at each and every planning instant $t_0 \in [0, T - x]$,

$$\max \int_{t_0}^{t_0+x} \exp[-\rho(t - t_0)] u[c(t)] \, dt, \tag{4}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + y(t) - c(t), \text{ for } t \in [t_0, t_0 + x], \tag{5}$$

$$k(t_0) = \int_0^{t_0} [y(t) - c_1^*(t;t)] \exp[r(t_0 - t)] dt, \tag{6}$$

$$k(t_0 + x) = 0, \tag{7}$$

where $y(t) \equiv (1 - \theta)w(t) - \gamma[w(t) - w(0)]$ is disposable income and where $u[c(t)]$ is the instantaneous utility function in general form with the properties $u_c[c(t)] > 0$ and $u_{cc}[c(t)] < 0$. Note that the symbol "$*$" denotes the *actual* path of a choice variable (consumption in this case) during the respective phase of the partitioned life cycle. Note also from (6) that actual consumption occurring in the past on the interval $[0, t_0]$ is taken into account in the decision problem from the perspective of any planning instant. This reflects the fact that the current state or balance of the liquid asset account is the result of all past consumption and saving choices.

The time-inconsistent dynamic optimization problem given by (4)–(7) can be solved via the *Maximum Principle* for one-stage optimal control problems with a fixed-endpoint condition. The maximum condition and costate equation together yield

$$\exp[-\rho(t-t_0)]u_c[c(t)] = \lambda(t_0)\exp[r(t_0-t)], \tag{8}$$

given a continuously differentiable costate variable $\lambda(t)$, where $\lambda(t_0)$ is a constant of integration. Given the assumed, standard properties of the instantaneous utility function, the marginal utility of consumption, $u_c[c(t)]$, is a one-to-one mapping from consumption. Therefore, $u_c[c(t)]$ has an inverse $u_c^{-1}[c(t)]$, and the planned consumption path is

$$\hat{c}_1(t;t_0) = u_c^{-1}\left[\lambda(t_0)\exp[(\rho-r)t+(r-\rho)t_0]\right], \text{ for } t\in[t_0,t_0+x], \tag{9}$$

conditional on the perspective of any planning instant $t_0 \in [0, T-x]$. Note that the symbol "$\wedge$" on a choice variable denotes the *planned* or *intended* time path that is never fully followed because of time-inconsistent dynamic re-optimization. Combining (9) with (5)–(6) yields the intended path of the liquid savings account balance

$$\hat{k}_1(t) = k(t_0)\exp[r(t-t_0)] + \int_{t_0}^{t}\left(y(v) - u_c^{-1}\left[\lambda(t_0)\exp[(\rho-r)v+(r-\rho)t_0]\right]\right)\exp[r(t-v)]dv, \tag{10}$$

for $t \in [t_0, t_0+x]$ conditional on the perspective of any $t_0 \in [0, T-x]$ where v is a dummy variable of integration. Using the fixed-endpoint condition (7) yields

$$k(t_0)\exp[-rt_0] + \int_{t_0}^{t_0+x} y(v)\exp[-rv]dv = \int_{t_0}^{t_0+x} u_c^{-1}\left[\lambda(t_0)\exp[(\rho-r)v+(r-\rho)t_0]\right]\exp[-rv]dv \tag{11}$$

which would definitize $\lambda(t_0)$ in (9) if an explicit functional form is selected for $u[c(t)]$.

The consumption program that a given individual perceives to be optimal and intends to follow for all $t \in [t_0, t_0+x]$, conditional on the perspective of $t_0 \in [0, T-x]$, is given by equation (9) in which $\lambda(t_0)$ is definitized in principle. Yet, a defining feature of the model is that the individual is free to re-optimize as age advances, given that new information is taken into account as an individual's planning horizon slides forward with age. Therefore, the *actual* consumption of a given individual at the instant of planning is $\hat{c}_1(t_0;t_0)$ which is the result of evaluating $t = t_0$ in (9). It should be mentioned again that t_0 represents any arbitrary vantage point of decision making on the interval

$[0, T-x]$. This suggests that the actual consumption of the individual for all $t \in [0, T-x]$ can be identified by replacing all t_0 in $\hat{c}_1(t_0; t_0)$ with t, which we denote as

$$c_1^*(t; t) = u_c^{-1}\left[\lambda(t)\right], \text{ for } t \in [0, T-x], \tag{12}$$

given $\lambda(t)$ which solves (11) with $t_0 = t$,

$$k\left(t\right)\exp[-rt] + \int_t^{t+x} y(v)\exp[-rv]dv = \int_t^{t+x} u_c^{-1}\left[\lambda(t)\exp[(\rho - r)v + (r-\rho)t]\right]\exp[-rv]dv \tag{13}$$

Equation (12) identifies a given individual's actual consumption at each and every age $t \in [0, T-x]$. It is important to recognize that this expression is, in fact, an implicit function of the actual liquid savings account balance at time t, which in turn is a function of the time path of actual consumption on the interval $[0, t]$ via

$$\frac{dk_1^*(t)}{dt} = rk_1^*(t) + y(t) - c_1^*(t; t). \tag{14}$$

where $k_1^*(t)$ is the *actual* time path of the liquid savings account balance during Phase 1. Therefore, to completely identify the actual consumption of a given individual at each and every age $t \in [0, T-x]$, the system of equations needs to be solved: (12) given (13), along with (14) given $k_1^*(0) = 0$. Solving this system of equations characterizes the entire time paths of the *actual* consumption and saving choices of the individual. Further progress requires an explicit form for $u[c(t)]$.

2.2 Phase 2

A given individual solves the following interior optimal control problem at each and every planning instant $t_0 \in [T-x, T]$,

$$\max \int_{t_0}^{t_0+x} \exp[-\rho(t-t_0)]u[c(t)]\ dt, \tag{15}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + y(t) - c(t) \quad \text{for } t \in [t_0, T] \tag{16}$$

$$\frac{dk(t)}{dt} = rk(t) + A + b - c(t) \quad \text{for } t \in [T, t_0 + x] \tag{17}$$

$$k\left(t_0\right)=\int_0^{T-x}\left[y(t)-c_1^*(t;t)\right]\exp[r(t_0-t)]dt+\int_{T-x}^{t_0}\left[y(t)-c_2^*(t;t)\right]\exp[r(t_0-t)]dt, \tag{18}$$

$$k(t_0+x)=0. \tag{19}$$

Using the *Maximum Principle* for two-stage fixed-endpoint optimal control problems, the maximum conditions, costate equations, and switch-point condition together yield

$$\exp[-\rho(t-t_0)]u_c[c(t)]=\lambda(t_0)\exp[r(t_0-t)], \tag{20}$$

given a continuously differentiable costate variable $\lambda(t)$, where $\lambda(t_0)$ is again a constant of integration.[6] With $u_c^{-1}[c(t)]$ again denoting the inverse of marginal utility, the planned consumption path is

$$\hat{c}_2(t;t_0)=u_c^{-1}\left[\lambda(t_0)\exp[(\rho-r)t+(r-\rho)t_0]\right], \text{ for } t\in[t_0,t_0+x], \tag{21}$$

from the perspective of any planning instant $t_0\in[T-x,T]$. Combining (21) with (16)–(19) yields the intended path for the liquid savings account balance,

$$\hat{k}_2(t)=k\left(t_0\right)\exp[r(t-t_0)]+\int_{t_0}^{t}\left(y(v)-u_c^{-1}\left[\lambda(t_0)\exp[(\rho-r)v+(r-\rho)t_0]\right]\right)\exp[r(t-v)]dv, \tag{22}$$

for $t\in[t_0,T]$, and

$$\hat{k}_2(t)=\int_{t_0+x}^{t}\left(A+b-u_c^{-1}\left[\lambda(t_0)\exp[(\rho-r)v+(r-\rho)t_0]\right]\right)\exp[r(t-v)]dv, \tag{23}$$

for $t\in[T,t_0+x]$, conditional on the perspective of any planning instant $t_0\in[T-x,T]$. Evaluating (22) and (23) at $t=T$ and then equating yields

$$k\left(t_0\right)\exp[-rt_0]+\int_{t_0}^{T}y(v)\exp[-rv]dv+\int_{T}^{t_0+x}(A+b)\exp[-rv]dv$$

$$=\int_{t_0}^{t_0+x}u_c^{-1}\left[\lambda(t_0)\exp[(\rho-r)v+(r-\rho)t_0]\right]\exp[-rv]dv \tag{24}$$

which definitizes $\lambda(t_0)$ in (21) if an explicit functional form is selected for $u[c(t)]$.

[6] See Tomiyama (1985) and Caliendo and Pande (2005) for more on the technical details of two-stage optimal control problems.

Planned consumption, $\hat{c}_2(t;t_0)$, as given by (21) in which $\lambda(t_0)$ is definitized in principle, is the consumption program that a given individual perceives to be optimal and plans on following for all $t \in [t_0, t_0 + x]$ conditional on the planning perspective of $t_0 \in [T - x, T]$. Yet, remember that the individual is free to re-optimize as the short planning horizon slides forward with age and incorporates new information. Following the process described in Phase 1, actual consumption during Phase 2 is therefore

$$c_2^*(t;t) = u_c^{-1}\left[\lambda(t)\right], \text{ for } t \in [T - x, T], \tag{25}$$

given $\lambda(t)$ which solves (24) with $t_0 = t$,

$$k\left(t\right)\exp[-rt] + \int_t^T y(v)\exp[-rv]dv + \int_T^{t+x}(A + b)\exp[-rv]dv$$
$$= \int_t^{t+x} u_c^{-1}\left[\lambda(t)\exp[(\rho - r)v + (r - \rho)t]\right]\exp[-rv]dv. \tag{26}$$

Recognize that (25) identifies a given individual's actual consumption at each and every age $t \in [T - x, T]$. But it is also important to also recognize that this expression is an implicit function of the actual liquid savings account balance at time t, which in turn is a function of the time path of actual consumption via

$$\frac{dk_2^*(t)}{dt} = rk_2^*(t) + y(t) - c_2^*(t;t). \tag{27}$$

Therefore, to completely identify actual consumption at each and every age $t \in [T - x, T]$, the system of equations needs to be solved: (25) given (26), along with (27) given the true initial condition $k_1^*(T - x) = k_2^*(T - x)$. Solving this system of equations characterizes the time paths of the *actual* consumption and saving choices during Phase 2. Further progress requires an explicit form for $u[c(t)]$.

2.3 Phase 3

A given individual solves the following interior optimal control problem at each and every planning instant $t_0 \in [T, \bar{T} - x]$,

$$\max \int_{t_0}^{t_0+x} \exp[-\rho(t - t_0)]u[c(t)]\ dt, \tag{28}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + A + b - c(t), \text{ for } t \in [t_0, t_0 + x], \tag{29}$$

$$\begin{aligned} k(t_0) &= \int_0^{T-x} [y(t) - c_1^*(t;t)] \exp[r(t_0 - t)]dt + \int_{T-x}^{T} [y(t) - c_2^*(t;t)] \exp[r(t_0 - t)]dt \\ &\quad + \int_T^{t_0} [A + b - c_3^*(t;t)] \exp[r(t_0 - t)]dt, \end{aligned} \tag{30}$$

$$k(t_0 + x) = 0. \tag{31}$$

Again, similar to the procedure outlined in Phase 1 and Phase 2 above, applying the Maximum Principle yields the intended consumption path,

$$\hat{c}_3(t;t_0) = u_c^{-1} [\lambda(t_0) \exp[(\rho - r)t + (r - \rho)t_0]], \text{ for } t \in [t_0, t_0 + x], \tag{32}$$

conditional on the perspective of any planning instant $t_0 \in [T, \bar{T} - x]$. Combining (32) with (29)–(30) yields the intended path for the liquid savings account balance,

$$\hat{k}_3(t) = k(t_0) \exp[r(t - t_0)] + \int_{t_0}^{t} \left(A + b - u_c^{-1} [\lambda(t_0) \exp[(\rho - r)v + (r - \rho)t_0]]\right) \exp[r(t - v)]dv, \tag{33}$$

for $t \in [t_0, t_0 + x]$ conditional on the perspective of any $t_0 \in [T, \bar{T} - x]$. Using the endpoint condition, (31), yields

$$k(t_0) \exp[-rt_0] + \int_{t_0}^{t_0+x} (A + b) \exp[-rv]dv = \int_{t_0}^{t_0+x} u_c^{-1} [\lambda(t_0) \exp[(\rho - r)v + (r - \rho)t_0]] \exp[-rv]dv \tag{34}$$

which definitizes $\lambda(t_0)$ in (32) given an explicit form for $u[c(t)]$.

Similar to the process outlined in Phase 1 and Phase 2 above, the actual consumption path is

$$c_3^*(t;t) = u_c^{-1} [\lambda(t)], \text{ for } t \in [T, \bar{T} - x], \tag{35}$$

given $\lambda(t)$ which solves (34) with $t_0 = t$,

$$k(t) \exp[-rt] + \int_t^{t+x} (A + b) \exp[-rv]dv = \int_t^{t+x} u_c^{-1} [\lambda(t) \exp[(\rho - r)v + (r - \rho)t]] \exp[-rv]dv. \tag{36}$$

With $\lambda(t)$ definitized in principle, $c_3^*(t;t)$ is an implicit function of the actual liquid savings account balance at time t, which in turn is a function of the time path of actual consumption via

$$\frac{dk_3^*(t)}{dt} = rk_3^*(t) + A + b - c_3^*(t;t). \tag{37}$$

Therefore, to completely identify the time paths of the actual consumption and saving choices at each and every age $t \in \left[T, \bar{T} - x\right]$, the system of equations needs to be solved: (35) given (36), along with (37) given the true initial condition, $k_2^*(T) = k_3^*(T)$. Further progress again requires an explicit form for $u[c(t)]$.

2.4 Phase 4

Given our assumption that the endpoint on the planning horizon does not extend beyond the date of death, at $t_0 = \bar{T} - x$ a given individual solves a standard optimal control problem

$$\max \int_{\bar{T}-x}^{\bar{T}} \exp[-\rho(t - (\bar{T} - x))]u[c(t)]\ dt, \tag{38}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + A + b - c(t), \text{ for } t \in \left[\bar{T} - x, \bar{T}\right], \tag{39}$$

$$\begin{aligned} k\left(\bar{T} - x\right) \quad = \quad & \int_0^{T-x} [y(t) - c_1^*(t;t)] \exp[r(\bar{T} - x - t)]dt + \int_{T-x}^{T} [y(t) - c_2^*(t;t)] \exp[r(\bar{T} - x - t)]dt \\ & + \int_T^{\bar{T}-x} [A + b - c_3^*(t;t)] \exp[r(\bar{T} - x - t)]dt, \end{aligned} \tag{40}$$

$$k(\overline{T}) = 0. \tag{41}$$

The solution to this control problem is the actual consumption path. A straightforward method to obtain this path is by setting $t_0 = \bar{T} - x$ in (32) wherein $\lambda(\bar{T} - x)$ is definitized in principle. This yields

$$c_4^*(t) = \hat{c}_3(t; \bar{T} - x) = u_c^{-1}\left[\lambda(\bar{T} - x) \exp[(\rho - r)t + (r - \rho)(\bar{T} - x)]\right], \text{ for } t \in [\bar{T} - x, \bar{T}]. \tag{42}$$

With $\lambda(\bar{T}-x)$ definitized in principle, the actual path for the liquid savings account balance is

$$\begin{aligned} k_4^*(t) &= k\left(\bar{T}-x\right)\exp[r(t-(\bar{T}-x))] \\ &\quad + \int_{\bar{T}-x}^{t}\left(A+b-u_c^{-1}\left[\lambda(\bar{T}-x)\exp[(\rho-r)v+(r-\rho)(\bar{T}-x)]\right]\right)\exp[r(t-v)]dv, \quad (43) \end{aligned}$$

for $t \in [\bar{T}-x, \bar{T}]$. But, of course, an explicit form for $u[c(t)]$ is needed in order to definitize $\lambda(\bar{T}-x)$.

3 Analytical Work and Numerical Examples

3.1 Specific Functional Forms

Given that the solutions to our theoretical model are general enough for any instantaneous utility function with the properties $u_c[c(t)] > 0$ and $u_{cc}[c(t)] < 0$, all that is needed for quantitative simulation is to specify the form of the function. For the remainder of this paper, we assume that the utility function takes the isoelastic form commonly used in quantitative research,

$$u[c(t)] = \frac{c(t)^{1-\phi}-1}{1-\phi}, \quad (44)$$

in which ϕ is the inverse elasticity of intertemporal substitution. Therefore, the solutions for the paths of consumption and the liquid asset account balance across each of the four phases of the life cycle can be solved explicitly and expressed analytically, as documented in the Appendix.

Phase 1

$$\hat{c}_1(t) = \exp[((r-\rho)/\phi)t]\left(\frac{k(t_0)\exp[-rt_0]+\int_{t_0}^{t_0+x} y(v)\exp[-rv]dv}{\int_{t_0}^{t_0+x}\exp[((r-\rho)/\phi-r)v]dv}\right), \text{ for } t \in [t_0, t_0+x], \quad (45)$$

$$c_1^*(t) = \exp[((r-\rho)/\phi)t]\left(\frac{k_1^*(t)\exp[-rt]+\int_{t}^{t+x} y(v)\exp[-rv]dv}{\int_{t}^{t+x}\exp[((r-\rho)/\phi-r)v]dv}\right), \text{ for } t \in [0, T-x], \quad (46)$$

$$
\begin{aligned}
k_1^*(t) \;=\; & \exp\left[\int_0^t \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right) dj\right] \\
& \times \left(\int_0^t \left[y(v) - \left(\frac{\exp[((r-\rho)/\phi)v]\int_v^{v+x} y(s)\exp[-rs]ds}{\int_v^{v+x}\exp[((r-\rho)/\phi - r)s]ds}\right)\right]\right. \\
& \left.\times \exp\left[-\int_0^v \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right) dj\right] dv\right), \text{ for } t \in [0, T-x]. \quad (47)
\end{aligned}
$$

Phase 2

$$
\hat{c}_2(t) = \exp[((r-\rho)/\phi)t]\left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{T}\exp[-rv]y(v)dv + \int_{T}^{t_0+x}\exp[-rv](A+b)dv}{\int_{t_0}^{t_0+x}\exp[((r-\rho)/\phi - r)v]dv}\right), \quad (48)
$$

for $t \in [t_0, t_0 + x]$,

$$
c_2^*(t) = \exp[((r-\rho)/\phi)t]\left(\frac{k_2^*(t)\exp[-rt] + \int_{t}^{T}\exp[-rv]y(v)dv + \int_{T}^{t+x}\exp[-rv](A+b)dv}{\int_{t}^{t+x}\exp[((r-\rho)/\phi - r)v]dv}\right), \quad (49)
$$

for $t \in [T - x, T]$,

$$
\begin{aligned}
k_2^*(t) = & \exp\left[\int_{T-x}^t \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right) dj\right] k_2^*(T-x) \\
& + \left(\int_{T-x}^t \left[y(v) - \frac{\left(\int_v^T y(s)\exp[-rs]ds + \int_T^{v+x}(A+b)\exp[-rs]ds\right)\exp[((r-\rho)/\phi)v]}{\int_v^{v+x}\exp[((r-\rho)/\phi - r)s]ds}\right]\right. \\
& \left.\times \exp\left[-\int_{T-x}^v \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right) dj\right] dv\right) \\
& \times \exp\left[\int_{T-x}^t \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right) dj\right], \text{ for } t \in [T-x, T]. \quad (50)
\end{aligned}
$$

Phase 3

$$
\hat{c}_3(t) = \exp[((r-\rho)/\phi)t]\left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{t_0+x}(A+b)\exp[-rv]dv}{\int_{t_0}^{t_0+x}\exp[((r-\rho)/\phi - r)v]dv}\right), \text{ for } t \in [t_0, t_0 + x], \quad (51)
$$

$$c_3^*(t) = \exp[((r-\rho)/\phi)t] \left(\frac{k_3^*(t)\exp[-rt] + \int_t^{t+x}(A+b)\exp[-rv]dv}{\int_t^{t+x}\exp[((r-\rho)/\phi - r)v]dv} \right), \text{ for } t \in [T, \bar{T}-x], \tag{52}$$

$$\begin{aligned} k_3^*(t) = & \exp\left[\int_T^t \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds} \right) dj \right] \\ & \times \left(k_3^*(T) + \int_T^t \left[A + b - \frac{\exp[((r-\rho)/\phi)v]\int_v^{v+x}(A+b)\exp[-rs]ds}{\int_v^{v+x}\exp[((r-\rho)/\phi - r)s]ds} \right] \right. \\ & \left. \times \exp\left[-\int_T^v \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds} \right) dj \right] dv \right), \text{ for } t \in [T, \bar{T}-x]. \end{aligned} \tag{53}$$

Phase 4

$$c_4^*(t) = \exp[(r-\rho)/\phi)t] \left(\frac{k_4^*(\bar{T}-x)\exp[rx] + \int_{\bar{T}-x}^{\bar{T}}(A+b)\exp[r(\bar{T}-v)]dv}{\int_{\bar{T}-x}^{\bar{T}}\exp[((r-\rho)/\phi - r)v + r\bar{T}]dv} \right), \text{ for } t \in [\bar{T}-x, \bar{T}], \tag{54}$$

$$\begin{aligned} k_4^*(t) \quad = \quad & k_4^*\left(\bar{T}-x\right)\exp[r(t-(\bar{T}-x))] + \int_{\bar{T}-x}^t (A+b)\exp[r(t-v)]dv \\ & + \int_t^{\bar{T}-x} \left(\frac{k_4^*\left(\bar{T}-x\right)\exp[rx] + \int_{\bar{T}-x}^{\bar{T}}(A+b)\exp[r(\bar{T}-s)]ds}{\int_{\bar{T}-x}^{\bar{T}}\exp[((r-\rho)/\phi - r)s + ((\rho-r)/\phi)(\bar{T}-x) + r\bar{T}]ds} \right) \\ & \times \exp[((r-\rho)/\phi - r)v + ((\rho-r)/\phi)(\bar{T}-x) + rt] \ dv, \text{ for } t \in [\bar{T}-x, \bar{T}]. \end{aligned} \tag{55}$$

3.2 Parameterization

Baseline parameter values are summarized in Table 1. Following Findley and Caliendo (2007), we set $T = 40$ and $\bar{T} = 55$ in order to model a given individual who starts work at age twenty-five, retires at age sixty-five, and passes away at eighty. We set the real interest rate, r, and the discount rate, ρ, to 3.5 percent. We set the inverse elasticity of intertemporal substitution to $\phi = 1$ (implying that instantaneous utility is logarithmic), which is conventional (e.g., Gourinchas and Parker (2002), Bullard and Feigenbaum (2007), and Feigenbaum (2008)). The wage profile, $w(t) = wq(t)$, is parameterized with a normalization of $w = 1$, and the longitudinal age-efficiency profile, $q(t)$, is modeled as a fourth-order polynomial based on the parameter values reported in Feigenbaum (2008) and Feigenbaum and Caliendo (2010),

$$q(t) = 1 + 0.018095t + 0.000817t^2 - 5.1 \times 10^{-5}t^3 + 5.36 \times 10^{-7}t^4. \quad (56)$$

The baseline value for the social security tax rate, θ, is 10.6 percent, which reflects both the employer and employee contributions to the program. This value for the tax rate implies a benefit replacement rate of 35.5 percent relative to the initial wage. The social security program is financed on a pay-as-you-go basis. As such, it provides a below-market internal rate of rate of return from participation, which is conventional to study.

3.3 Life-cycle Consumption Profiles

It is useful to examine generated profiles for life-cycle consumption and for the liquid savings asset account balance. As an example, we consider the life-cycle consumption paths of shortsighted individuals who do not participate in a social security program and who do not participate in a SMarT program. Figure 1 plots the wage income profile as a solid blue line and the life-cycle consumption profiles as dashed lines for otherwise identical individuals who differ only with respect to the lengths of their planning horizons (yellow dashed line corresponds to a 1-year planning horizon; green dashed line corresponds to a 5-year horizon; red dashed line corresponds to a 10-year horizon; purple dashed line corresponds to a 15-year horizon). The life-cycle consumption profiles track the income profile closely when the planning horizon is short. For longer planning horizons, the consumption profiles are somewhat smoother. Consider an individual who has a 5-year planning horizon (green dashed line). Consumption closely follows income for the first thirty-five years (from age twenty-five to age sixty). But when the endpoint on the planning horizon crosses over the retirement threshold at age sixty (meaning that Phase 1 ends and Phase 2 begins given this particular planning horizon length), then the individual starts to scale back consumption and increase savings in the liquid account due to anticipating the loss of wage-income in retirement. And once the future date of death is anticipated (Phase 3 ends and Phase 4 begins), the individual smooths consumption over the short remainder of the life cycle. The lesson to learn from Figure 1 is that individuals with longer planning horizons are in a better position to have a life-cycle consumption profile that is relatively smoother, on account that they anticipate retirement sooner, and thus experience a smaller decline in consumption upon reaching retirement.

The life-cycle profiles for the liquid savings account balances of individuals who do not participate in social security and the SMarT program are presented in Figure 2. These depicted paths for the liquid savings account balances correspond to the consumption paths depicted in Figure 1. An individual with a shorter planning horizon does not borrow or save as much in the liquid savings account compared to an otherwise identical individual who has a longer planning horizon. This is due to the characteristic of the model that shortsighted individuals plan to deplete savings (or repay any accumulated debt) by that date in the future that corresponds to the endpoint on their respective planning horizon. Consider again a shortsighted individual who has a 5-year planning horizon (green dashed line). The individual borrows during the early years of the life cycle because wages initially increase with age. But then the individual starts to pay off debt and accumulate savings in the liquid asset account. As described above, the individual begins to save drastically when the future date of retirement enters the planning horizon. And the balance of the liquid savings account reaches its peak value at the date of retirement as the individual retires and begins to draw down the account balance.

In Figure 3 we plot the life-cycle consumption profiles of otherwise identical individuals who participate in the SMarT program under different parametric assumptions, yet they do not participate in a social security program. These individuals all have 5-year planning horizons, but they participate in the SMarT program with different contribution rates (the yellow dashed line corresponds to a SMarT contribution rate of 1 percent; the green dashed line corresponds to a SMarT rate of 10 percent; the red dashed line corresponds to a SMarT rate of 25 percent; the purple dashed line corresponds to a SMarT rate of 50 percent). It is readily apparent that shortsighted individuals with higher SMarT saving rates (those who contribute a higher fraction of pay raises into the illiquid asset account that is employer-sponsored) will have life-cycle consumption profiles that move in the direction of becoming somewhat smoother, which is due to the fact that they contribute more while young and working, but then they receive a larger payout from their illiquid asset account during retirement. We should mention that individuals have access to perfect credit markets in the model, such that they can borrow against their future wealth. However, because individuals are shortsighted, their wealth received during retirement is not salient to their decision making in the present, given that their retirement wealth lies beyond the endpoint of their short planning horizons for most of the working years. Thus, shortsighted individuals do not borrow against their future retirement wealth in the SMarT program even though they are able to do so, which means that they do not generally unwind their accumulated savings that result from participation.

4 Optimal SMarT Rate for Shortsighted Individuals

We are interested in examining whether or not participation in the SMarT program can improve the well-being of shortsighted individuals. One way to demonstrate the impact of the SMarT program on well-being is to identify the particular value of the SMarT contribution rate that maximizes the lifetime welfare of a shortsighted individual. If this welfare-maximizing contribution rate is positive, then the SMarT program improves well-being given the assumptions of the model.

Formally, we define this optimal (welfare-maximizing) SMarT contribution rate as

$$\gamma_{\max} \equiv \underset{\gamma \in [0,1]}{\arg\max} \{V\} \tag{57}$$

where

$$\begin{aligned} V \equiv & \int_0^{T-x} \exp[-\beta t] \frac{c_1^*(t)^{1-\phi} - 1}{1-\phi} dt + \int_{T-x}^{T} \exp[-\beta t] \frac{c_2^*(t)^{1-\phi} - 1}{1-\phi} dt \\ & + \int_T^{\bar{T}-x} \exp[-\beta t] \frac{c_3^*(t)^{1-\phi} - 1}{1-\phi} dt + \int_{\bar{T}-x}^{\bar{T}} \exp[-\beta t] \frac{c_4^*(t)^{1-\phi} - 1}{1-\phi} dt \end{aligned} \tag{58}$$

given a social discount rate, β. Note that lifetime well-being, as denoted by (58), is defined using *actual* consumption as arguments, which is consistent with a long tradition in behavioral economics of using experienced utility when conducting normative analysis.[7] Indeed, Akerlof (2002; p.423) states,

> A key theoretical innovation permitting systematic analysis of time-inconsistent behavior is the recognition that individuals may maximize a utility function that is divorced from that representing "true welfare". Once this distinction is accepted, "saving too little"

[7]For more on welfare and normative analysis in behavioral economic models, see Samuelson (1975), Harsanyi (1977), Feldstein (1985), Kahneman, Wakker, and Sarin (1997), O'Donoghue and Rabin (1999), Akerlof (2002), Docquier (2002), Kanbur, Pirttilä, and Tuomala (2006), Rubinstein (2006), Hurst and Willen (2007), Cremer, De Donder, Maldonado, and Pestieau (2008, 2009), Pestieau and Possen (2008), Cremer and Pestieau (2011), Boadway (2012), and Winter, Schlafmann, and Rodepeter (2012), and Love (2013), among many others.

> becomes a meaningful concept...Determining whether people save too much or too little involves asking whether people...have one (intertemporal) utility function which describes their welfare, but maximize another. Such evidence as there is suggests potentially large difference between the two concepts.

The welfare-maximizing value of the SMarT contribution rate depends on the planning horizon length, x, the social security tax rate, θ, in addition to the other parameters of the model.

With the social discount rate set to $\beta = 0.035$ (which matches the private discount rate and the interest rate), we search over the parameter space $x \in [1, 15]$, $\theta \in [0, 1]$, and we find $\gamma_{\max} \in [0, 0.65)$ as reported in Figure 4. The main intuition for an optimal SMarT rate that is positive is because shortsighted individuals do not anticipate their future retirement needs when they are making decisions during the early working years of the life cycle. Thus, they do not save for retirement on their own without assistance, at least until the later working years arrive (when the future interval of retirement enters into the planning horizon). The well-being of shortsighted individuals is improved as a result of participating in the SMarT program because participation successfully transfers resources from the working years to the retirement years, which helps to make the age-consumption profile become smoother.

A welfare-maximizing SMarT contribution rate that is as large as 65 percent may seem excessively high at first glance. However, this upper bound is actually lower than the implied SMarT rate from real-world pilot implementations of the program. Thaler and Benartzi (2004) report that participation in the SMarT program increased the flow of contributions into employer-sponsored savings plans from a rate of 3.5 percent to 13.6 percent over a three and half year period, while the wages of employees increased by around 3.25 percent to 3.5 percent over the same time frame. This indicates that employees contributed approximately 79 percent to 85 percent of their wage raises, which implies that the actual SMarT contribution rate in the real-world pilot implementations of the SMarT program was approximately 79 percent to 85 percent. Thus, even an upper bound of 65 percent on the optimal (welfare-maximizing) SMarT contribution rate would appear to be quite feasible given this observed evidence.

Regarding parameters of the model, the optimal SMarT contribution rate decreases in the length of the planning horizon. This is due to the idea that individuals with shorter planning horizons benefit from a relatively higher SMarT rate because they fail to save adequately on their own. The optimal

SMarT contribution rate also decreases in the size of the social security program. The optimal SMarT rate is over 60 percent for all planning horizons that we consider when there is no social security program (i.e., when $\theta = 0$). Likewise, the optimal SMarT rate decreases as the size of the social security program increases. This happens because social security is also able to transfer some resources from the working years to the retirement years, and thus, individuals do not need to contribute nearly as much into an employer-sponsored savings plan in order to meet retirement needs. Indeed, the welfare-maximizing SMarT contribution rate is between 38 percent and 41 percent when the social security program in the model is parameterized to the size of the U.S. program (i.e., $\theta = 0.106$). Furthermore, the optimal SMarT rate falls to zero when the social security program in the model is almost three times as large as the current program ($\theta > 0.27$). Lastly, the optimal SMarT contribution rate increases as the social discount rate, β, decreases. This is due to the fact that experienced utility late in life (when the annuity benefits to consumption are realized as a result of participation in the SMarT program) is not discounted as heavily. Indeed, we find that the optimal SMarT contribution rate equals 100 percent for all planning horizons considered when the social discount rate is zero, as long as $\theta < 0.15$ like in the case of the U.S. social security program.[8]

5 Exercises

In addition to calculating the optimal SMarT contribution rate, we examine the welfare effects that result from participation in the SMarT program. We do this by conducting a series of counterfactual experiments in which we calculate the lifetime well-being of a shortsighted individual who participates, and then we compare it to the lifetime well-being of an otherwise-identical individual who lives in a state of the world absent the program. We introduce the following notation: $c_1^*(t;\gamma,\theta)$, $c_2^*(t;\gamma,\theta)$, $c_3^*(t;\gamma,\theta)$, and $c_4^*(t;\gamma,\theta)$ is the life-cycle consumption program of a shortsighted individual, corresponding to a state of the world in which the SMarT program and social security are both operative; and, $c_1^*(t;0,0)$, $c_2^*(t;0,0)$, $c_3^*(t;0,0)$, and $c_4^*(t;0,0)$ is the life-cycle consumption program of an otherwise-identical individual, corresponding to a state of the world in which neither program exists. We also conduct experiments in which just one of the two programs is operative while the other program is not.

[8] A social discount rate of $\beta > 0$ is inconsistent with the criterion of Ramsey (1928) which precludes social discounting of the future on the basis of pure time preference. Ramsey describes such practices as “ethically indefensible” that results from a “failure of the imagination”.

5.1 SMarT Program Participation without Social Security

First, we compare the lifetime well-being of a shortsighted individual who participates in the SMarT program to the well-being of an otherwise-identical individual who does not participate. And in this first quantitative experiment, we assume the absence of a government-operated social security program (i.e., $\theta = 0$). As such, the liquid savings asset account balance, $k(t)$, is the only means through which the individual accumulates savings for retirement aside from the possibility of the SMarT program. We employ the standard consumption equivalent variation technique in order to assess the welfare effects that result from participation in the SMarT program.

In this first experiment, the consumption equivalent variation (CEV) is defined as the fraction of lifetime consumption that would need to be given up by an individual who lives in a state of the world absent the SMarT program in order to equate lifetime well-being to that of an otherwise-identical individual who participates in the SMarT program. A positive value for the CEV indicates that the individual is better off as a result of participation in the SMarT program. Recalling notation, $c_i^*(t;\gamma,0)$ for $i \in \{1,2,3,4\}$ is the life-cycle consumption program of an individual who participates in the SMarT program, and $c_i^*(t;0,0)$ for $i \in \{1,2,3,4\}$ is the life-cycle consumption program of an otherwise-identical individual who does not participate. In both cases, the government does not operate any type of social security program. Formally, the CEV in this experiment is denoted as Δ which is the percentage amount of consumption that would need to be removed (i.e., "paid") each period in order to equate the left-hand side of (59) with the right-hand side of (59),

$$\begin{aligned} &\int_0^{T-x} \exp[-\beta t]\frac{[c_1^*(t;\gamma,0)(1-\Delta)]^{1-\phi}-1}{1-\phi}dt + \int_{T-x}^{T} \exp[-\beta t]\frac{[c_2^*(t;\gamma,0)(1-\Delta)]^{1-\phi}-1}{1-\phi}dt \\ &+ \int_{T}^{\bar{T}-x} \exp[-\beta t]\frac{[c_3^*(t;\gamma,0)(1-\Delta)]^{1-\phi}-1}{1-\phi}dt + \int_{\bar{T}-x}^{\bar{T}} \exp[-\beta t]\frac{[c_4^*(t;\gamma,0)(1-\Delta)]^{1-\phi}-1}{1-\phi}dt \\ =\; &\int_0^{T-x} \exp[-\beta t]\frac{c_1^*(t;0,0)^{1-\phi}-1}{1-\phi}dt + \int_{T-x}^{T} \exp[-\beta t]\frac{c_2^*(t;0,0)^{1-\phi}-1}{1-\phi}dt \\ &+ \int_{T}^{\bar{T}-x} \exp[-\beta t]\frac{c_3^*(t;0,0)^{1-\phi}-1}{1-\phi}dt + \int_{\bar{T}-x}^{\bar{T}} \exp[-\beta t]\frac{c_4^*(t;0,0)^{1-\phi}-1}{1-\phi}dt \qquad (59) \end{aligned}$$

The calculated values of this particular CEV measure are presented in Table 2 and in Figure 5. The CEV is a function of the SMarT contribution rate, γ, and of the planning horizon, x. The welfare benefits from participation in the SMarT program decrease in the planning horizon. If the planning horizon is short, then the welfare benefits from participation are large. Indeed, shortsighted individuals who have a 1-year planning horizon would need to give up approximately half of their lifetime consumption in order to be as well off as they would be if they were to participate in the SMarT program. These large calculated values for the CEV are the result of shortsighted individuals not anticipating and not planning for retirement until just shortly before retirement actually occurs. Consumption falls near to zero during retirement in the absence of participating in the SMarT program. Thus, the welfare gains are large from even just small contributions into the SMarT program in order to avoid sizable reductions in consumption at retirement.[9]

The CEV is still sizable for individuals who have longer planning horizons, meaning that the SMarT program increases well-being for those individuals as well. For example, the CEV is 13.2 percent for individuals who have 5-year planning horizons coupled with a SMarT contribution rate of 60 percent. Similarly, the CEV is 3.1 percent for individuals who have 15-year planning horizons at the same contribution rate. Lastly, the CEV takes on its highest value when $\gamma = \gamma_{\max}$ for a given planning horizon length.

5.2 SMarT Program Participation in place of Social Security

We now examine the welfare effects that result from participation in the SMarT program compared to a benchmark of participation in a government-operated social security program. This exercise provides insight into whether the SMarT program outperforms a social security program when it comes to assisting with retirement preparation. In essence, this particular experiment assesses whether or not the SMarT program is a good substitute for a social security program. This is important to understand given that government-operated social security programs are frequently justified on the basis that they help shortsighted individuals to prepare adequately for retirement.[10] Similarly, the SMarT program is also motivated on the basis that it can help individuals to save more than what they would save otherwise for retirement (Thaler and Benartzi 2004).

In this second experiment, the consumption equivalent variation (CEV) is defined as the fraction of lifetime consumption that would need to be given up by an individual who participates in a social

[9] Of course, the magnitudes of these welfare gains are predicated on the principle of concave utility, which is a standard assumption.

[10] See Findley and Caliendo (2008) for an overview.

security program in order to equate lifetime well-being to that of an otherwise-identical individual who participates in the SMarT program instead. A positive value for the CEV indicates that the individual is better off as a result of participation in the SMarT program. Specifically, $c_i^*(t;\gamma,0)$ for $i \in \{1,2,3,4\}$ is the life-cycle consumption program of an individual who participates in the SMarT program, but does not participate in social security. Moreover, $c_i^*(t;0,\theta)$ for $i \in \{1,2,3,4\}$ is the life-cycle consumption program of an otherwise-identical individual who participates in a social security program, but does not participate in the SMarT program. Formally, the CEV in this second experiment is denoted as Ω which is the percentage amount of consumption that would need to be removed each period in order to equate the left-hand side of (60) with the right-hand side of (60),

$$\begin{aligned} & \int_0^{T-x} \exp[-\beta t] \frac{[c_1^*(t;\gamma,0)(1-\Omega)]^{1-\phi}-1}{1-\phi} dt + \int_{T-x}^{T} \exp[-\beta t] \frac{[c_2^*(t;\gamma,0)(1-\Omega)]^{1-\phi}-1}{1-\phi} dt \\ & + \int_{T}^{\bar{T}-x} \exp[-\beta t] \frac{[c_3^*(t;\gamma,0)(1-\Omega)]^{1-\phi}-1}{1-\phi} dt + \int_{\bar{T}-x}^{\bar{T}} \exp[-\beta t] \frac{[c_4^*(t;\gamma,0)(1-\Omega)]^{1-\phi}-1}{1-\phi} dt \\ = & \int_0^{T-x} \exp[-\beta t] \frac{c_1^*(t;0,\theta)^{1-\phi}-1}{1-\phi} dt + \int_{T-x}^{T} \exp[-\beta t] \frac{c_2^*(t;0,\theta)^{1-\phi}-1}{1-\phi} dt \\ & + \int_{T}^{\bar{T}-x} \exp[-\beta t] \frac{c_3^*(t;0,\theta)^{1-\phi}-1}{1-\phi} dt + \int_{\bar{T}-x}^{\bar{T}} \exp[-\beta t] \frac{c_4^*(t;0,\theta)^{1-\phi}-1}{1-\phi} dt \end{aligned} \tag{60}$$

The calculated values for this CEV in this experiment are reported in Table 3 and in Figure 6. A calculated value that is positive suggests that individuals benefit more from participation in the SMarT program than they benefit from participation in a social security program. Similar to our findings reported in the previous section, the largest welfare gains from participation in the SMarT program accrue to individuals who have shorter planning horizons, given that the SMarT contribution rate is sufficiently large. For example, the highest calculated value for the CEV that results from participation in the SMarT program (as opposed to participation in social security) is 11.3 percent of lifetime consumption for individuals with 1-year planning horizons, whereas the highest calculated value of this CEV is only 7.8 percent for individuals with 15-year planning horizons.

We find that the calculated values for this CEV measure can be negative at cases in which individuals have short planning horizons and they participate in the SMarT program at low contribution rates. More specifically, if the SMarT contribution rate is small and if individuals are very short-sighted, then a social security program is better able to transfer consumption from the working years to retirement, compared to the SMarT program. In these quantitative experiments, the social security tax rate is set to the US rate of $\theta = 0.106$ which yields a social security benefit during retirement that replaces around 35.5 percent of initial wage-income. If the SMarT contribution rate is fairly small in tandem with shorter planning horizon lengths, then the SMarT program is not successful at transferring enough consumption from the working years to retirement, and therefore, it does not succeed at smoothing consumption by as much as social security succeeds. However, participation in the SMarT program at *any* contribution rate makes individuals strictly better off than participating in social security, if individuals have planning horizons that are greater than 6.7 years in length. This is due to the fact that the social security program in the model is unfunded and offers a below-market internal rate of return, which represents the real-world program in the United States. As such, individuals who plan at least 6.7 years into the future are better off participating in the SMarT program that earns the market rate on contributions from wage-income raises, compared to participating in a tax and transfer program that yields a below-market return. The lesson learned from this experiment is that the Save More Tomorrow program is not a good substitute for a social security program if people are sufficiently shortsighted and if they participate with small contribution rates. Otherwise, the SMarT program outperforms social security when it comes to improving lifetime well-being.

5.3 SMarT Program Participation with Social Security

Finally, we compare the effects on well-being that result from participation in the SMarT program in tandem with participation in social security, compared to a benchmark of participation in social security only. This provides insight into whether or not the SMarT program is a good complement to a government-operated social security program. In this third experiment, the consumption equivalent variation is defined as the fraction of lifetime consumption that would need to be given up by an individual who participates in a social security program in order to equate lifetime well-being to that of an otherwise-identical individual who participates in the social security program *and* in the SMarT program together. A positive value for the CEV indicates that the individual is better off as a result of additional participation in the SMarT program. Specifically, $c_i^*(t;\gamma,\theta)$ for $i \in \{1,2,3,4\}$ is the life-cycle consumption program of an individual who participates in the SMarT program *and* in social security. Alternatively, $c_i^*(t;0,\theta)$ for $i \in \{1,2,3,4\}$ is the life-cycle consumption program of an otherwise-identical individual who participates in a social security program only. Formally, the CEV in this third experiment is denoted as Λ which is the percentage amount of consumption that would need to be removed each period in order to equate the left-hand side of (61) with the right-hand side of (61),

$$\begin{aligned}
&\int_0^{T-x} \exp[-\beta t]\frac{[c_1^*(t;\gamma,\theta)(1-\Lambda)]^{1-\phi}-1}{1-\phi}dt + \int_{T-x}^{T} \exp[-\beta t]\frac{[c_2^*(t;\gamma,\theta)(1-\Lambda)]^{1-\phi}-1}{1-\phi}dt \\
&+ \int_{T}^{\bar{T}-x} \exp[-\beta t]\frac{[c_3^*(t;\gamma,\theta)(1-\Lambda)]^{1-\phi}-1}{1-\phi}dt + \int_{\bar{T}-x}^{\bar{T}} \exp[-\beta t]\frac{[c_4^*(t;\gamma,\theta)(1-\Lambda)]^{1-\phi}-1}{1-\phi}dt \\
= &\int_0^{T-x} \exp[-\beta t]\frac{c_1^*(t;0,\theta)^{1-\phi}-1}{1-\phi}dt + \int_{T-x}^{T} \exp[-\beta t]\frac{c_2^*(t;0,\theta)^{1-\phi}-1}{1-\phi}dt \\
&+ \int_{T}^{\bar{T}-x} \exp[-\beta t]\frac{c_3^*(t;0,\theta)^{1-\phi}-1}{1-\phi}dt + \int_{\bar{T}-x}^{\bar{T}} \exp[-\beta t]\frac{c_4^*(t;0,\theta)^{1-\phi}-1}{1-\phi}dt
\end{aligned} \tag{61}$$

The calculated values for this CEV in this experiment are reported in Table 4 and in Figure 7. As reported, we find that the calculated values are positive and sizable for most planning horizon lengths and SMarT contribution rates. For example, individuals with a 1-year planning horizon can be better off by as much as 5 percent of lifetime consumption if they participate in the SMarT program also, as compared to participating in social security only. Similar to our findings in the previous experiments reported above, the largest calculated values for this CEV correspond to individuals who are more shortsighted. Consistent with this finding is the fact that the calculated values for this CEV can be negative for shortsighted individuals who participate in the SMarT program with very high contributions rates, in the neighborhood of contributing 85 percent to 100 percent of wage-income raises. For example, if the SMarT contribution rate is 90 percent, then individuals will not benefit from additional participation in the SMarT program is they have a planning horizon of greater than 8.2 years in length. Similarly, if the SMarT contribution rate is 100 percent of wage raises, then individuals will not benefit from additional participation in the SMarT program is they have a planning horizon of greater than 3.3 years in length. The lesson learned from this experiment is that participation in the Save More Tomorrow program is not a good complement to participation in a social security program if participation in the SMarT program is overly aggressive with extremely high contribution rates on wage raises. Otherwise, when it comes to improving well-being, the SMarT program is an excellent complement to the operation of a social security program, at least for the vast majority of the parameter space that we examine.

6 Robustness

To examine the robustness of our quantitative findings, we consider alternative values for the parameters of the model. The inverse elasticity of intertemporal substitution, ϕ, has a meaningful impact on the welfare comparisons reported above. In the baseline parameterization, we set $\phi = 1$, but we also perform calculations with $\phi = 2$ meaning that the curvature of the instantaneous utility function increases. We find that the relative welfare gains that result from participation in the SMarT program are relatively higher compared to our baseline calculations, which means that consumption is somewhat smoother over the life cycle if $\phi = 2$. For example, given individuals with 5-year planning horizons, we find that the calculated values for the consumption equivalent variation Δ (which compares the value of participation in the SMarT program relative to no participation at all) is between 30 percent to 40 percent if $\phi = 2$, compared to only 9 percent to 13 percent if $\phi = 1$. Similarly, we find that the calculated values for the consumption equivalent variation Ω (which compares the value of participation in the SMarT program relative to participation in social security) and for Λ (which compares the value of participation in the SMarT program in tandem with social security participation relative to participation in social security only) are both larger if $\phi = 2$. We also mention that the calculated values of the welfare-maximizing SMarT contribution rate, $\gamma_{\max}$, are very similar if $\phi = 2$ relative to baseline cases with $\phi = 1$.

The interest rate, r, and social discount rate, β, are both key parameters in our baseline welfare calculations reported above. Intuitively, larger social discount rates decrease the calculated values for the optimal SMarT contribution rate and for the calculated values of the consumption equivalent variation measures, on account that the consumption benefits from SMarT participation are realized later in life. For example, if $r = \beta = 0.05$, then the highest calculated value of the optimal SMarT contribution rate (corresponding to individuals with 1-year planning horizons and no social security participation) is $\gamma_{\max} = 0.45$ compared to $\gamma_{\max} = 0.65$ in our baseline calculations with $r = \beta = 0.035$. But, the reverse is also true, namely that the welfare gains that result from participation are higher if the social discount rate is lower: the highest calculated value of the optimal SMarT contribution rate is $\gamma_{\max} = 0.91$ if $r = \beta = 0.02$.

We also explore the possibility of the interest rate being greater than the private and social discount rates, and vice versa. When the interest rate is higher than the private and social discount rates, the welfare benefits that result from participation in the SMarT program (earned interest on accumulated savings) are greater than the relative welfare benefits that result from consumption earlier in life. Thus, saving for retirement via SMarT program participation is more desirable in these cases, as compared to cases in which the interest rate and the discount rates are equal. As a consequence, the calculated values for the optimal SMarT contribution rate and for the calculated values of the consumption equivalent variation measures are larger. The reverse is true for cases in which the discount rates are higher than the interest rate.

7 Concluding Remarks

Various applications of the Save More Tomorrow (SMarT) program of Thaler and Benartzi (2004) have been implemented widely by 401(k) retirement plan sponsors within the United States. The SMarT program has been heralded as a way to help individuals overcome behavioral biases in order to become better prepared for retirement, and it appears that the program is successful at increasing contributions by participants into employer-sponsored retirement savings plans. The principal innovation of the SMarT program is that it is engineered to account for and counteract the effects of present bias (hyperbolic discounting). However, it has been documented recently that, in the absence of sizable credit market imperfections, the SMarT program is theoretically unable to help such individuals become better prepared for retirement. The intuition for this finding reported in the literature is that, even though contributions into employer-sponsored retirement accounts might increase as a result of SMarT program participation, these contributions are offset by reductions in other forms of saving and/or by increases in debt accumulation.

The contribution of this study is that we document theoretically that individuals who are short-sighted (myopic) can benefit significantly from participation in the Save More Tomorrow program. Of notable importance is that the improvements in well-being occur in the absence of credit market imperfections. Alternatively stated, shortsighted individuals increase their contributions into employer-sponsored retirement saving plans via SMarT program participation, while at the same time they successfully maintain their holdings of other financial assets. As such, the Save More Tomorrow program is able to successfully transfer consumption resources from the working years to the retirement years, which acts to better smooth consumption over the life cycle. Moreover, achieving a smoother life-cycle consumption profile acts to improve lifetime well-being, at least under the assumption of a concave period utility function which is standard.

The notion that individuals have short planning horizons when making financial decisions has a history of support in the literature (e.g., see Strotz (1956); Friedman (1963); Carroll and Summers (1991); Carroll (2001); among others). We construct a life-cycle model of consumption and saving that represents shortsighted individuals who participate in the SMarT program. The novel feature of this model is that individuals formulate financial plans that are relevant for only a fixed number of periods into the future. The use of short planning horizons by individuals triggers time-inconsistent decision making, on account that shortsighted individuals fail to account for that part of the future which is beyond the endpoint of their planning horizons. More specifically, as age advances over the life cycle, the planning horizon slides forward and incorporates additional information that was previously unanticipated, which leads to frequent re-optimization.

Within this framework, shortsighted individuals do not anticipate retirement until just a few years before retirement occurs (when the future date of retirement enters into the planning horizon). Thus, in the absence of some type of external help, such individuals fail to save enough on their own in order to smooth consumption over the life cycle, which is manifest by a significant drop in consumption at retirement. We find that shortsighted individuals, as modeled, can benefit theoretically from participation in the SMarT program, because participation increases their total asset holdings with little or no reduction in the holdings of other forms of retirement savings. This is due to the fact that such individuals do not anticipate the annuity benefits from participation until they are near retirement. Indeed, unlike private liquid savings accounts that shortsighted individuals can still manage and see every day, the accumulated assets from participation in the SMarT program remain largely out of sight until retirement approaches. Shortsighted individuals are aware that they are making SMarT contributions into employer-sponsored retirement plans. However, since the accumulated funds are not available until retirement, these funds are just not salient to shortsighted individuals from the vantage point of when retirement is still a ways off into the future (outside of their respective planning horizons). Thus, shortsighted individuals do not offset SMarT program contributions into retirement accounts, and therefore, the Save More Tomorrow program acts as an unofficial commitment device that helps individuals to become better prepared for retirement.

Specifically, we find that the welfare-maximizing SMarT contribution rate is approximately 40 percent of wage raises for shortsighted individuals who participate in the SMarT program while also participating in a government-operated social security program that is parameterized to the size of the current program in the United States. We also find that this welfare-maximizing contribution rate is higher in a state of the world in which social security programs are not available. We also conduct three types of counterfactual experiments in order to better understand by how much shortsighted individuals could gain by participating in the SMarT program. First, we compare the lifetime well-being of a shortsighted individual who participates in the SMarT program to the well-being of an otherwise-identical individual who does not participate, both in the absence of participation in a social security program. This helps us to identify the pure effects of participation in the SMarT program. Using standard consumption equivalent variation measures, we calculate that shortsighted individuals are better off as a result of participation on the order of 3 percent of lifetime consumption (corresponding to shortsighted individuals with 15-year planning horizons) all the way up to 55 percent of lifetime consumption (corresponding to individuals with 1-year planning horizons). Second, we compare the welfare effects that result from participation in the SMarT program relative to participation in a social security program. We find that shortsighted individuals are better off from SMarT program participation by an order of 8 percent of lifetime consumption (corresponding to individuals with 15-year planning horizons) all the way to 11 percent of lifetime consumption (corresponding to individuals with 1-year planning horizons). This finding indicates that the SMarT program can be a good substitute for a social security program, if the contributions rates into the program are not too small. Lastly, we compare the welfare effects that result from participation in the SMarT program and in a social security program, compared to participation in a social security program only. This calculation provides insight into whether or not the SMarT program is a good complement to the operation of a social security program. We calculate that the welfare benefits are positive for all shortsighted individuals, as long as contribution rates are not too aggressive (below the neighborhood of 85 percent to 100 percent of wage-income raises), and that the welfare benefit can be as high as 4.6 percent of lifetime consumption for some individuals. This suggests that the Save More Tomorrow program can be a good complement to government-operated social security programs if parameterized appropriately. In summary, our model and quantitative calculations suggest that the adoption and implementation of the Save More Tomorrow program can be a viable way to overcome the behavioral bias of shortsightedness in a way that the program acts as an unofficial commitment device that successfully helps with financial preparation for retirement.

Appendix

Derivation of Specific Functional Forms for Phase 1

Given the functional form of utility, (44), equation (10) becomes

$$\hat{k}_1(t) = k(t_0)\exp[r(t-t_0)] + \int_{t_0}^{t}\left(y(v) - [\lambda(t_0)\exp[(\rho - r)v + (r-\rho)t_0]]^{\frac{-1}{\phi}}\right)\exp[r(t-v)]dv, \quad \text{(A1)}$$

for $t \in [t_0, t_0 + x]$. Evaluating (A1) at $k(t_0 + x) = 0$ yields the the constant of integration,

$$\lambda(t_0) = \left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{t_0+x} y(v)\exp[-rv]dv}{\int_{t_0}^{t_0+x}\exp[((\rho-r)/\phi - r)v]dv}\right)^{-\phi}. \quad \text{(A2)}$$

Thus, planned consumption is

$$\hat{c}_1(t) = \exp[((\rho-r)/\phi)t]\left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{t_0+x} y(v)\exp[-rv]dv}{\int_{t_0}^{t_0+x}\exp[((\rho-r)/\phi - r)v]dv}\right), \text{ for } t \in [t_0, t_0+x], \quad \text{(A3)}$$

which is equation (45) in the main text above, and actual consumption during Phase 1 is

$$c_1^*(t) = \exp[((\rho-r)/\phi)t]\left(\frac{k_1^*(t)\exp[-rt] + \int_{t}^{t+x} y(v)\exp[-rv]dv}{\int_{t}^{t+x}\exp[((\rho-r)/\phi - r)v]dv}\right), \text{ for } t \in [0, T-x]. \quad \text{(A4)}$$

which is equation (46) in the text above and is an implicit function of the liquid savings account balance. To find the closed-form, analytical solution, we rewrite (A4) and insert it into (14),

$$\frac{dk_1^*(t)}{dt} = rk_1^*(t) + y(t) - \frac{\exp[((\rho-r)/\phi - r)t]}{\int_{t}^{t+x}\exp[((\rho-r)/\phi - r)v]dv}k_1^*(t) - \frac{\int_{t}^{t+x} y(v)\exp[((\rho-r)/\phi - r)v]dv}{\int_{t}^{t+x}\exp[((\rho-r)/\phi - r)v]dv}, \quad \text{(A5)}$$

for $t \in [0, T-x]$, which can be written compactly as

$$\frac{dk_1^*(t)}{dt} = \sigma_1(t)k_1^*(t) + y(t) - \eta_1(t), \text{ for } t \in [0, T-x], \quad \text{(A6)}$$

where

$$\sigma_1(t) = r - \frac{\exp[((\rho - r)/\phi - r)t]}{\int_t^{t+x} \exp[((\rho - r)/\phi - r)s]ds}, \tag{A7}$$

$$\eta_1(t) = \frac{\int_t^{t+x} y(s)\exp[((\rho - r)/\phi - r)s]ds}{\int_t^{t+x} \exp[((\rho - r)/\phi - r)s]ds}. \tag{A8}$$

Note that the dummy of integration v has been changed to s for convenience in what follows.

The general solution to this differential equation is

$$k_1^*(t) = \exp\left[\int^t \sigma_1(j)dj\right]\left(z_1 + \int^t [y(v) - \eta_1(v)]\exp\left[-\int^v \sigma_1(j)dj\right]dv\right), \text{ for } t \in [0, T - x], \tag{A9}$$

where z_1 is a constant of integration. Using the initial condition $k_1^*(0) = 0$, the particular solution is

$$k_1^*(t) = \exp\left[\int^t \sigma_1(j)dj\right]\left(\int_0^t [y(v) - \eta_1(v)]\exp\left[-\int^v \sigma_1(j)dj\right]dv\right), \text{ for } t \in [0, T - x], \tag{A10}$$

which can be rewritten by multiplying both sides by $\exp[-\int^0 \sigma_1(j)dj]$,

$$k_1^*(t) = \exp\left[\int_0^t \sigma_1(j)dj\right]\left(\int_0^t [y(v) - \eta_1(v)]\exp\left[-\int_0^v \sigma_1(j)dj\right]dv\right), \text{ for } t \in [0, T - x]. \tag{A11}$$

Substituting (A7) and (A8) into (A11) yields

$$\begin{aligned} k_1^*(t) \;=\; & \exp\left[\int_0^t \left(r - \frac{\exp[((r - \rho)/\phi - r)j]}{\int_j^{j+x} \exp[((r - \rho)/\phi - r)s]ds}\right)dj\right] \\ & \times \left(\int_0^t \left[y(v) - \left(\frac{\exp[((r - \rho)/\phi)v]\int_v^{v+x} y(s)\exp[-rs]ds}{\int_v^{v+x} \exp[((r - \rho)/\phi - r)s]ds}\right)\right]\right. \\ & \left. \times \exp\left[-\int_0^v \left(r - \frac{\exp[((r - \rho)/\phi - r)j]}{\int_j^{j+x} \exp[((r - \rho)/\phi - r)s]ds}\right)dj\right]dv\right), \text{ for } t \in [0, T - x], \end{aligned} \tag{A12}$$

which is equation (47) in the main text above.

Derivation of Specific Functional Forms for Phase 2

Given the functional form of utility, (44), equation (24) becomes

$$k(t_0)\exp[-rt_0] + \int_{t_0}^T y(v)\exp[-rv]dv + \int_T^{t_0+x} (A + b)\exp[-rv]dv$$

$$= \int_{t_0}^{t_0+x} [\lambda(t_0)\exp[(\rho - r)v + (r-\rho)t_0]]^{\frac{-1}{\phi}} \exp[-rv]dv\,, \tag{A13}$$

which can be solved for the constant of integration,

$$\lambda(t_0) = \left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{T} y(v)\exp[-rv]dv + \int_{T}^{t_0+x}(A+b)\exp[-rv]dv}{\exp[\ \rho t_0/\phi]\int_{t_0}^{t_0+x}\exp^{((r-\rho)/\phi - r)v}]dv} \right)^{-\phi}. \tag{A14}$$

Thus, planned consumption is

$$\hat{c}_2(t) = \exp[((r-\rho)/\phi)t] \left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{T} y(v)\exp[-rv]dv + \int_{T}^{t_0+x}(A+b)\exp[-rv]dv}{\int_{t_0}^{t_0+x}\exp[((r-\rho)/\phi - r)v]dv} \right), \tag{A15}$$

for $t \in [t_0, t_0 + x]$, which is equation (48). Actual consumption is

$$c_2^*(t) = \exp[((r-\rho)/\phi)t] \left(\frac{k_2^*(t)\exp[-rt] + \int_{t}^{T} y(v)\exp[-rv]dv + \int_{T}^{t+x}(A+b)\exp[-rv]dv}{\int_{t}^{t+x}\exp[((r-\rho)/\phi - r)v]dv} \right), \tag{A16}$$

for $t \in [T - x, T]$, which is equation (49).

Equation (A16) is an implicit function of the liquid savings account balance. To again find the closed-form, analytical solution, rewrite (A16) and insert it into (27),

$$\frac{dk_2^*(t)}{dt} = \sigma_2(t)k_2^*(t) + y(t) - \eta_2(t), \text{ for } t \in [T-x, T], \tag{A17}$$

where

$$\sigma_2(t) = r - \frac{\exp[((r-\rho)/\phi - r)t]}{\int_t^{t+x}\exp[((r-\rho)/\phi - r)s]ds}, \tag{A18}$$

$$\eta_2(t) = \frac{\left(\int_t^T y(s)\exp[-rs]ds + \int_T^{t+x}(A+b)\exp[-rs]ds\right)\exp[((r-\rho)/\phi)t]}{\int_t^{t+x}\exp[((r-\rho)/\phi - r)s]ds}. \tag{A19}$$

Note that the dummy of integration have been changed from v to s for convenience again.

The general solution to this differential equation is

$$k_2^*(t) = \exp\left[\int^t \sigma_2(j)dj\right]\left(z_2 + \int^t [y(v) - \eta_2(v)]\exp\left[-\int^v \sigma_2(j)dj\right]dv\right), \text{ for } t \in [T-x, T], \tag{A20}$$

where z_2 is a constant of integration. Using the boundary condition, $k_1^*(T-x) = k_2^*(T-x)$, the particular solution is

$$k_2^*(t) = \exp\left[\int_{T-x}^{t} \sigma_2(j)dj\right]\left(k_2^*(T-x) + \int_{T-x}^{t}[y(v)-\eta_2(v)]\exp\left[-\int_{T-x}^{v}\sigma_2(j)dj\right]dv\right), \tag{A21}$$

for $t \in [T-x, T]$. Substituting (A18) and (A19) into (A21) yields

$$\begin{aligned} k_2^*(t) = {} & \exp\left[\int_{T-x}^{t}\left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right)dj\right] k_2^*(T-x) \\ & + \left(\int_{T-x}^{t}\left[y(v) - \frac{\left(\int_v^T y(s)\exp[-rs]ds + \int_T^{v+x}(A+b)\exp[-rs]ds\right)\exp[((r-\rho)/\phi)v]}{\int_v^{v+x}\exp[((r-\rho)/\phi - r)s]ds}\right]\right. \\ & \times \left.\exp\left[-\int_{T-x}^{v}\left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right)dj\right]dv\right) \\ & \times \exp\left[\int_{T-x}^{t}\left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x}\exp[((r-\rho)/\phi - r)s]ds}\right)dj\right], \text{ for } t \in [T-x, T], \end{aligned} \tag{A22}$$

which is equation (50) in the main text above.

Derivation of Specific Functional Forms for Phase 3

Given the functional form of utility, (44), equation (33) becomes

$$\hat{k}_3(t) = k(t_0)\exp[r(t-t_0)] + \int_{t_0}^{t}\left(A + b - [\lambda(t_0)\exp[(\rho - r)v + (r-\rho)t_0]]^{\frac{-1}{\phi}}\right)\exp[r(t-v)]dv, \tag{A23}$$

for $t \in [t_0, t_0 + x]$. Evaluating (A23) at $k(t_0 + x) = 0$ yields the the constant of integration,

$$\lambda(t_0) = \left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{t_0+x}(A+b)\exp[-rv]dv}{\int_{t_0}^{t_0+x}\exp[((\rho - r)/\phi - r)v]dv}\right)^{-\phi}. \tag{A24}$$

Thus, planned consumption is

$$\hat{c}_3(t) = \exp[((\rho - r)/\phi)t]\left(\frac{k(t_0)\exp[-rt_0] + \int_{t_0}^{t_0+x}(A+b)\exp[-rv]dv}{\int_{t_0}^{t_0+x}\exp[((\rho - r)/\phi - r)v]dv}\right), \text{ for } t \in [t_0, t_0 + x], \tag{A25}$$

which is equation (51) in the main text above, and actual consumption during Phase 3 is

$$c_3^*(t) = \exp[((\rho - r)/\phi)t] \left(\frac{k_3^*(t)\exp[-rt] + \int_t^{t+x} (A+b)\exp[-rv]dv}{\int_t^{t+x} \exp[((\rho - r)/\phi - r)v]dv} \right), \text{ for } t \in [T, \bar{T} - x]. \tag{A26}$$

which is equation (52) in the text above.

Equation (A26) is an implicit function of the liquid savings account balance. Again, to find the closed-form, analytical solution, we rewrite (A26) and insert it into (37),

$$\frac{dk_3^*(t)}{dt} = \sigma_3(t)k_3^*(t) + A + b - \eta_3(t), \text{ for } t \in [T, \bar{T} - x], \tag{A27}$$

where

$$\sigma_3(t) = r - \frac{\exp[((r-\rho)/\phi - r)t]}{\int_t^{t+x} \exp[((r-\rho)/\phi - r)s]ds}, \tag{A28}$$

$$\eta_3(t) = \frac{\exp[((r-\rho)/\phi)t] \int_t^{t+x} (A+b)\exp[-rs]ds}{\int_t^{t+x} \exp[((r-\rho)/\phi - r)s]ds}. \tag{A29}$$

The general solution to this differential equation is

$$k_3^*(t) = \exp\left[\int^t \sigma_3(j)dj\right] \left(z_3 + \int^t [A + b - \eta_3(v)] \exp\left[-\int^v \sigma_3(j)dj\right] dv \right), \text{ for } t \in [T, \bar{T} - x], \tag{A30}$$

where z_3 is a constant. Using the boundary condition $k_2^*(T) = k_3^*(T)$, the particular solution is

$$k_3^*(t) = \exp\left[\int_T^t \sigma_3(j)dj\right] \left(k_3^*(T) + \int_T^t [A + b - \eta_3(v)] \exp\left[-\int_T^v \sigma_3(j)dj\right] dv \right), \text{ for } t \in [T, \bar{T} - x]. \tag{A31}$$

Substituting (A28) and (A29) into (A31) yields

$$\begin{aligned} k_3^*(t) = \exp &\left[\int_T^t \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x} \exp[((r-\rho)/\phi - r)s]ds} \right) dj \right] \\ \times &\left(k_3^*(T) + \int_T^t \left[A + b - \frac{\exp[((r-\rho)/\phi)v] \int_v^{v+x} (A+b)\exp[-rs]ds}{\int_v^{v+x} \exp[((r-\rho)/\phi - r)s]ds} \right] \right. \\ \times \exp &\left. \left[-\int_T^v \left(r - \frac{\exp[((r-\rho)/\phi - r)j]}{\int_j^{j+x} \exp[((r-\rho)/\phi - r)s]ds} \right) dj \right] dv \right), \text{ for } t \in [T, \bar{T} - x], \end{aligned} \tag{A32}$$

which is equation (53) in the main text above.

Derivation of Specific Functional Forms for Phase 4

Given the functional form of utility, (44), which allows us to analytically defintize the unknown constant of integration, equation (42) becomes

$$c_4^*(t) = \exp[(r-\rho)/\phi)t]\left(\frac{k_4^*(\bar{T}-x)\exp[rx] + \int_{\bar{T}-x}^{\bar{T}}(A+b)\exp[r(\bar{T}-v)]dv}{\int_{\bar{T}-x}^{\bar{T}}\exp[((r-\rho)/\phi - r)v + r\bar{T}]dv}\right), \text{ for } t \in [\bar{T}-x, \bar{T}], \tag{A33}$$

given the boundary condition $k_3^*(\bar{T}-x) = k_4^*(\bar{T}-x)$. Equation (A33) is equation (54) in the text above. This also implies that equation (43) becomes

$$\begin{aligned} k_4^*(t) \quad = \quad & k_4^*\left(\bar{T}-x\right)\exp[r(t-(\bar{T}-x))] + \int_{\bar{T}-x}^{t}(A+b)\exp[r(t-v)]dv \\ & + \int_{t}^{\bar{T}-x}\left(\frac{k_4^*\left(\bar{T}-x\right)\exp[rx] + \int_{\bar{T}-x}^{\bar{T}}(A+b)\exp[r(\bar{T}-s)]ds}{\int_{\bar{T}-x}^{\bar{T}}\exp[((r-\rho)/\phi - r)s + ((\rho-r)/\phi)(\bar{T}-x) + r\bar{T}]ds}\right) \\ & \times \exp[((r-\rho)/\phi - r)v + ((\rho-r)/\phi)(\bar{T}-x) + rt]\ dv, \text{ for } t \in [\bar{T}-x, \bar{T}], \end{aligned} \tag{A34}$$

which is equation (55) in the main text above.

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Table 1. Baseline Parameter Values.

Parameter Name	Symbol	Value
Date of retirement	T	40 (age 65)
Date of death	$\overline{T}$	55 (age 80)
Real interest rate	r	0.035
Discount rate	ρ	0.035
Inverse elasticity of intertemporal substitution	ϕ	1
Initial wage	w	1
Social Security tax rate	θ	0.106
SMarT contribution rate	γ	0.00

Table 2. Consumption Equivalent Variation (Δ) from SMarT Program Participation without Social Security.

x	γ: 0.1	0.2	0.4	0.6	0.8	1
1	0.4938	0.5227	0.5436	0.5491	0.5477	0.5422
3	0.1477	0.1853	0.2152	0.2234	0.2214	0.2125
5	0.0678	0.0976	0.1242	0.1320	0.1298	0.1209
10	0.0210	0.0351	0.0508	0.0560	0.0540	0.0466
15	0.0104	0.0181	0.0277	0.0311	0.0295	0.0238

Table 3. Consumption Equivalent Variation (Ω) from SMarT Program Participation in place of Social Security.

x	γ: 0.1	0.2	0.4	0.6	0.8	1
1	0.0042	0.0610	0.1021	0.1129	0.1103	0.0993
3	0.0191	0.0623	0.0967	0.1062	0.1038	0.0937
5	0.0325	0.0634	0.0910	0.0991	0.0969	0.0875
10	0.0514	0.0650	0.0803	0.0853	0.0834	0.0761
15	0.0585	0.0659	0.0750	0.0781	0.0767	0.0712

Table 4. Consumption Equivalent Variation (Λ) from SMarT Program Participation with Social Security.

x	γ: 0.1	0.2	0.4	0.6	0.8	1
1	0.0262	0.0398	0.0483	0.0426	0.0274	0.0048
3	0.0219	0.0337	0.0412	0.0359	0.0219	0.0006
5	0.0175	0.0274	0.0337	0.0287	0.0157	-0.0039
10	0.0095	0.0153	0.0190	0.0149	0.0046	-0.0110
15	0.0056	0.0092	0.0115	0.0084	0.0005	-0.0116

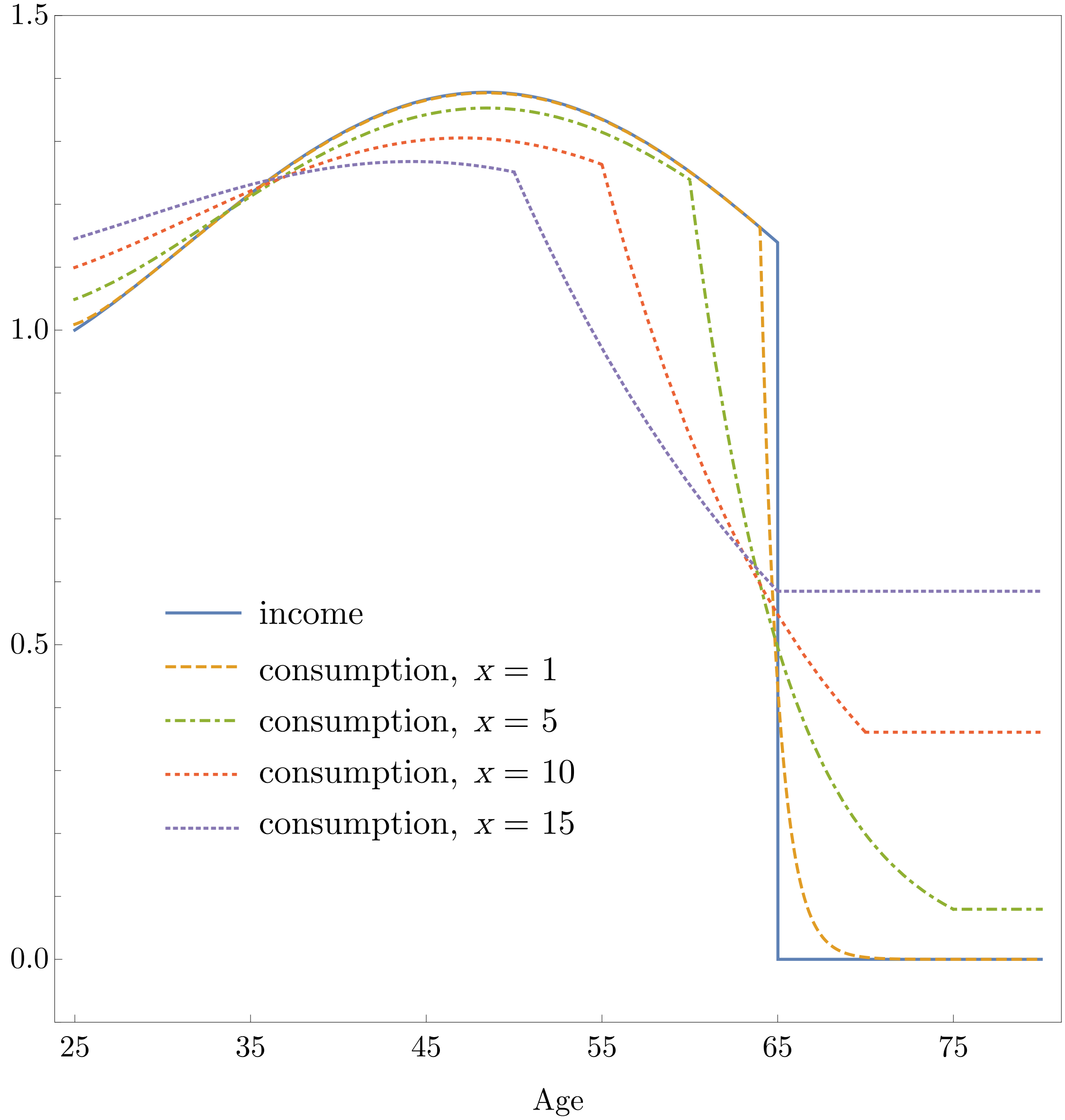

Figure 1. Life-cycle consumption profiles of shortsighted individuals who do not participate in the SMarT program and do not participate in a social security program. Income path is the solid-blue line. The dashed lines denote consumption paths for different planning horizons: yellow-dashed line corresponds to a 1-year planning horizon, green-dashed-dotted line corresponds to a 5-year horizon, red-dotted line corresponds to a 10-year horizon, and purple-dotted line corresponds to a 15-year horizon.

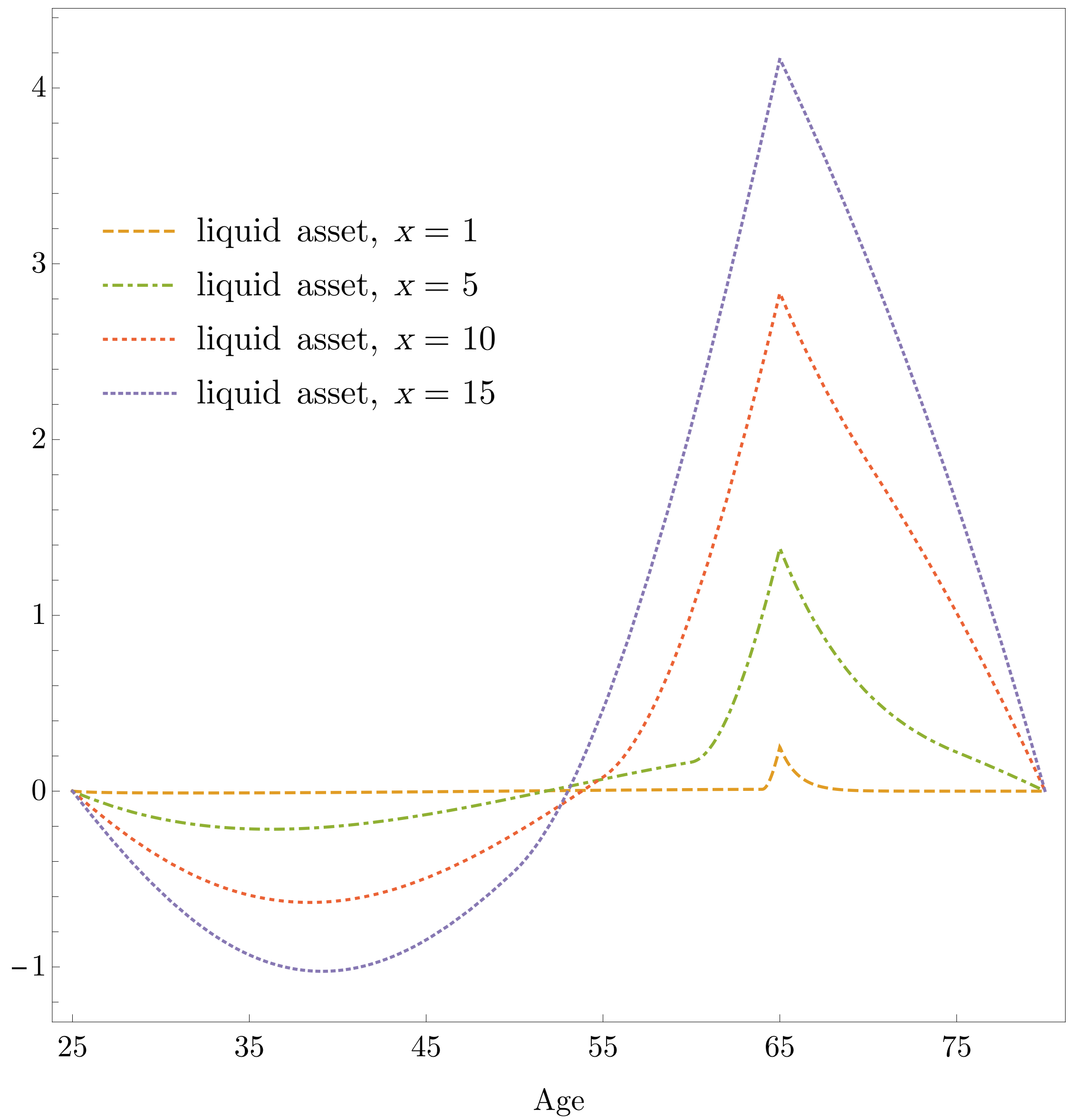

Figure 2. Life-cycle profiles of the liquid savings asset for shortsighted individuals who do not participate in the SMarT program and do not participate in a social security program. The yellow-dashed line corresponds to a 1-year planning horizon, green-dashed-dotted line corresponds to a 5-year horizon, red-dotted line corresponds to a 10-year horizon, and purple-dotted line corresponds to a 15-year horizon.

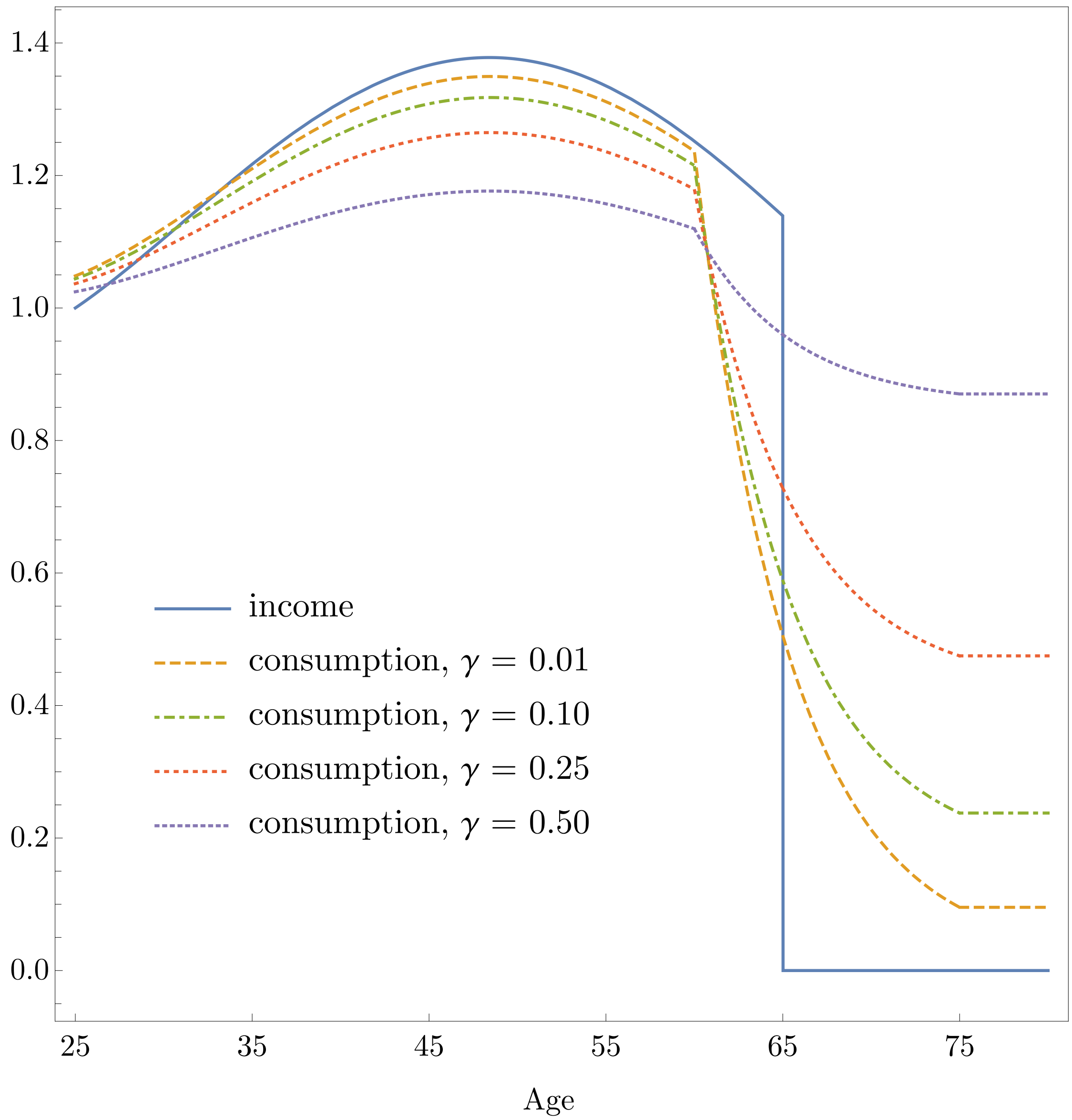

Figure 3. Life-cycle consumption profil es of shortsighted individuals with a 5-year planning horizon who participate in the SMarT program with different contribution rates but do not participate in a social security program. Income path is the solid-blue line. The dashed lines denote consumption paths for different SMarT contribution rates: yellow-dashed line corresponds to a 0.01 contribution rate, green-dashed-dotted line corresponds to a 0.10 rate, red-dotted line corresponds to a 0.25 rate, and purple-dotted line corresponds to a 0.50 rate.

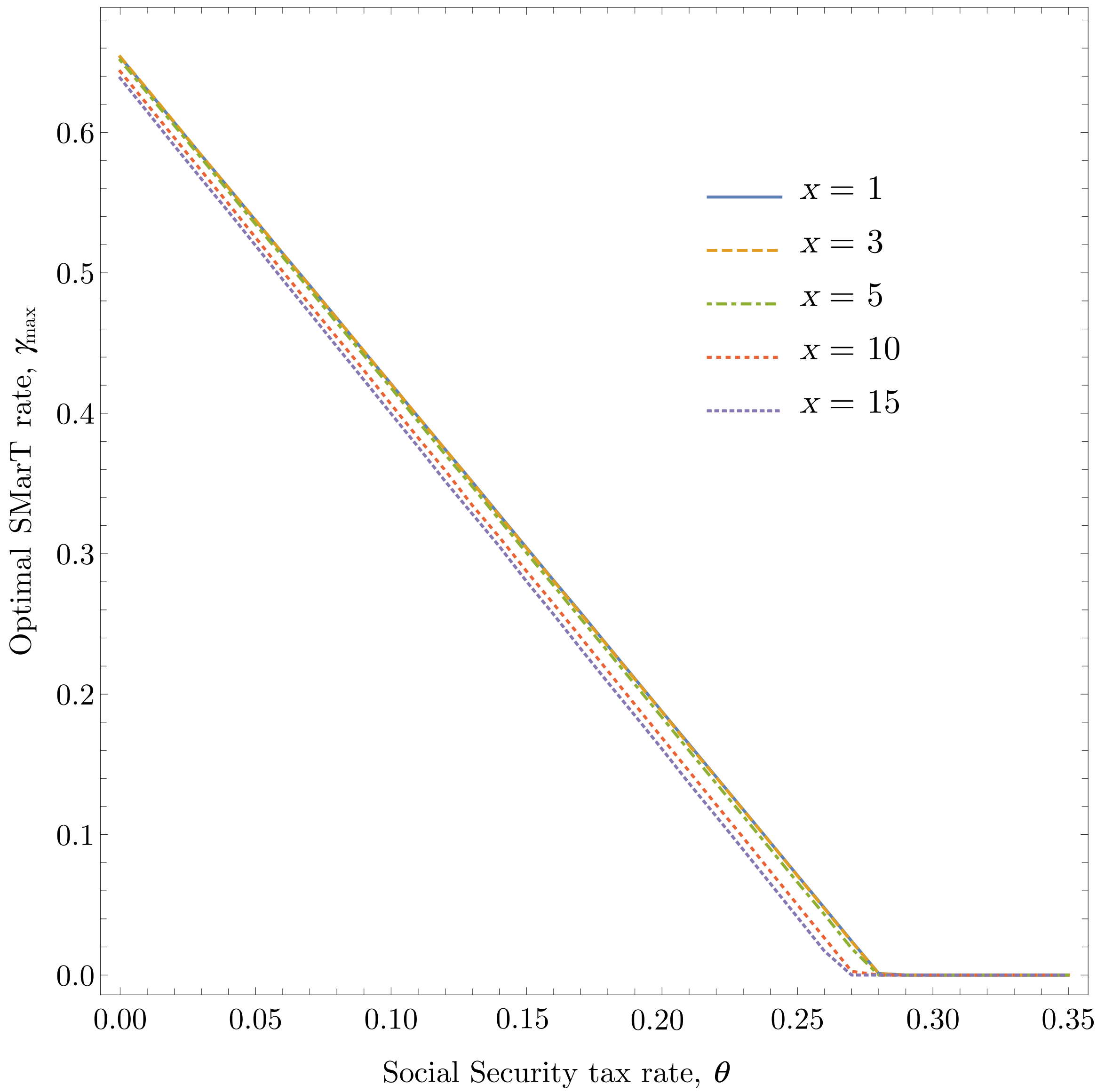

Figure 4. Optimal SMarT contribution rate for different social security tax rates and different planning horizons, given the parameterization $\beta = \rho = r = 0.035$. The solid-blue line corresponds to a 1-year planning horizon, the yellow-dashed line corresponds to a 3-year horizon, green-dashed-dotted line corresponds to a 5-year horizon, red-dotted line corresponds to a 10-year horizon, and purple-dotted line corresponds to a 15-year horizon.

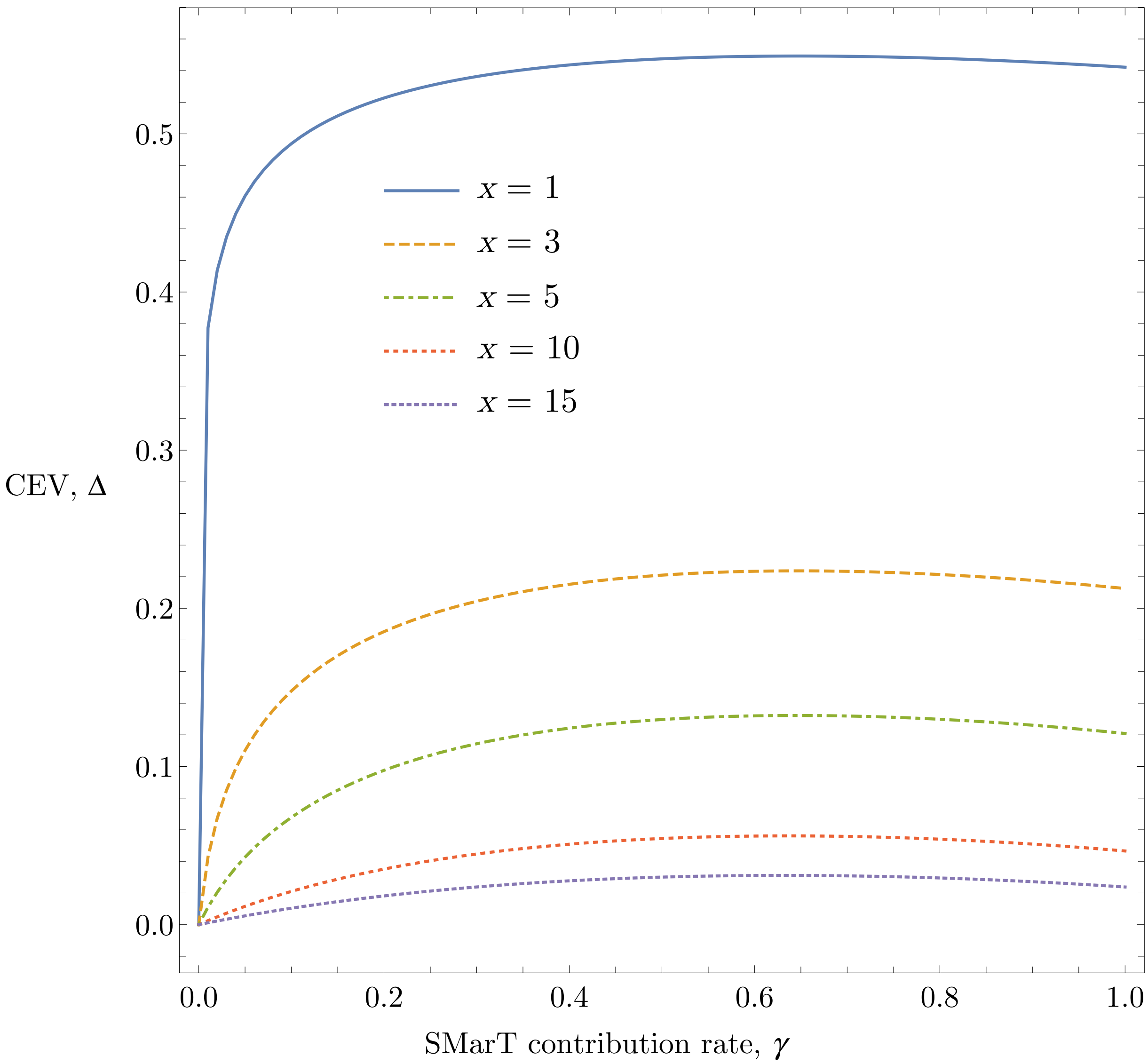

Figure 5. Consumption Equivalent Variation, Δ, for different SMarT contribution rates and different planning horizons. The solid-blue line corresponds to a 1-year planning horizon, the yellow-dashed line corresponds to a 3-year horizon, green-dashed-dotted line corresponds to a 5-year horizon, red-dotted line corresponds to a 10-year horizon, and purple-dotted line corresponds to a 15-year horizon.

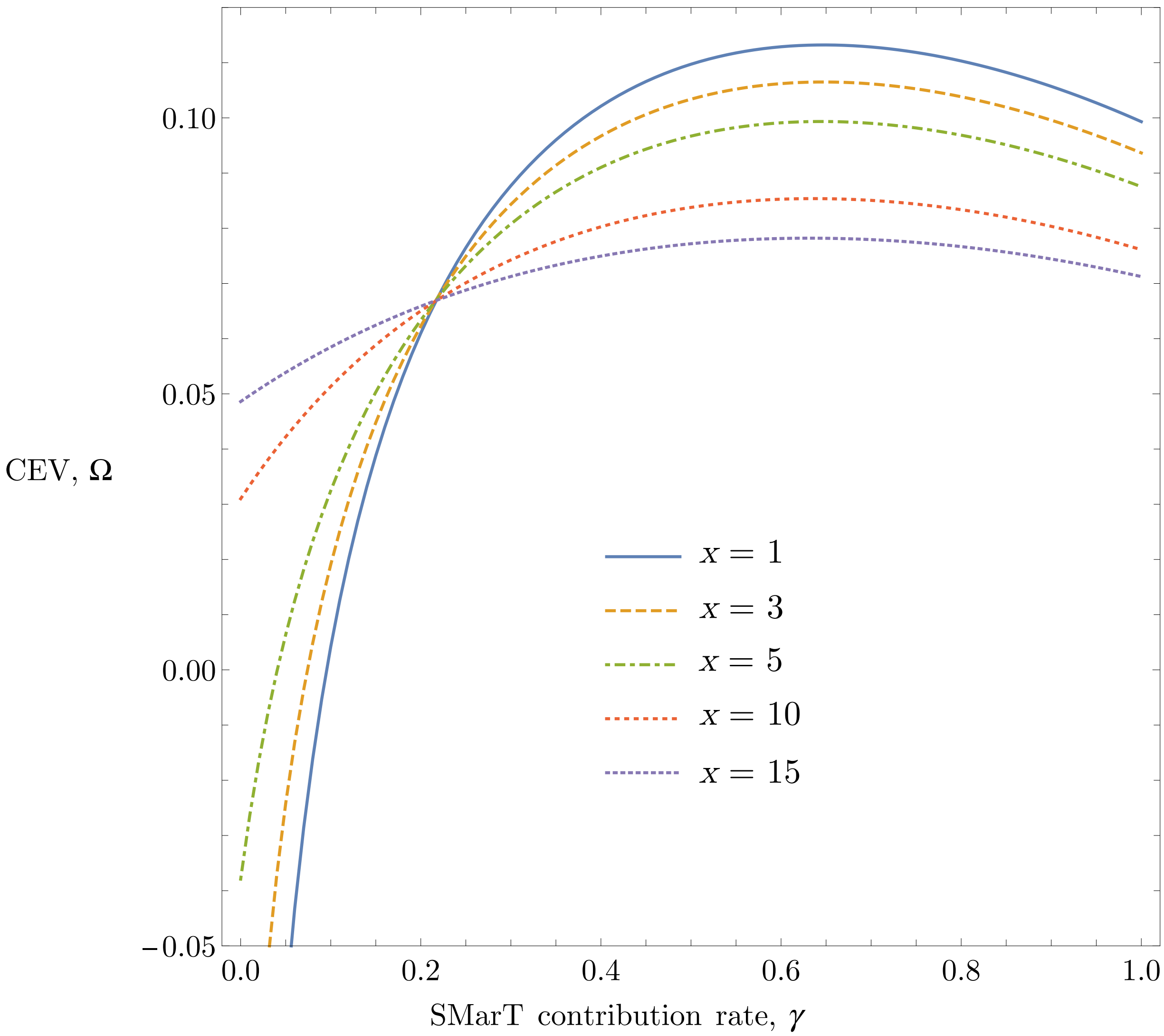

Figure 6. Consumption Equivalent Variation, $\mathbf{\Omega}$, for different SMarT contribution rates and different planning horizons. The solid-blue line corresponds to a 1-year planning horizon, the yellow-dashed line corresponds to a 3-year horizon, green-dashed-dotted line corresponds to a 5-year horizon, red-dotted line corresponds to a 10-year horizon, and purple-dotted line corresponds to a 15-year horizon.

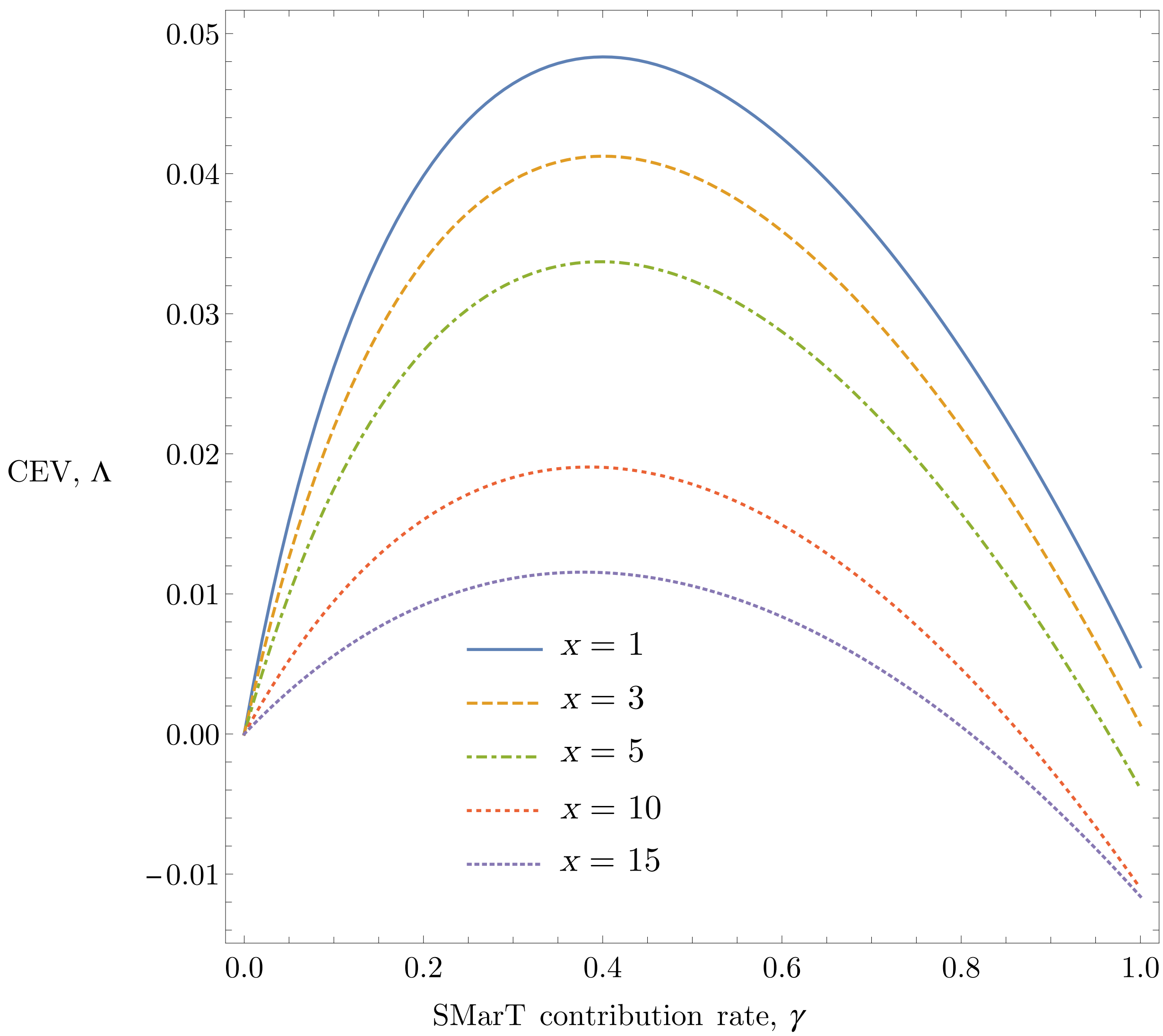

Figure 7. Consumption Equivalent Variation, Λ, for different SMarT contribution rates and different planning horizons. The solid-blue line corresponds to a 1-year planning horizon, the yellow-dashed line corresponds to a 3-year horizon, green-dashed-dotted line corresponds to a 5-year horizon, red-dotted line corresponds to a 10-year horizon, and purple-dotted line corresponds to a 15-year horizon.