SOLUTIONS

1. Calculate the hydraulic residence times (the retention time) for Lake Superior and for Lake Erie using data in the following Table.

   ![Table 4.3](Image)  
<table>
<thead>
<tr>
<th>Lake</th>
<th>Volume $10^9$ m$^3$</th>
<th>Outflow $10^9$ m$^3$/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superior</td>
<td>12,000</td>
<td>67</td>
</tr>
<tr>
<td>Michigan</td>
<td>4,900</td>
<td>36</td>
</tr>
<tr>
<td>Huron</td>
<td>3,500</td>
<td>161</td>
</tr>
<tr>
<td>Erie</td>
<td>468</td>
<td>182</td>
</tr>
<tr>
<td>Ontario</td>
<td>1,634</td>
<td>211</td>
</tr>
</tbody>
</table>

2. The Rappahannock River near Warrenton, VA, has a flow rate of 3.00 m$^3$·s$^{-1}$. Tin Pot Run (a pristine stream) discharges into the Rappahannock at a flow rate of 0.05 m$^3$·s$^{-1}$. To study mixing of the stream and river, a conservative tracer is added to Tin Pot Run. If the instruments that measure the tracer can detect a concentration of 1.00 mg·L$^{-1}$, what minimum concentration must be achieved in Tin Pot Run so that 1.00 mg·L$^{-1}$ of the tracer can be measured after the river and stream mix?

   Assume that the 1.00 mg·L$^{-1}$ of tracer is to be measured after complete mixing of the stream and Rappahannock has been achieved and that no tracer is in Tin Pot Run or the Rappahannock above the point where the two streams mix. What mass rate (in kilogram per day) of tracer must be added to Tin Pot Run? (Answer 263.52 or 264 kg·day$^{-1}$)

3. Five million gallons per day (MGD) of a conservative substance, with concentration 10.0 mg/L, is released into a stream having an upstream flow of 10 MGD and substance concentration of 3.0 mg/L. Assume complete mixing.
   a. What is the concentration in ppm just downstream?
   b. How many pounds of substance per day pass a given spot downstream?
4. The two-pond system shown in the following figure is fed by a stream with a flow rate of 1.0 MGD and a BOD (nonconservative pollutant) concentration of 20 mg/L. The rate of decay is 0.30/day. The volume of the first pond is 5.0 million gallons and the second is 3.0 million gallons. Solve for $C_1$ and $C_2$ in mg/L.

![Diagram of two-pond system](image)

5. Poorly treated municipal wastewater is discharged to a stream. The river flow rate upstream of the discharge point is $Q_{us} = 8.7 \text{ m}^3/\text{s}$. The discharge occurs at a flow of $Q_d = 0.9 \text{ m}^3/\text{s}$ and has a BOD concentration of 50.0 mg/L. Assuming that the upstream BOD concentration is negligible. (a) What is the BOD concentration just downstream of the discharge point?

6. A river with 400 ppm of salts (a conservative substance) and an upstream flow of 25 m$^3$/s receives an agricultural discharge of 5.0 m$^3$/s carrying 2000 mg/L of salts (see figure below, P1.7). The salts quickly become uniformly distributed in the river. A municipality just downstream withdraws water and mixes it with enough pure water (no salt) from another source to deliver water having no more than 500 ppm salts to its customers. 

   What should the mixture ratio ($F$) of pure water to the river water be?

![Diagram of river and discharge](image)

7. A pollutant which degrades according to the equation $\frac{dC}{dt} = -kC$ is said to react according to ______________________ kinetics.

8. In a material balance problem, the situation where the rate of accumulation is zero is referred to as _______________________.

2
1. Calculate the hydraulic residence times (the retention time) for Lake Superior and for Lake Erie using data in Table 4-3 (Mihelcic and Zimmerman).

**Solution:**

Using information provided in Table 4.3.

\[
\theta = \frac{V}{Q} \Rightarrow (\text{for Superior}) = \frac{12,000 \times 10^9 \text{ m}^3}{67 \times 10^9 \text{ m}^3/\text{yr}} = 180 \text{ yr}
\]

\[
(\text{for Erie}) = \frac{468 \times 10^9 \text{ m}^3}{182 \times 10^9 \text{ m}^3/\text{yr}} = 2.6 \text{ yr}
\]
2. The Rappahannock River near Warrenton, VA, has a flow rate of 3.00 m$^3$/s$^{-1}$. Tin Pot Run (a pristine stream) discharges into the Rappahannock at a flow rate of 0.05 m$^3$/s$^{-1}$. To study mixing of the stream and river, a conservative tracer is added to Tin Pot Run. If the instruments that measure the tracer can detect a concentration of 1.00 mg·L$^{-1}$, what minimum concentration must be achieved in Tin Pot Run so that 1.00 mg·L$^{-1}$ of the tracer can be measure after the river and stream mix? Assume that the 1.00 mg·L$^{-1}$ of tracer is to be measure after complete mixing of the stream and Rappahannock has been achieved and that no tracer is in Tin Pot Run or the Rappahannock above the point where the two streams mix. What mass rater (in kilogram per day) of tracer must be added to Tin Pot Run? (Answer 263.52 or 264 kg·day$^{-1}$)

Given: Q$_{RR}$ = 3.00 m$^3$/s, Q$_{TPR}$ = 0.05 m$^3$/s, detection limit = 1.0 mg/L

Solution:

a. Mass balance diagram  (NOTE: Q$_{out}$ = Q$_{RR}$ + Q$_{TPR}$ = 3.05 m$^3$/s)

b. Mass balance equation

\[ C_{RR}Q_{RR} + C_{TPR}Q_{TPR} = C_{out}Q_{out} \]

Since C$_{RR}$ in = 0 this equation reduces to: \[ C_{TPR}Q_{TPR} = C_{out}Q_{out} \] (if no tracer is added to Rappahannock, the C$_{RR}$ would be 0)

\[ C_{TPR} = \frac{1 \text{mg/L} \left(3.05 \text{m}^3/\text{s}\right)}{0.05 \text{m}^3/\text{s}} = 61 \text{ mg/L} \]

c. Mass Rate of Tin Pot is = C$_{TPR}$Q$_{TPR}$ = (61 mg/L) 0.05 m$^3$/s \[ \left[ \frac{1000 \text{L/m}^3 \left(86,400 \text{sec/d}\right)}{1,000,000 \text{mg/kg}} \right] = 264 \text{ kg/d} \]
3. Five million gallons per day (MGD) of a conservative substance, with concentration 10.0 mg/L, is released into a stream having an upstream flow of 10 MGD and substance concentration of 3.0 mg/L. Assume complete mixing.
   a. What is the concentration in ppm just downstream?
   b. How many pounds of substance per day pass a given spot downstream?

\[ Q_2 = 10 \text{ MGD} \]
\[ C_2 = 2 \text{ MGD} \]
\[ Q_1 = 5 \text{ MGD} \]
\[ C_1 = 10 \text{ MGD} \]
\[ Q_a = ? \]
\[ C_{a2} = ? \]
\[ \text{Mass Rate?} \]

Given: Mixing 10 MGD, 3.0 mg/L, with 5 MGD, 10 mg/L:

Solution:

a. \[ 10 \text{ MGD} \times 3.0 \text{ mg/L} + 5 \text{ MGD} \times 10 \text{ mg/L} = (10 + 5) \text{ MGD} \times C \text{ mg/L} \]
   \[ C = \frac{80}{15} = 5.33 \text{ mg/L} \]

b. \[ 5.33 \text{ mg/L} \times (15 \times 10^6 \text{ gal/day}) \times 3.785 \text{ L/gal} \times (1 \text{ kg/10}^6 \text{ mg}) \times (2.2 \text{ lb/kg}) = 666 \text{ lb/day} \]
4. The two-pond system shown in the following figure is fed by a stream with a flow rate of 1.0 MGD and a BOD (nonconservative pollutant) concentration of 20 mg/L. The rate of decay is 0.30/day. The volume of the first pond is 5.0 million gallons and the second is 3.0 million gallons. Solve for $C_1$ and $C_2$ in mg/L.

Lake 1: $\text{Input} = \text{Output} + KCV$

$1 \text{ MGD} \times 20 \text{ mg/L} = 1 \text{ MGD} \times C_1 + 0.3/\text{day} \times 5 \text{ MG} \times C_1$

$C_1 = \frac{20}{1+1.5} = 8.0 \text{ mg/L}$

Lake 2: $1 \text{ MGD} \times 8.0 \text{ mg/L} = 1 \text{ MGD} \times C_2 + 0.3/\text{day} \times 3 \text{ MG} \times C_2$

$C_2 = \frac{8.0}{1+0.9} = 4.2 \text{ mg/L}$
5. Poorly treated municipal wastewater is discharged to a stream. The river flow rate upstream of the discharge point is \( Q_{u/s} = 8.7 \, \text{m}^3/\text{s} \). The discharge occurs at a flow of \( Q_d = 0.9 \, \text{m}^3/\text{s} \) and has a BOD concentration of 50.0 mg/L. Assuming that the upstream BOD concentration is negligible. (a) What is the BOD concentration just downstream of the discharge point?

**Solution:**

\[
Q_{u/s} (0) + Q_d (C_d) = Q_{total} C_{down}
\]

\[
0.9 \, \text{m}^3/\text{s} \times 50 \, \text{mg/L} = \left(8.7 \, \text{m}^3/\text{s} + 0.9 \, \text{m}^3/\text{s}\right) \times C_{down}
\]

\[
C_{down} = 4.7 \, \text{mg/L}
\]
6. A river with 400 ppm of salts (a conservative substance) and an up stream flow of 25 m$^3$/s receives an agricultural discharge of 5.0 m$^3$/s carrying 2000 mg/L of salts (see figure below, P1.7). The salts quickly become uniformly distributed in the river. A municipality just downstream withdraws water and mixes it with enough pure water (no salt) from another source to deliver water having no more than 500 ppm salts to its customers.

*What should the mixture ratio ($F$) of pure water to the river water be?*

\[ F = \frac{667}{500} - 1 = 0.333 \quad \text{(that is, 1/3 pure water)} \]

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**Figure P1.7**

\[ \begin{align*}
25 \text{ m}^3/\text{s}, 400 \text{ ppm} & \quad \overset{Q}{\longrightarrow} \quad 500 \text{ ppm} \\
\Downarrow & \\
5 \text{ m}^3/\text{s}, 2000 \text{ ppm} & \quad \overset{Q \text{ @ } 0 \text{ ppm}}{\longrightarrow} \quad FQ \text{ @ } 0 \text{ ppm}
\end{align*} \]
7. A pollutant which degrades according to the equation \( \frac{dC}{dt} = -kC \) is said to react according to _______ First Order _______ kinetics.

8. In a material balance problem, the situation where the rate of accumulation is zero is referred to as _______ Steady State _______.