Landau's MFT of Free Energy And Probability

Mini-talk 6

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Agenda

- Landau's
  - Free energy definition
  - Why significant advance in concept?
- Validity of MFT
- RG? Not ready
- Intro: probability
- Next time
  - Time dependent Phase Transition

Generalized MFT

- *Existence* of a free energy functional $F[\varphi(r)]$
- $\varphi(\mathrm{r}) \equiv <\varphi(\mathrm{r})> + \delta\varphi(\mathrm{r})$
- *Near* critical point, can be taylor series expanded in power of $\varphi(\mathrm{r})$

$$F = \int dr \left[a - h\varphi(r) + t\varphi^2(r) + c\varphi^4(r) + \left(\nabla \varphi(r)\right)^2 \right]$$

- *Equilibrium* when F is minimized $\Rightarrow \delta F = 0$

$$-h + 2t\varphi(r) + 4c\varphi^3(r) - 2\nabla^2\varphi(r) = 0$$

What is your
Broken symmetry?
order parameter?

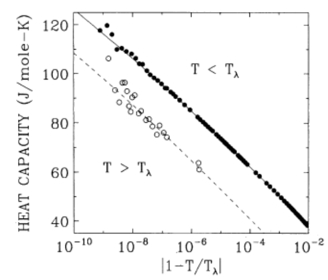
Dimension of space

Nothing New yet
GREAT ADVANCE In THINKING

Summary

- Universality: More is the same
- Symmetry: Broken
- Interactions: K=β J <nn>
- Scaling:
- Order parameter jump:
- Heat capacity jump:
- Correlation length: ξ

$$\frac{M}{\epsilon^{\beta}} = f\left(\frac{H}{\epsilon^{\beta\delta}}\right)$$



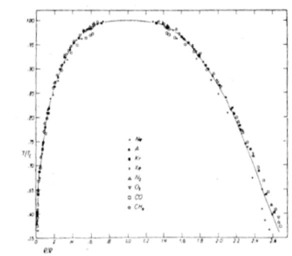


Figure 1.6 Reduced densities of coexisting liquid and gas phases for a number of simple molecular fluids (Guggenheim 1945). The experimental points support a law of corresponding states, but the universal curve is cubic rather than quadratic as required by van der Waals' theory.

Summary

- Correlation length: ξ

$$g(\vec{r},\vec{s}) \equiv \langle [\rho(\vec{r}) - \langle \rho \rangle][\rho(\vec{s}) - \langle \rho \rangle] \rangle = \frac{\partial}{\partial h(\vec{s})} \langle \sigma(\vec{r}) \rangle \Rightarrow$$

$$g(r,r') = \delta_{r,r'} + K \sum_{s \ \langle nn \rangle} g(s,r') - 3 \langle \sigma_r \rangle^2 g(r,r')$$

- *Fourier Transform*

$$g(\vec{r},\vec{r}\,') \equiv \int \frac{d^d \vec{k}}{(2\pi/a)^d} e^{i\vec{k}\bullet(\vec{r}-\vec{r}\,')} g(\vec{k}) \ \text{and} \ \delta_{r,r'} \equiv \int \frac{d^d \vec{k}}{(2\pi/a)^d} e^{i\vec{k}\bullet(\vec{r}-\vec{r}\,')} 1$$

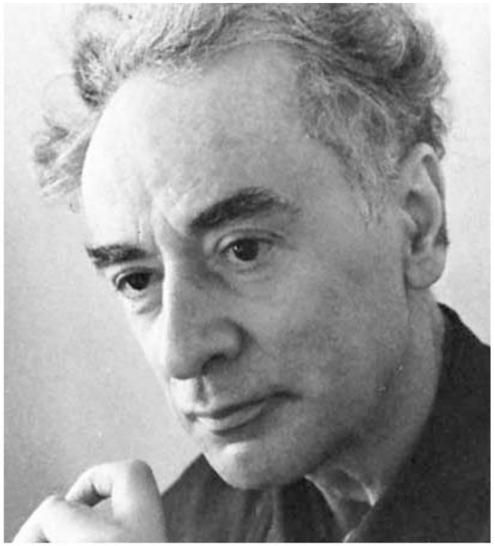
- $\xi \sim (T - T_c)^{-1/2}$

$$g(\vec{r},\vec{r}\,') \sim \left(\frac{a}{|r-r'|}\right)^{(d-2)+\eta} e^{-\frac{|r-r'|}{\xi}}$$

- $V_{Yukawa}(\vec{r},\vec{r}\,') \sim \frac{a}{|r-r'|} e^{-m|r-r'|}$

Landau (~1937) Order Parameter Concept

- 1st order Phase transitions are manifestations of a Broken Symmetry of an
- Order parameter
  - fluid density, magnetization
  - measures the extent of symmetry breaking
- Spatial and time dependence implicates
  - Correlation (scattering experiments) and
  - Dimensionality (next time)



When MFT fails?

$$Ginsburg: \; MFT \; correct \; if \; \frac{\delta\varphi}{<\varphi>} << 1; \; Near \; T_c,$$

- $\xi \sim (T - T_c)^{-1/2} \sim t^{-1/2}$
- $Consider \;\; |r - r'| \sim \xi$

$$g(\vec{r}, \vec{r}\,') \sim \left(\frac{1}{|r - r'|} \right)^{d-2} e^{-\frac{|r-r'|}{\xi}} \rightarrow \left(\frac{1}{|r - r'|} \right)^{d-2} \sim |t|^{d/2-1}$$

- $Since \;\; <\sigma_r> \sim |t|^{1/2} \Rightarrow \frac{g(\vec{r}, \vec{r}\,')}{<\sigma_r><\sigma_{r'}>} \sim |t|^{d/2-2}$

$If \;\; d/2 - 2 > 0, \; LHS \; is \; finite \; or$

$If \;\; d > 4 \; then \; MFT \;\; is \; correct.$

Universality?

- Ferromagnet
- Binary alloy
- Liquid-gas
- Surface adsorption
- Approximated models may be incorrect for critical phenomena

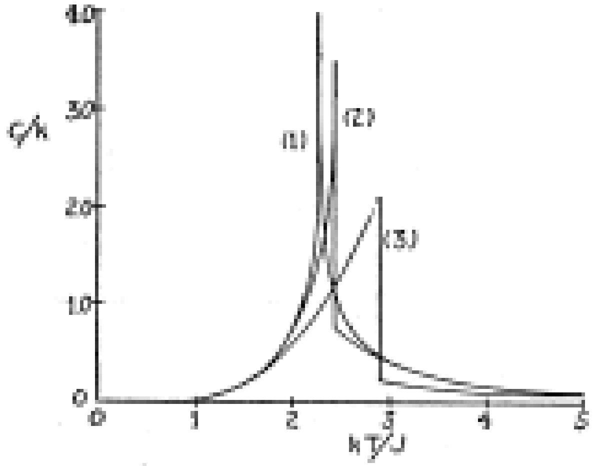


FIG. 1. The exact specific heat curve of the two-dimensional Ising lattice (curve 1) is compared with approximate curves of Kramer-Wannier (curve 2), and Bethe (curve 3) (see reference 5).

Intro: Probability I

1. Ideal gas, normal or Gaussian distributed in momentum

- $p(E) \sim e^{-\beta E} \sim e^{e^{-\frac{1}{2}\frac{mv^2}{kT}}}$; $a\ Gaussian$
- Math: $p(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
-

Intro: Probability II

- $\Gamma \equiv \{q, p\}$ the phase space
- *The* system is represented by a point in Γ
- *An* ensemble (Gibbs) is a swarm or density of points $\rho(\Gamma, t)$

with a volume Ω in Γ:

- $Boltzmann: \; S \equiv k \ln \Omega$
- $Boltzmann: \; S(t) = -k \int_{\Omega} d\Gamma \rho(\Gamma, t) \ln \rho(\Gamma, t)$
- Math: $p(x; \mu, \sigma^2) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
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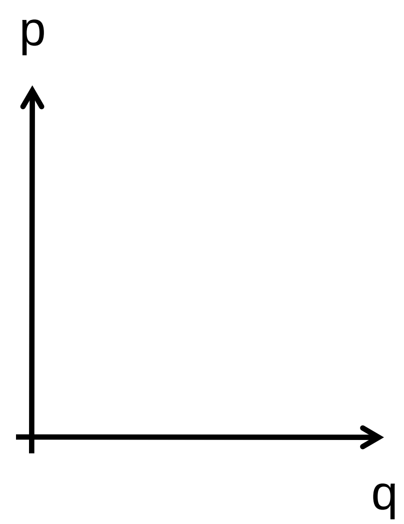


Intro: Probability III

- $p(x;0,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$?
- $I\equiv-\int dxp(x)\ln p(x)-\alpha\left[\int dxp(x)-1\right]-\beta\left[\int dx\, x^2 p(x)-\sigma^2\right]$
- $\delta I=0\ wrt\ \delta p\Rightarrow\int dx\left[-\ln p-1-\alpha-\beta x^2\right]\delta p(x)=0$
- $p(x)=e^{-1-\alpha-\beta x^2}$
- $\int dx\left[p(x)\right]\equiv1\ and\int dx\left[x^2 p(x)\right]\equiv\sigma^2$

$$p(x;0,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$$

A Gaussian distribution maximizes the Entropy

Q: Can one generate a power law, that follow from the scaling hypothesis?

Characteristic Functions

All you want to know but afraid to ask

Q: Uncertainty, $<(\delta x)^2>$?

- $$p(x;\mu,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$
- Calculate $$<e^{ikx}>\equiv\int_{-\infty}^{\infty}dxe^{ikx}p(x;\mu,\sigma^2)\equiv\tilde{p}(k;\mu,\sigma^2)=?$$
- $$<x^n>=\frac{1}{i^n}\frac{\partial^n\tilde{p}(k;\mu,\sigma^2)}{\partial k^n}\bigg|_{k\to 0}=?$$
- $$<x>=\frac{\partial\tilde{p}(k;\mu,\sigma^2)}{\partial(ik)}\bigg|_{k\to 0}=\mu$$

$$<x^2>=\frac{\partial\tilde{p}(k;\mu,\sigma^2)}{\partial(ik)}\bigg|_{k\to 0}=\sigma^2$$

Scale Free Probability VI

- $p(x;\beta,x_0) \sim \frac{1}{x^\beta}$?
- $I \equiv -\int dx p(x) \ln p(x) - \alpha\left[\int dx p(x) - 1\right] - \beta\left[\int dx\ \left[\ln(x+x_0)\right] p(x) - c\right]$
- $\delta I = 0\ wrt\ \delta p \Rightarrow \int dx\left[-\ln p - 1 - \alpha - \beta\ln(x+x_0)\right]\delta p(x) = 0$
- $p(x) = e^{-1-\alpha-\beta\ln(x+x_0)} = e^{-1-\alpha}\frac{1}{(x+x_0)^\beta}$
- $\int_0^\infty dx\left[p(x)\right] \equiv 1 \Rightarrow$

$$p(x;\beta,x_0) = \left(\frac{\beta-1}{x_0}\right)\frac{1}{\left(\frac{x}{x_0}+1\right)^\beta} \to \frac{1}{x^\beta}$$

A Gaussian distribution maximizes the Entropy

Q: Can one generate a power law, that follow from the scaling hypothesis?

1^{st} or 2^{nd} moment does not exist for a p(x)?

No, if a<2 for p(x)~1/x^(1+a).

Is this possible as a physical probability?
Yes.

St. Petersburg's Lemma:
H: 2 i.e. 1^{st} H
TH: 2^2 i.e. 2^{nd} H after 2 tosses etc
TTH: 2^3

$$\frac{1}{2}(2)+\frac{1}{4}(2^2)+\ldots=1+1+\ldots=$$

Challenge: Can you generate?

A Stretched Gaussian or exponential distribution that maximizes the Entropy?

$$p(x) \sim e^{-x^{2-\alpha}}$$

or

$$p(x) \sim e^{-\alpha t^{\beta}}$$

$$where\ 0 < \alpha, \beta < 1?$$

More technical considerations

A Variational Field $\phi(r,t)$ theory *Near Tc*:

- $F[\phi] \equiv \int d^3\vec{r} \left[a + h\phi + \frac{b}{2}\phi^2 + \frac{c}{4}\phi^4 + \frac{\gamma}{6}\phi^6 + p\left[\nabla\phi(r,t)\right]^2 \right]$

$h = external\ field$

$b \sim T - T_c$, the temperature parameter

- $\frac{\delta F[\phi]}{\delta\phi} \equiv 0 \Rightarrow MFT$
- A recipe for kinetic phase transition:

$$\frac{d\phi[t]}{dt} \equiv -\frac{\delta}{\delta t} F[\phi(r,t)] + \epsilon\eta(t,t')$$

Time Correlation Function

3) Electrical conduction of charge.

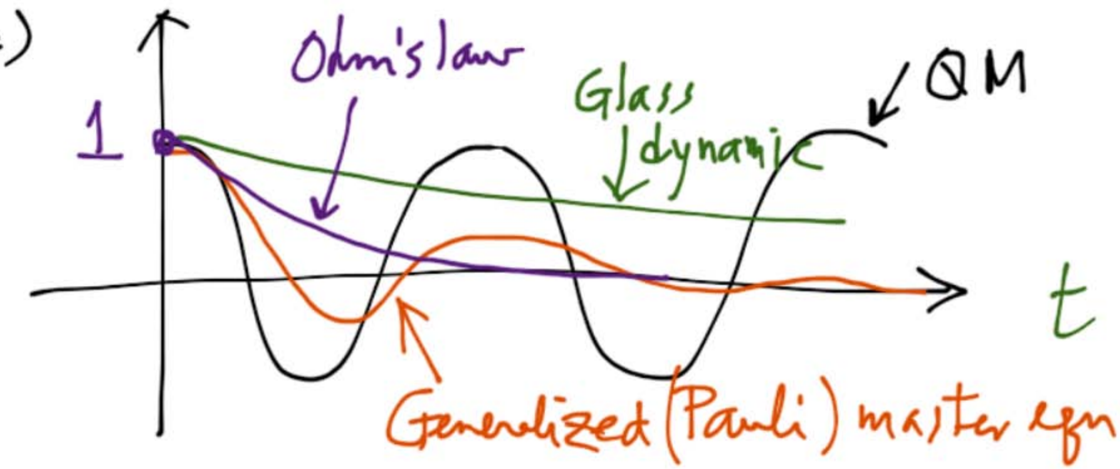
$i(t) \equiv q * v(t)$

(a) $v(t) = \langle v(t) \rangle + \varepsilon \eta(t)$

- $\varepsilon \ll 1$
- $\eta(t)$ noise

(b) $C(t,t') \equiv \langle v(t) v(t') \rangle = ?$

(c)



Summary

- Landau's
  - Order parameter
  - Broken Symmetry
  - Universality
  - Fluctuation
  - Correlation length
- Introduction to Probability
- Next time
  - Random walk
  - "Renormalization" Group????