Ubiquitous
of
Scale Free
Power Laws

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Scaling

- Physics foundation: Critical Phenomena
  - Thermodynamic Functions and Critical Indices
  - Scaling Hypothesis and Fat tail statistics
  - Universality
  - Symmetry and Broken symmetry
- Wide spread applications
  - Chemistry
  - Biology and Physiology
  - Engineering and Material science
  - Economic and Finance Time series
  - Sociology
  - Geology and Climate

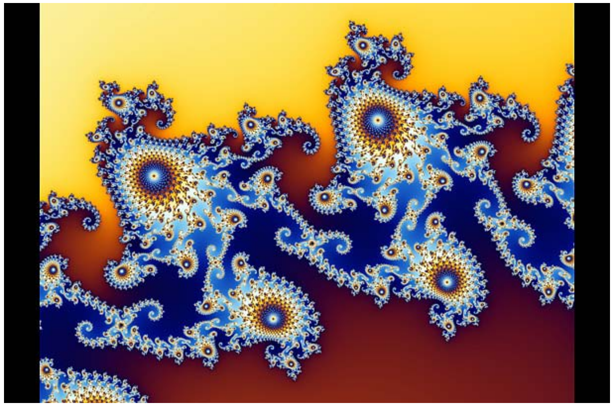
Physics

Fractal Geometry of Nature

What is the coastal length of Britain?

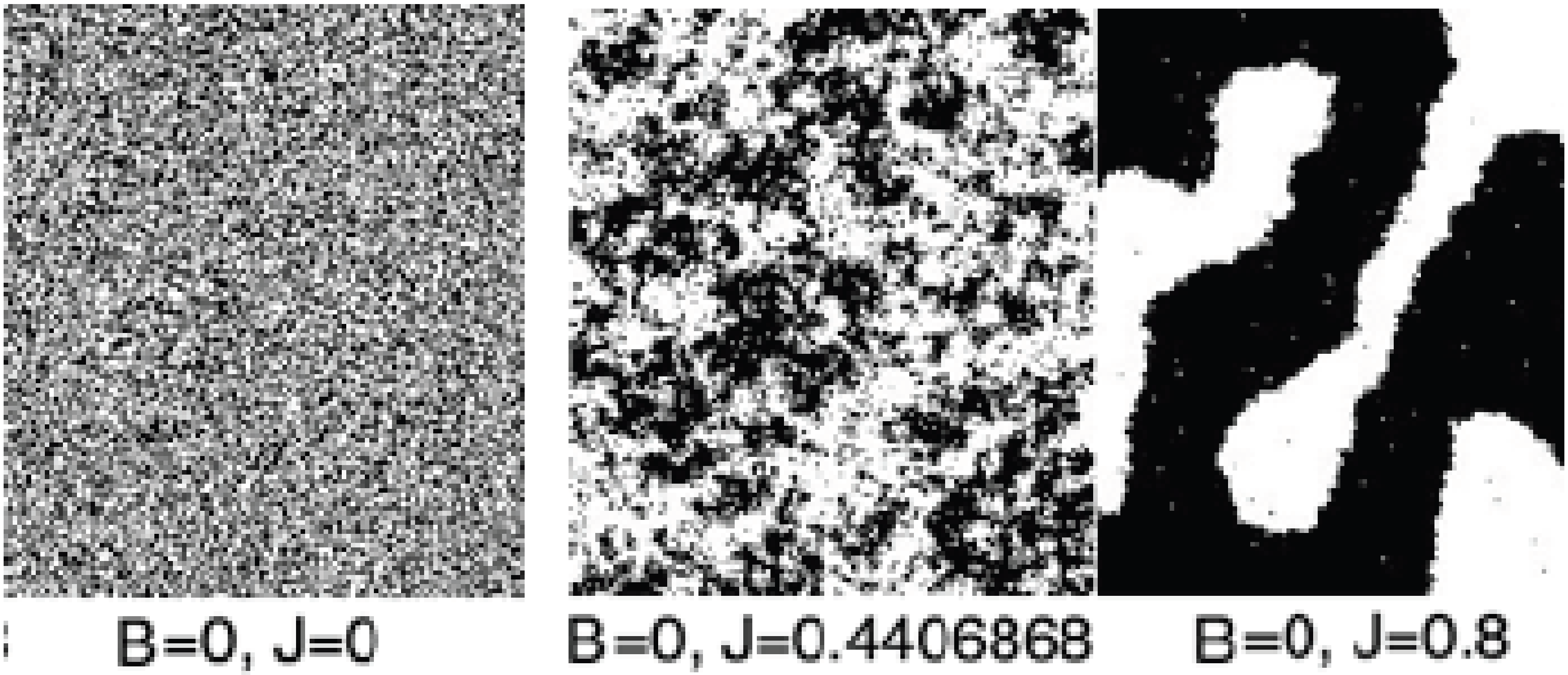
Unit = 200 km -> length = 2400 km (approx.)

Unit = 50 km -> length = 3400 km



Father of Fractal: B. Mandelbrolt

2D Ising model



Scaling in Order Parameter

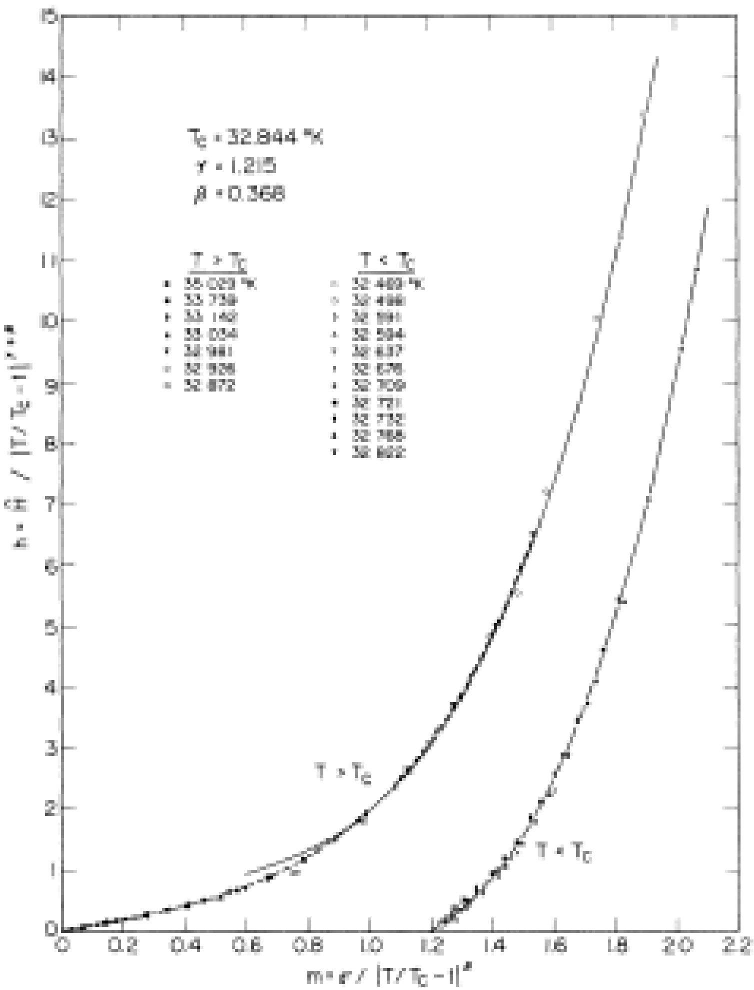


FIG. 1. A linear plot of scaled magnetic field $h(m)$ as a function of scaled magnetization m. The dashed line represents Eq. (1), and fits experimental points from $0 < m < 1.8$ when $T > T_c$. The solid lines are Eq. (2), and the dotted lines show Eq. (4) when it departs significantly from the experimental points. On this plot the origin is the critical isochore, the critical isotherm is at $h = m = \infty$, and the coexistence curve is represented by the point $h = 0$, $m \simeq 1.2$.

J.T. Ho and J.D. Litster, PRL, 22, 603 (1969)

Superfluid λ-Transition

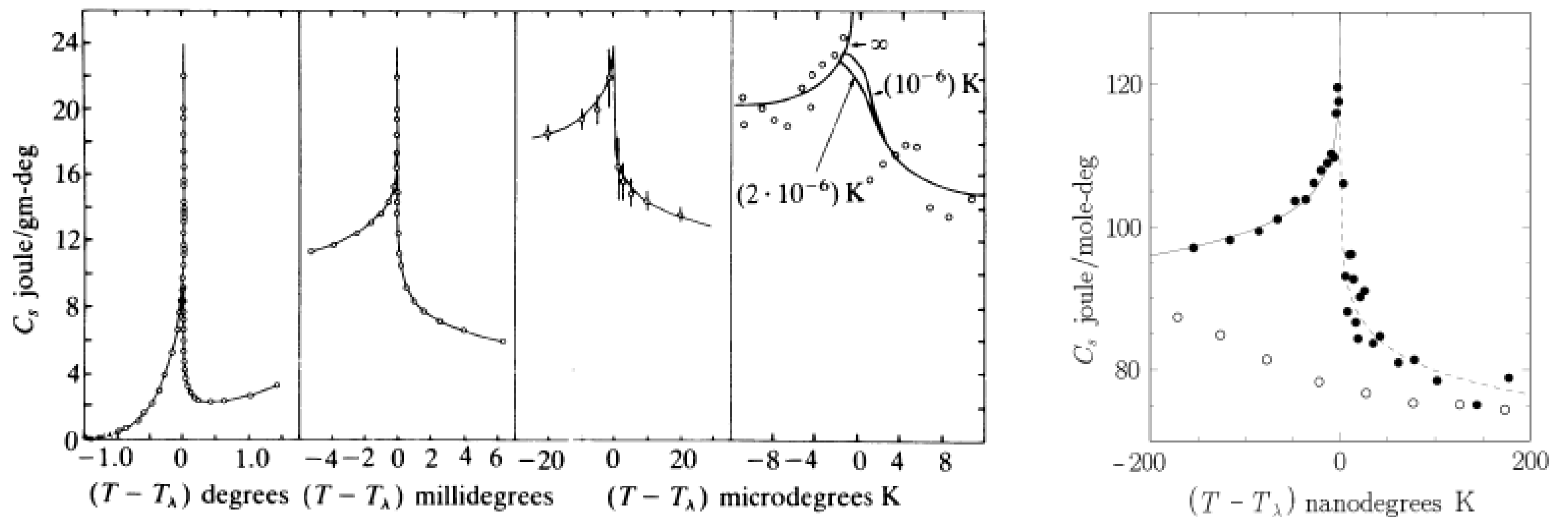


FIGURE 1.1 Specific heat near superfluid transition at $T_\lambda \approx 2.18$ K measured with increasing temperature resolutions. The curve has the typical λ-shape which is the reason for calling it λ-transition. Note that at higher resolutions, the left shoulder of the peak lies above the right shoulder. The data are from Ref. [3]. The forth plot is broadened by the pressure difference between top and bottom of the sample. This is removed by the microgravity experiment in the space shuttle yielding the last plot (open circles are irrelevant here) [4]. They show no pressure broadening even in the nK regime around the critical temperature.

Fairbank et al, Proc. 1965 Washington Conference on Critical Phenomena

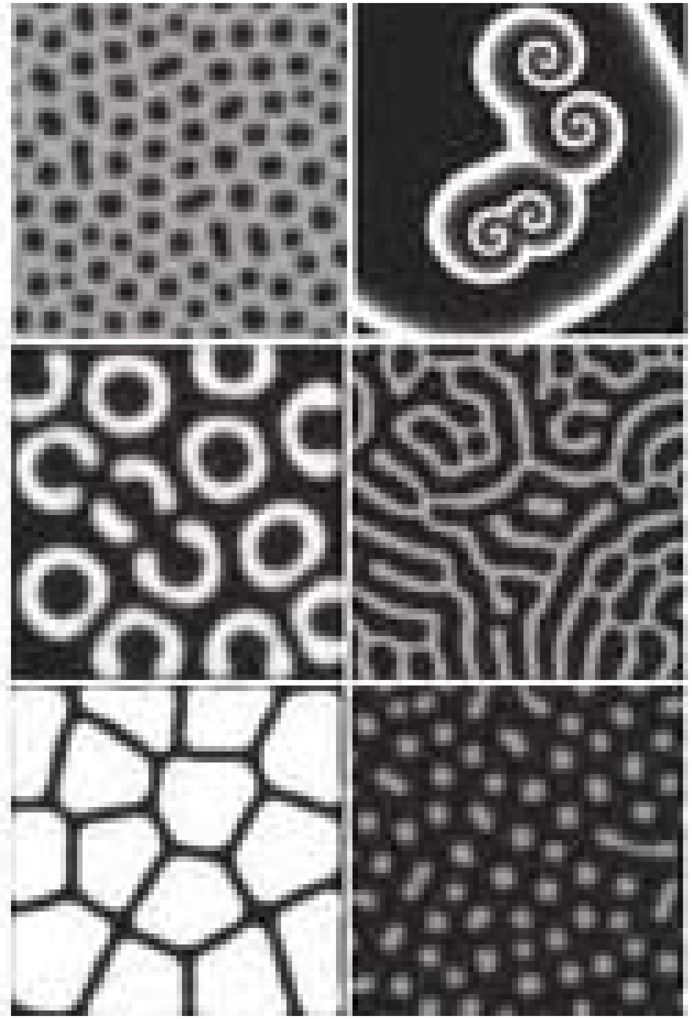
Chemistry

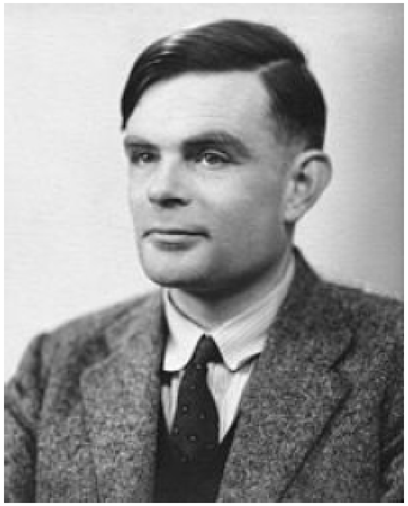
Complexity in Chemistry

George M. Whitesides* and Rustem F. Ismagilov

"Complexity" is a subJect that is beginning to be important in chemistry. Historically, chemistry has emphasized the approximation of complex nonlinear processes by simpler linear ones. Complexity is becoming a profitable approach to a wide range of problems, especially the understanding of life.

Complex pattern formation





Turing: Reaction-Diffusion Model as a Framework for Understanding Biological Pattern Formation

Van der Waal's Equation of State

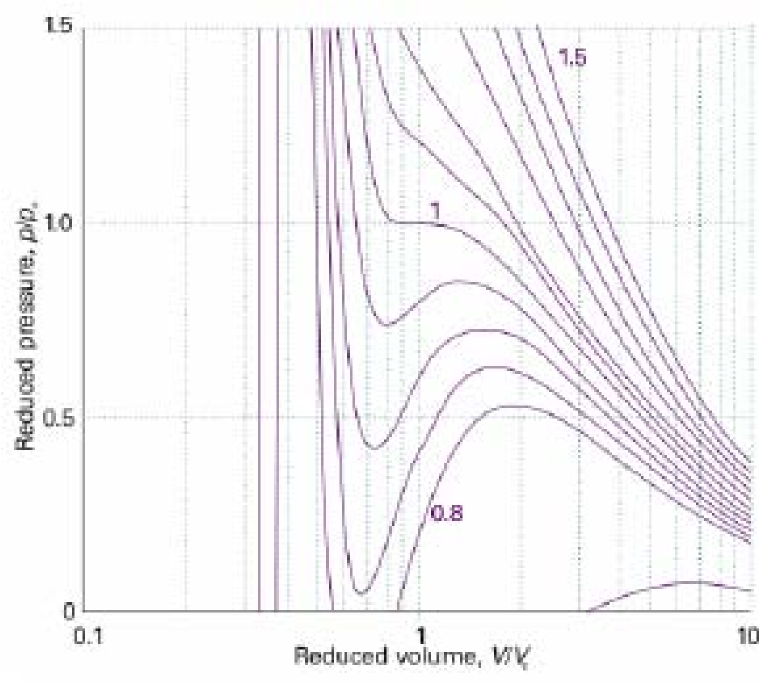


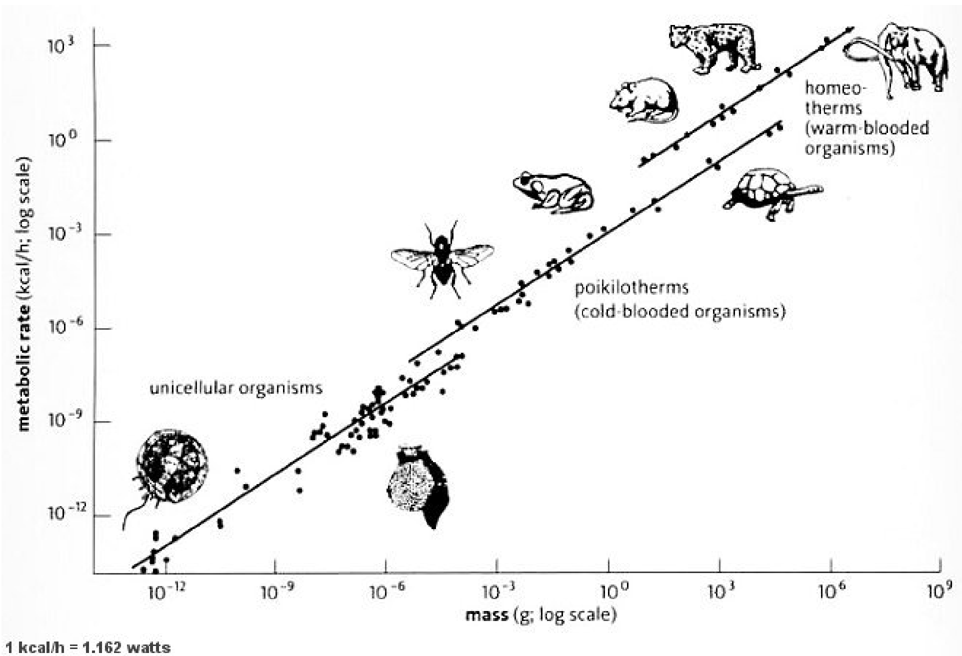
Figure 3: PVT curves predicted by the theory set up by van der Waals. The fluid is mechanically unstable whenever the pressure increases as the volume increases.

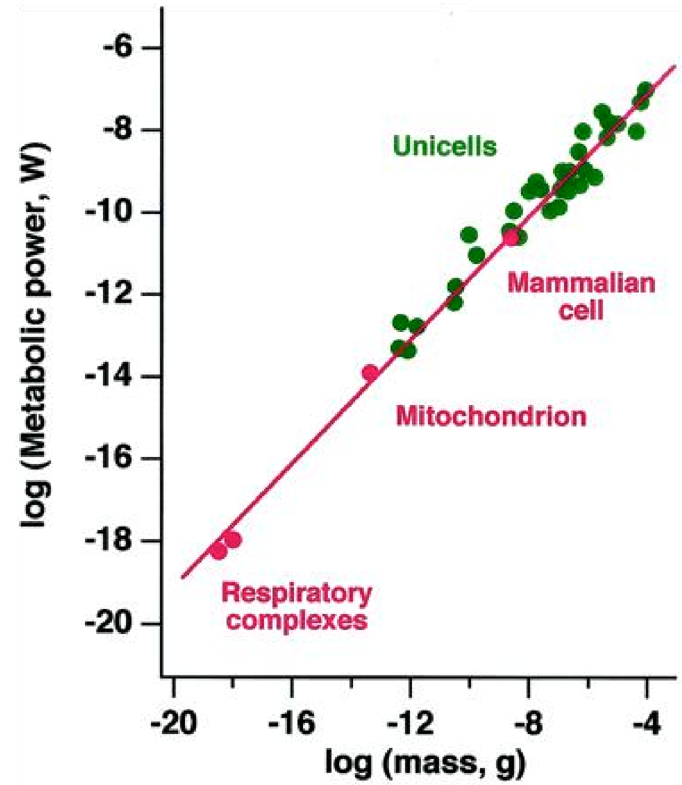
"Cubic" equation results in unstable isotherms below T_c

Appearance of a pair of complex conjugate roots.

Biology and Physiology

Scaling in (micro-) Biology

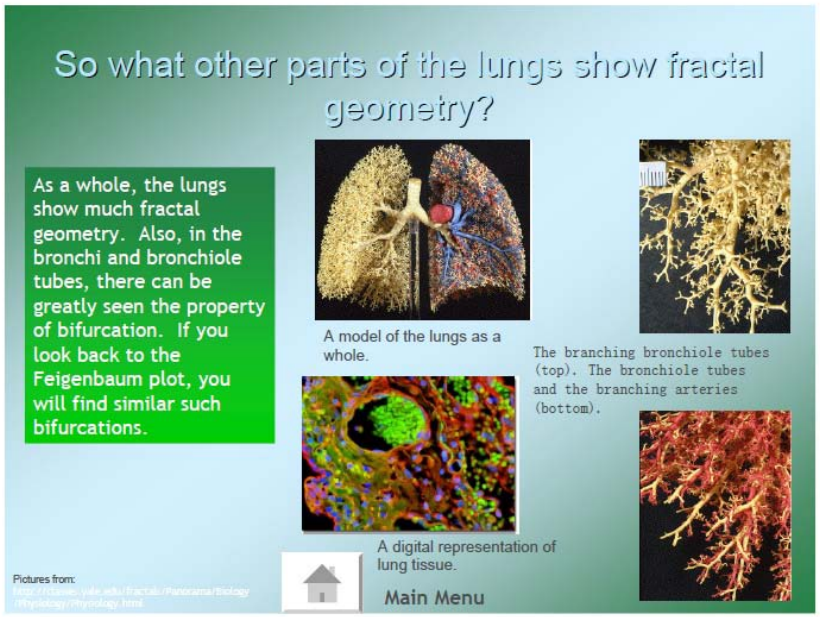


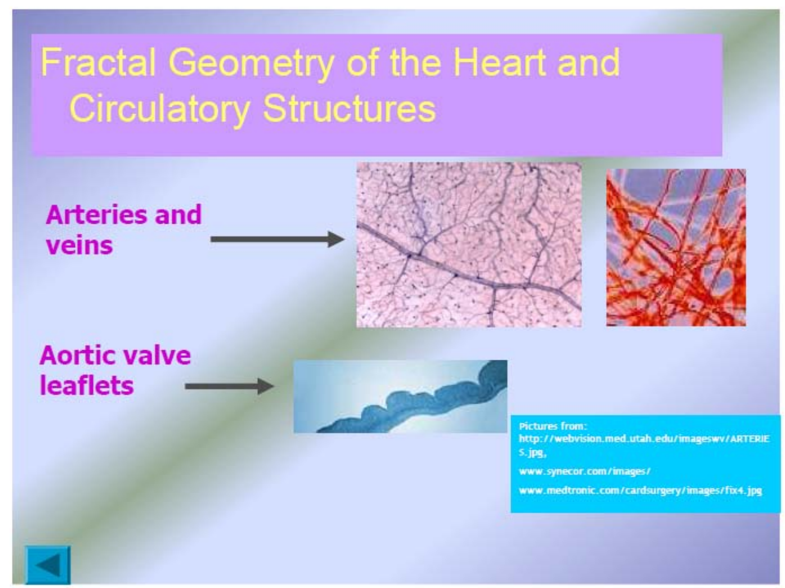


- $k \sim m^{\alpha} \quad (\alpha = 3/4)$
- *Physiology Origin*?

Wekipedia

Fractal Physiology

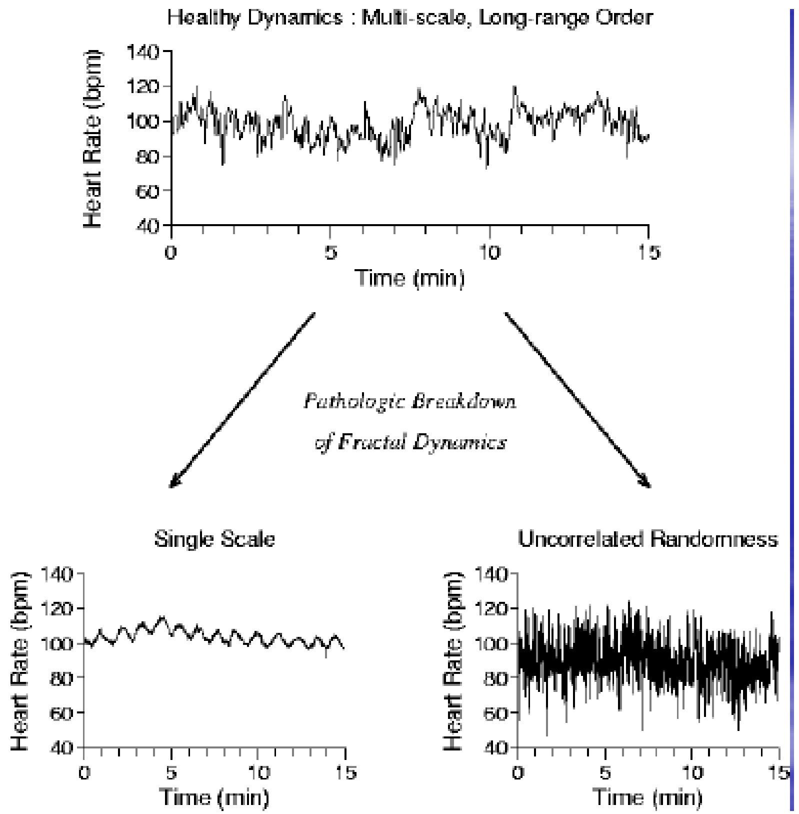




Is Chaos always Bad?

Both of these images show what happens as the heart goes out of its normal state. The bottom left graph shows a subject with heart failure. This graph has highly periodic values with little variation.

The bottom right graph shows a subject with atrial fibrillation. This heart rate is very erratic jumping from the high end of heart rate to the low end, with no particular pattern.



Goal of this interdisciplinary studies is
[1] Understand fractal nature of healthy heart
[2] Relate treatment response and loss of Fractality to Fatalities

Synchronization in Biology

- How thousands of autonomous agents (bird or fish) can behave a single unity without a central command?
- Can one design flock of nano-robots and send them to explore humanly unreachable regions, such as Mars or internal human organs?
- Modeling of heart beat?

http://www.youtube.com/watch?v=V71hz9wNsgs&feature=related

Engineering and Material Science

Glass Dynamics (Angell)

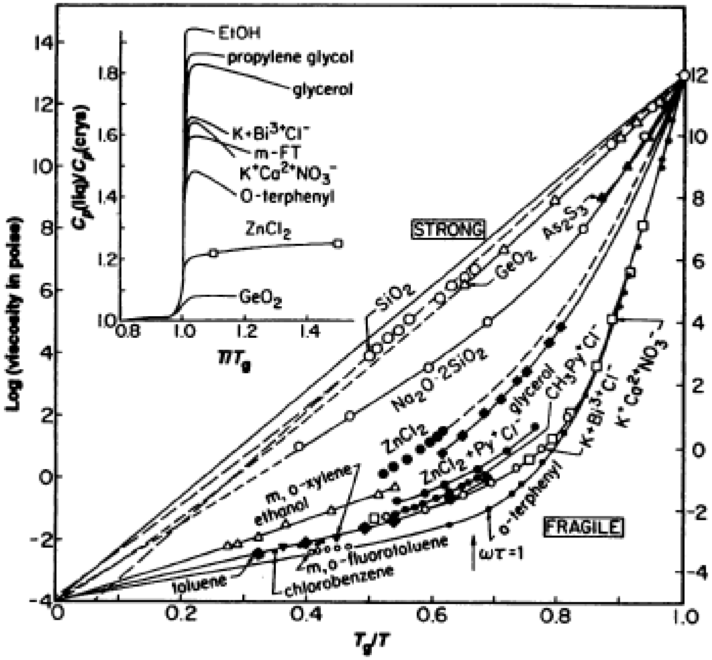


FIG. 3. T_g-scaled Arrhenius plots of viscosity data showing the "strong/fragile" pattern of liquid behavior on which the liquids classification of the same name is based. (*Inset*) The jump in heat capacity at T_g is generally large for the fragile liquids and small for strong liquids, though there are a number of exceptions to this generalization, particularly when hydrogen bonding is present. (Reproduced from ref. 29.)

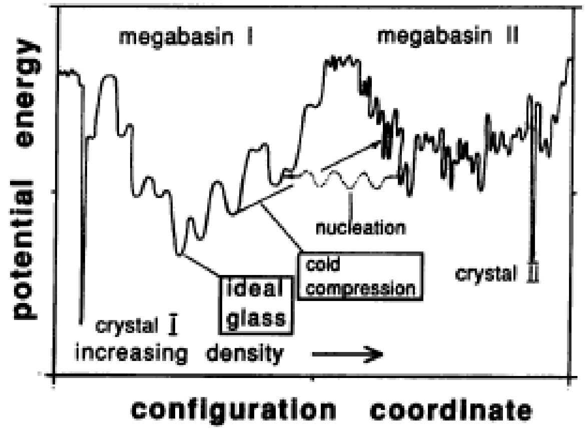
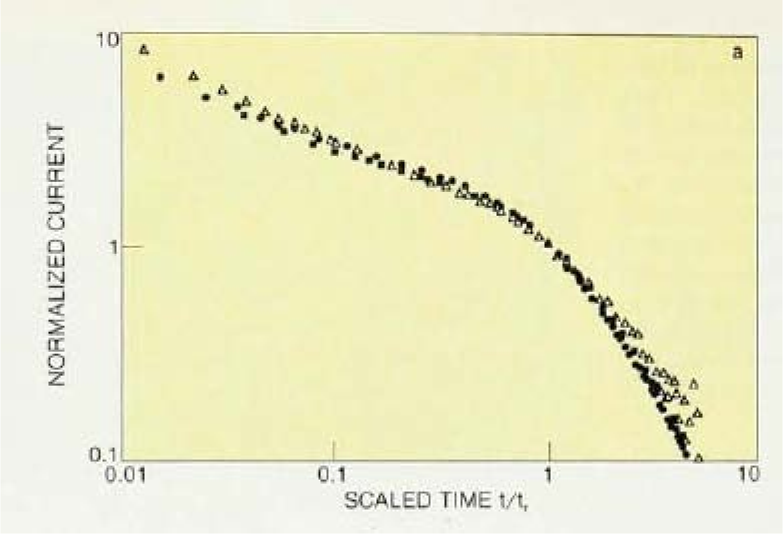
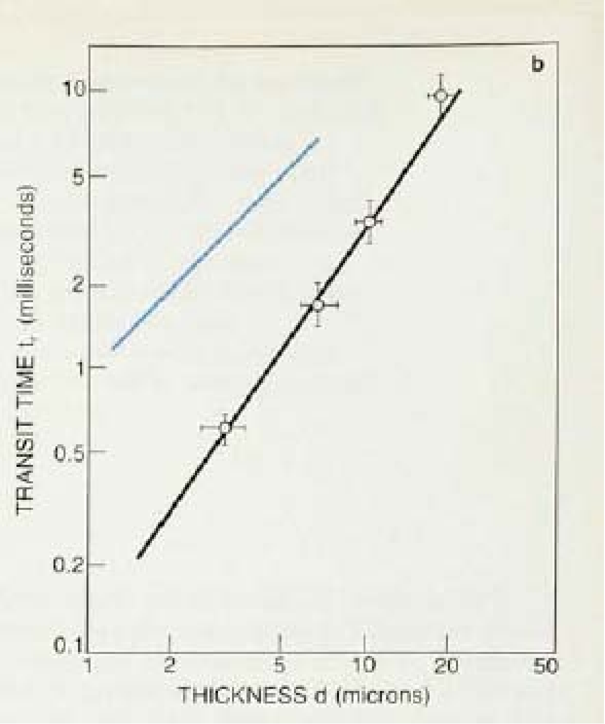


FIG. 7. Potential energy hypersurface showing megabasins needed to understand the existence of polyamorphic forms and the observed first-order-like phase transitions between them. The wavy horizontal arrow indicates narrow channel in configuration space (out of plane of paper) by which nucleation of the low-entropy phase can occur during cooling (strictly, the vertical axis should be a chemical potential in order for the horizontal transition to be appropriate). The inclined straight line schematizes how cold compression can lead to sudden (unnucleated, spinodal-like) collapse to a higher density glass. In addition to the article by Stillinger (49), a good discussion of the nature of the configuration space minima, with an emphasis on spin glasses, is given in an important review of the glassy state by Anderson (1). [Reproduced with permission from ref. 17 (copyright American Association for the Advancement of Science).]

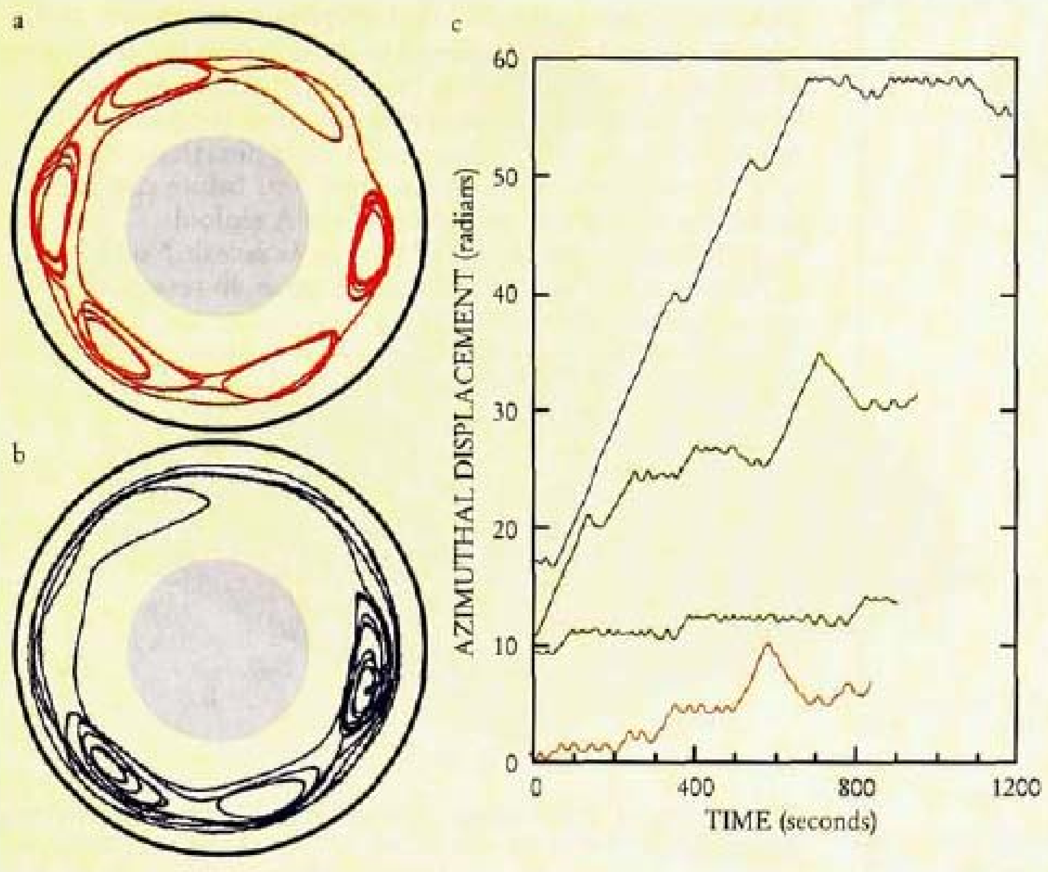
Electrical Conduction in Polymer





Signature of dispersive transport in polyvinylcarbazole and resulting scaling behavior of the transit time. **a:** Superposition master plot of I(*t*) obtained at a number of electric field strengths and temperatures. The value of the exponent β is 0.6. **b:** Dependence of transit time on sample thickness. The black line is drawn with slope $1/\beta$, where β is 0.6. The colored line has the slope of 1 that is expected for normal transport. (Adapted from ref. 7.) **Figure 6**

Levy Flight in Fluid (Swinney)



OBSERVING FLUID FLOW with tracer particles in the rotating-annulus apparatus shown in figure 2 reveals **a:** a state of six stable vortices that frequently trap the particle as it circles around the annulus. **b:** In another observed flow state, the particle spends less time trapped in vortices. **c:** Plotting azimuthal displacement of several particle trajectories against time shows that the **b** trajectory (purple), exhibiting long flights between vortex captures, gets around the annulus much faster than the **a** trajectory (red). (Courtesy of H. Swinney.) FIGURE 3

Material science-Optical Engineering

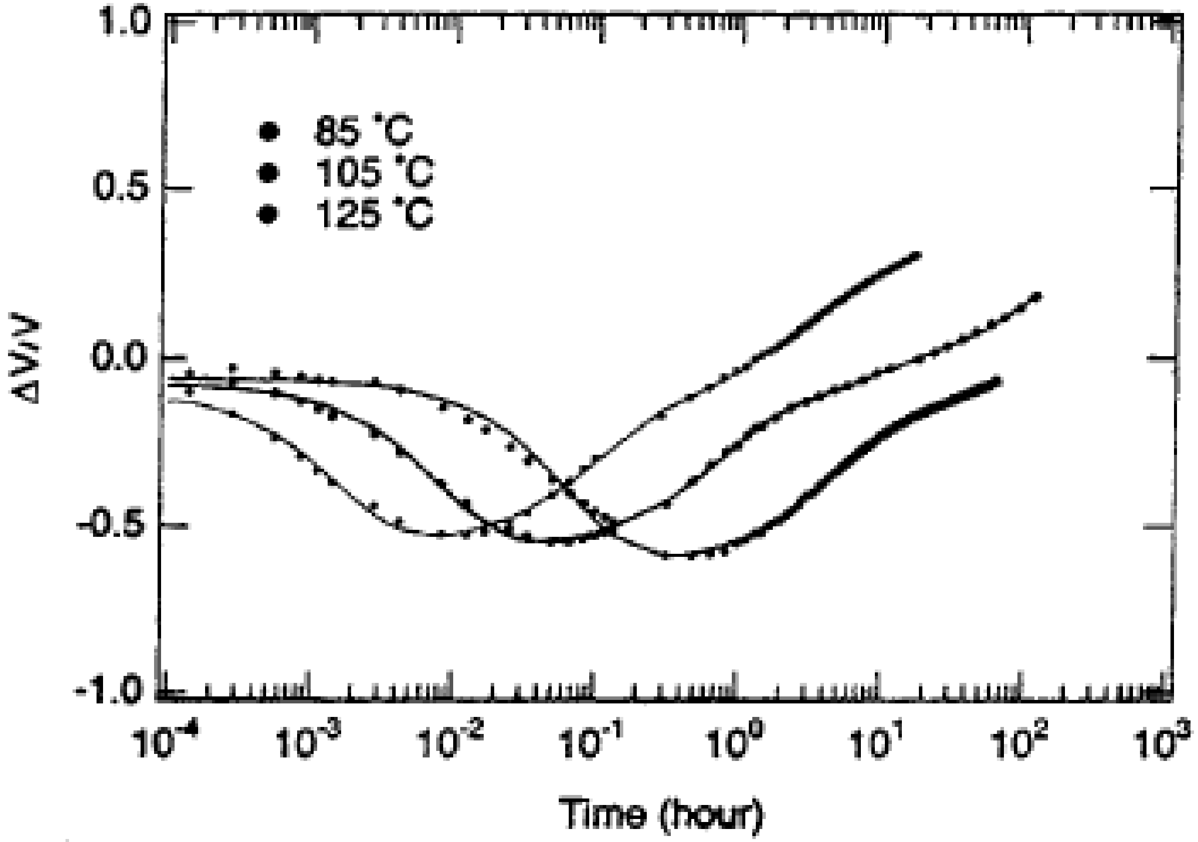
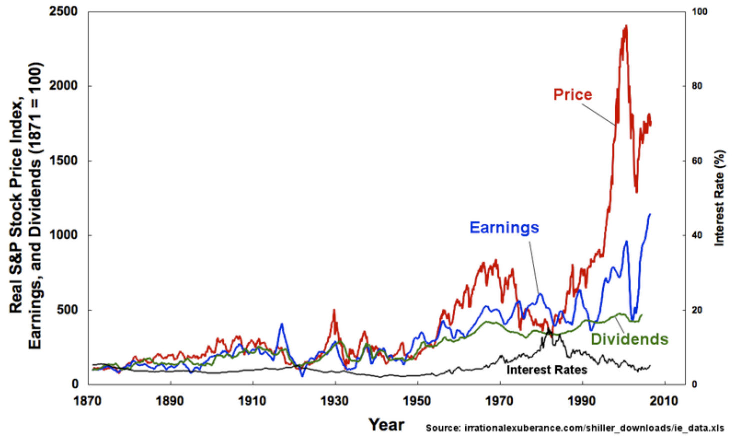


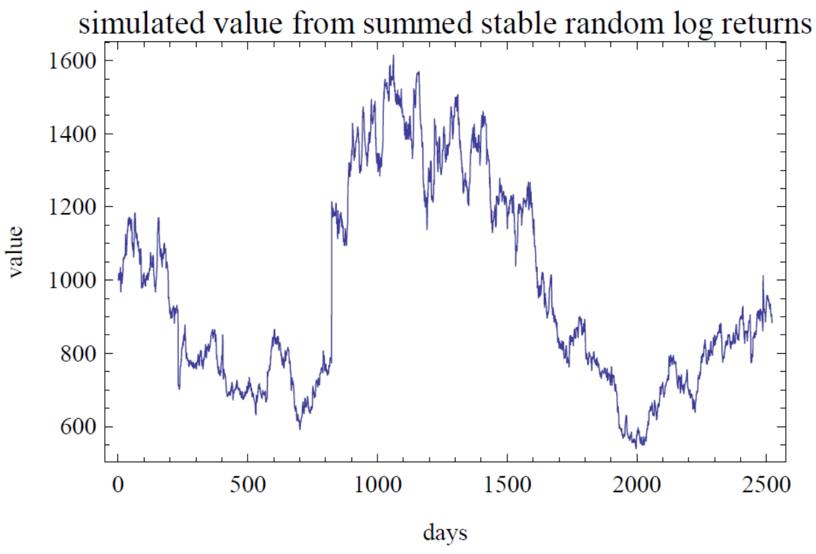
Fig. 13. Bias voltage drift at various temperatures, 10 Gb/s, z-cut, Ti modulators, from Minford [55].

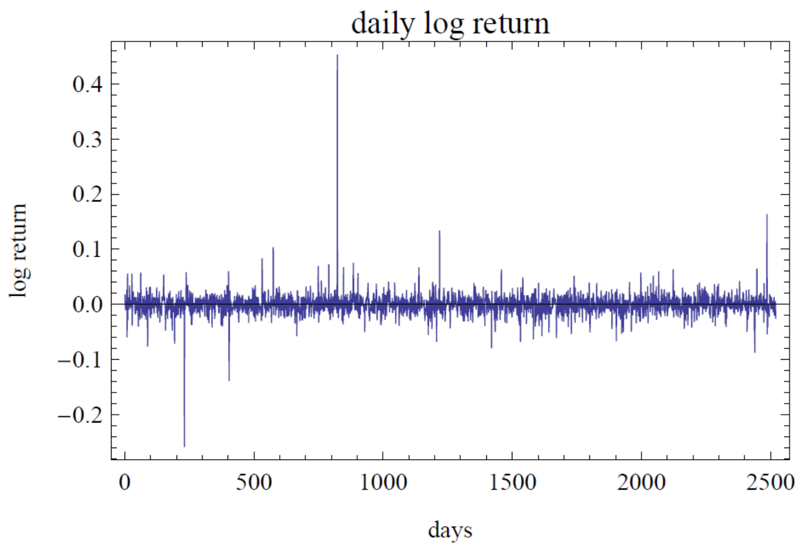
Economic and Finance Time Series

Financial Time series

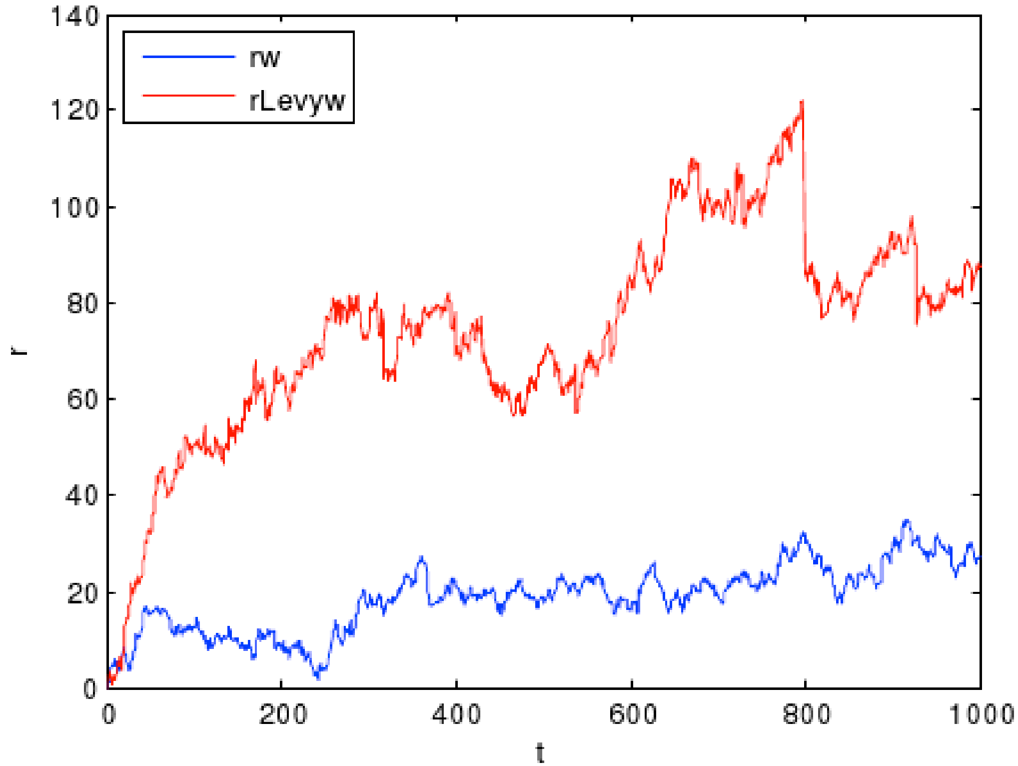
- Is the Market Efficient?
- Physical Reality of Technical Analysis?

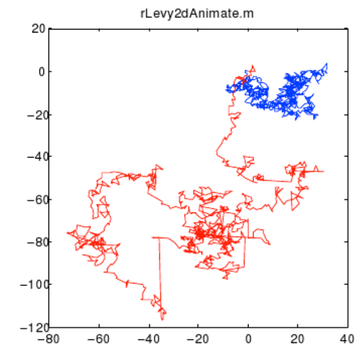




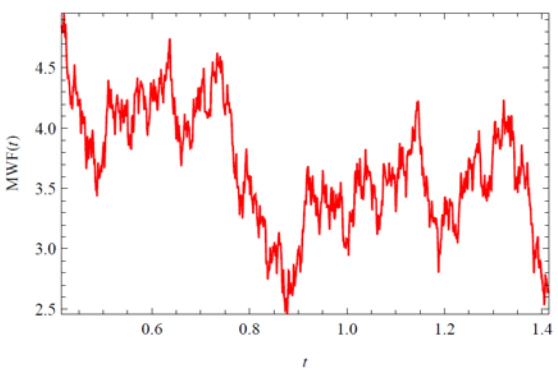


Random Walk Down Wall Street?





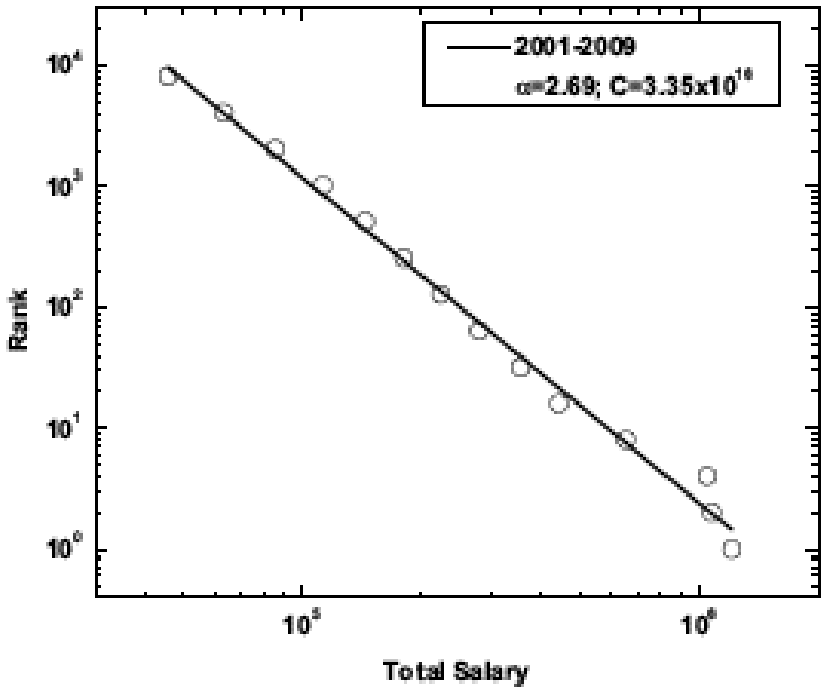
$$\mathrm{MWF}(t) = \sum_{n=1}^{m} 1/b^{n(2-d)}\,(1 - k\cos(b^n t + \phi))$$



1. Blue: standard Random Walk
2. Red: Levy Walk
3. Interaction between N Random Walks?

Sociology

Pareto: Income distribution

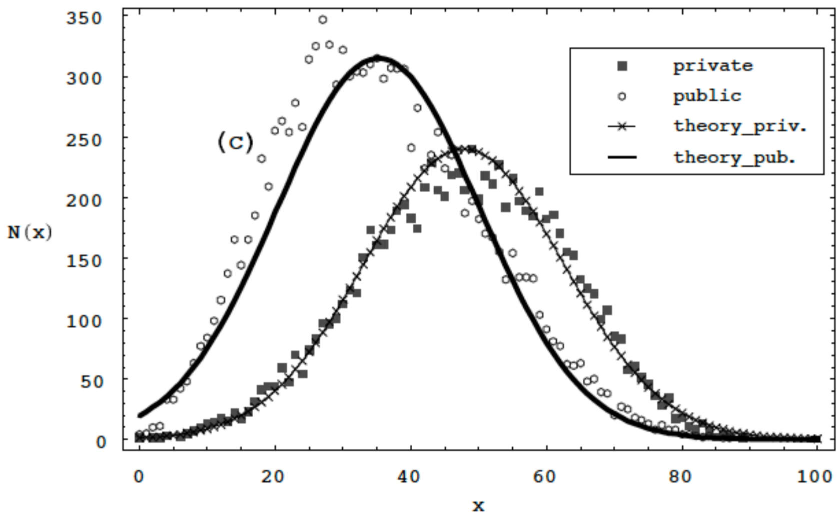


$$cdf(x) = cx^{-\alpha}$$

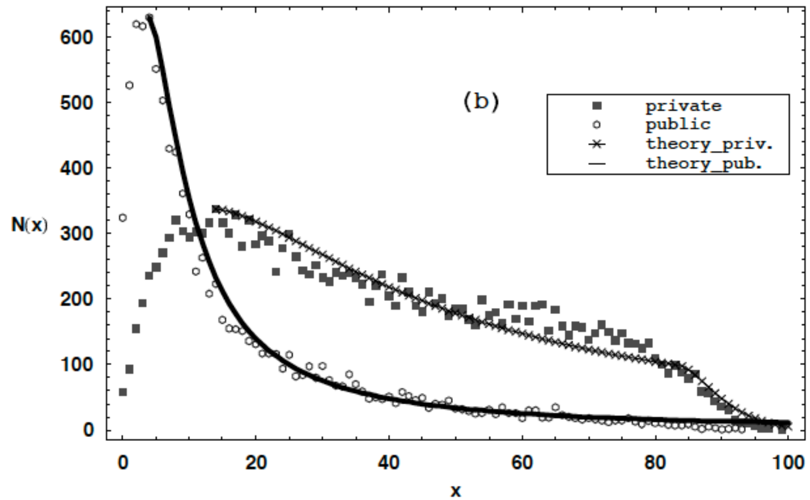
$$p(x) \sim \frac{1}{x^{1+\alpha}}$$

Italian economist Vilfredo Pareto, is a power law probability distribution that coincides with social, scientific, geophysical, actuarial, and many other types of observable phenomena.

Student's Grade distribution



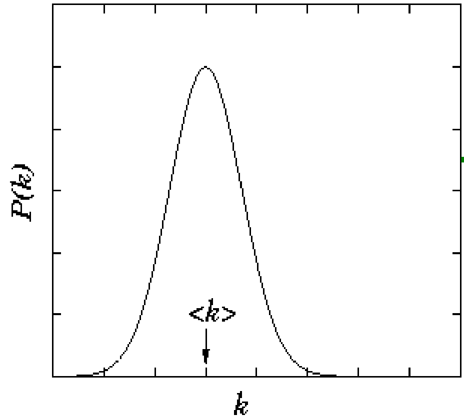
$$p(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



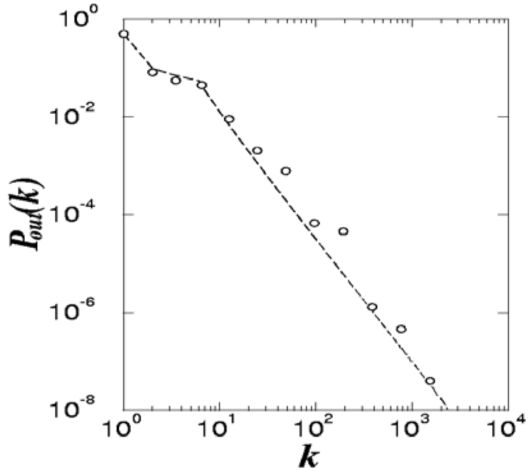
$$p(x;\beta,T) = \frac{(\beta-1)T^{\beta-1}}{(x+T)^{\beta}}$$

Gupta et al., arXiv.0301523

Network Systems



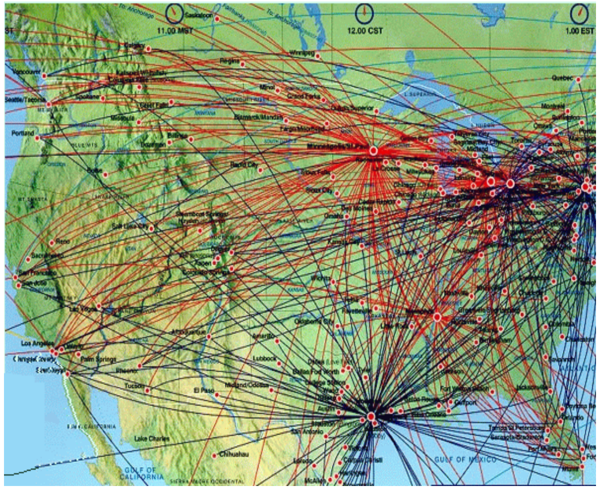
Expected

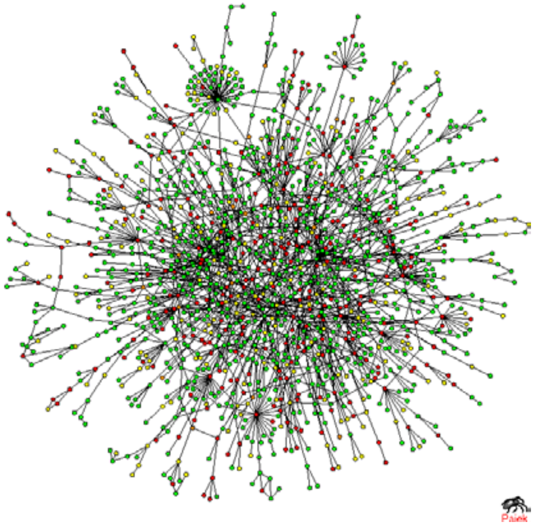


Found

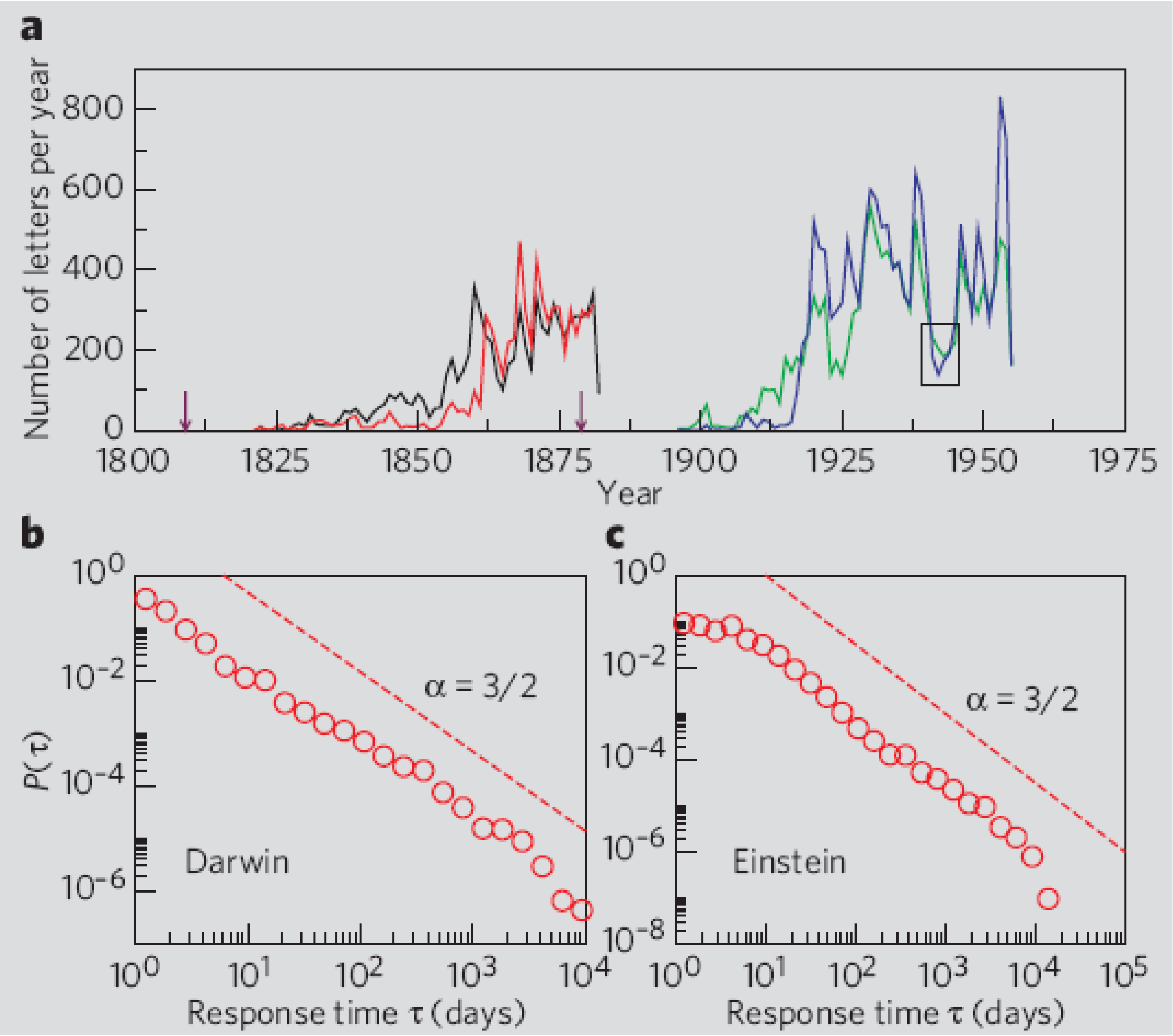
$P(k) \sim k^{-\gamma}$

Hub
within hub
within hub
…….





Darwin's/Einstein's Behavior



Geology, Climate etc

Modeling Earthquake (Turcotte)

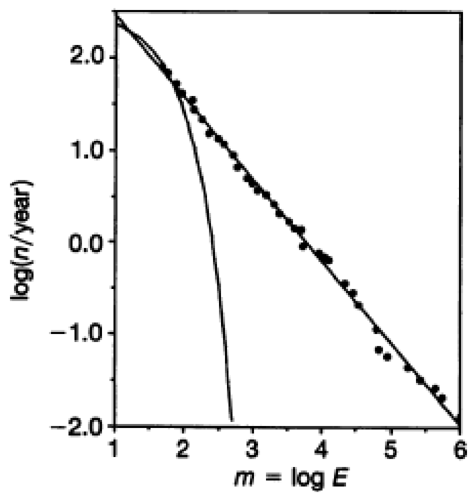


FIG. 1. Distribution of earthquakes in the New Madrid zone in the southeastern United States during the period 1974–1983, collected by Johnson and Nava (14). This power-law, scale-free behavior is compared to a Gaussian curve, which has a sharp cutoff.

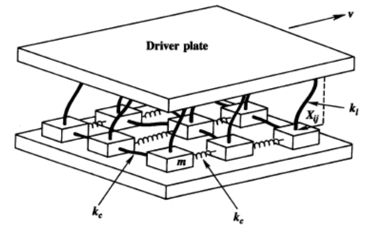


FIG. 9. Illustration of the two-dimensional slider-block model. An array of blocks, each with mass m, is pulled across a surface by a driver plate at a constant velocity v. Each block is coupled to the adjacent blocks with either leaf or coil springs (constant k_c), and to the driver plate with a leaf spring (constant k_l). The extension of the (i, j) pulling spring is x_{ij}.

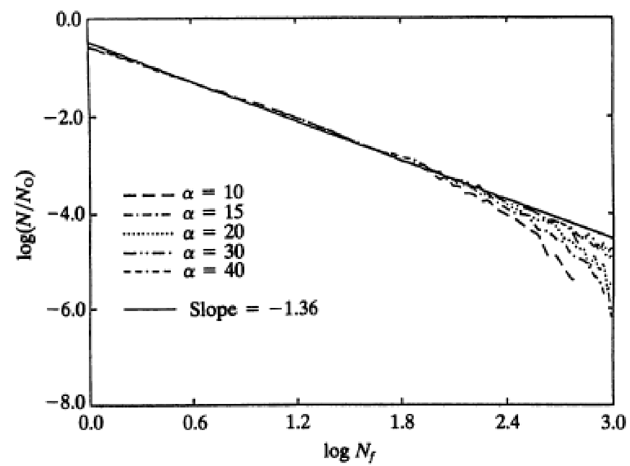


FIG. 10. The ratio of the number of events N with size N_f to the total number of events N_o is plotted against N_f (N_f is the number of blocks that participate in an event and is a measure of the area of an event). Results are given for $\phi = 1.5$ and $\alpha = 10, 15, 20, 30$, and 40. The solid line is the correlation with the power-law (fractal) relation Eq. 1.

Sandpile b/a Avalanche (Bak)

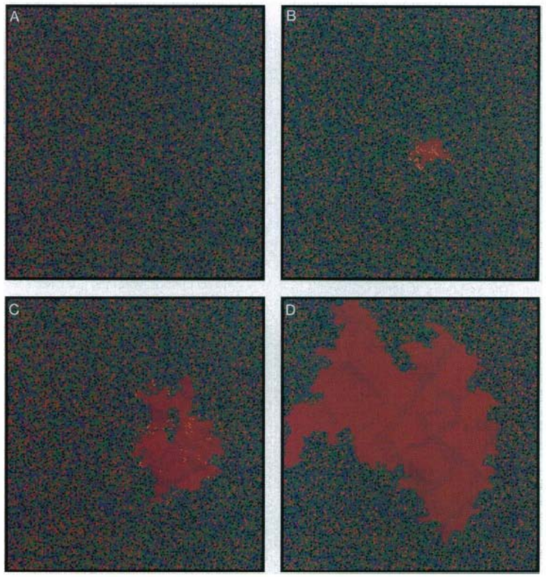


FIG. 3. Evolving avalanche in sandpile model. (*A*) The configuration before a grain of sand is dropped. The various colors indicate heights 0–3, with 3 being the critical height. (*B–D*) Snapshots during the avalanche. The red color indicates sites that have toppled. Yellow sites are active, toppling sites. (Figure courtesy of M. Creutz.)

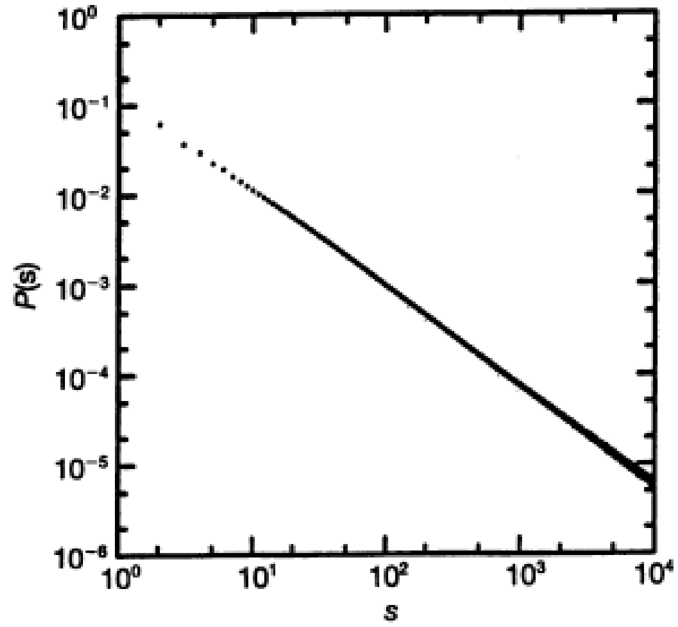


FIG. 2. Power law distribution for avalanches in the sandpile model. Power laws appear as straight lines in double logarithmic plots.