Differential Equations Project 2

In chapter 2, we learned about three different methods for approximating the solution to a differential equation we can't solve: Euler's method, the improved version of Euler's Method, and the Runge-Kutta method.

In this project, you will examine another numerical approximation technique: the Adams method, which is a multistep method. The purpose of the project is two-fold: it will give you experience with reading and interpreting a technical document on your own. In addition, it will give you another method for numerically approximating the solution to a first-order differential equation.

In order to be a truly excellent project, you should turn in a Mathematica notebook on Moodle, as well as a *handwritten* document carefully and thoughtfully answering the questions in part 1 below. All documents should be clearly laid out. I *will not* sift through your results to try to understand what's going on, so you should format your Mathematica notebook in a clear, natural way.

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1. Read the handout describing the Adams-Bashforth formula, and answer the following questions:
   (a) Explain why formula (2) makes sense. (hint: reread your notes from the Runge-Kutta method)
   (b) Why can we approximate $\Phi'(t)$ with a polynomial? (hint: read up on Taylor polynomials)
   (c) We could potentially replace $\Phi'(t)$ with a linear polynomial $ax+b$, a quadratic polynomial ax^2+bx+c, a cubic ax^3+bx^2+cx+d, etc. There is an advantage to using a higher-order polynomials; explain what the advantage is (again, Taylor polynomials will be helpful).
   (d) Why is replacing $\Phi'(t)$ with a polynomial beneficial, in terms of solving the problem (how does it make the computation easier)?
   (e) Consider the second-order Adams-Bashforth formula,
   $$y_{n+1} = y_n + \frac{3}{2}hf_n - \frac{3}{2}hf_{n-1}.$$
   When using the formula, we will be given an initial condition $y(x_0) = y_0$. Notice that the formula doesn't give us a way to find y_1–we must already have an approximation for y_1! What might be an appropriate way to approximate y_1?
2. In Mathematica, write a program for the second-order Adams-Bashforth formula, and use your program to approximate the solution to the initial value problem
   $$y' = -3x^2y,\ y(0) = 3.$$
   (a) Run the program with step size $h = .1$ and have Mathematica graph the approximation curve.
   (b) Run code for approximating the solution with $h = .1$, but using the Runge-Kutta method; again, have Mathematica graph the approximation curve.

(c) The actual solution to the equation is $y = 3e^{-x^3}$. Have Mathematica graph the solution curve along with your two approximation curves on the same axis. Which approximation technique did a better job?

3. For extra credit, write code for the fourth-order Adams-Bashforth formula, and use your code to approximate the solution to the above initial value problem; compare your approximation curve to the two you generated above, as well as to the actual solution curve. Which appears to be the most accurate approximation technique?