A Continuum Approach to Introductory Biomechanics

LAFAYETTE

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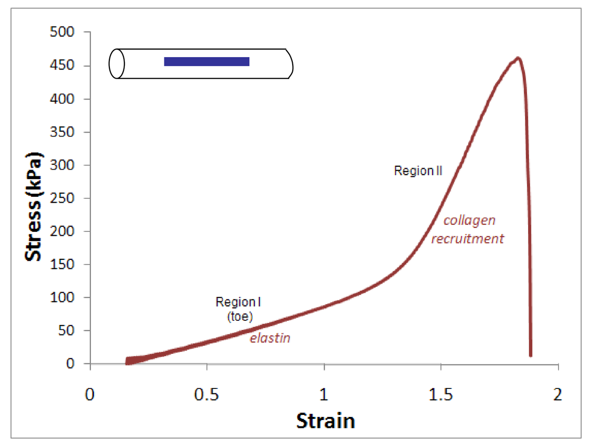
Abstract

An introduction to biomechanics based on a continuum mechanics approach is described. This approach introduces a spectrum of material behavior that has Hookean solids at one extreme, and Newtonian fluids at the other, with many interesting combinations such as biomaterials in between. By building progressively from one-dimensional to higher dimension formulations, this approach makes continuum concepts such as the Cartesian stress tensor accessible to early undergraduate students. From this gradual development of ideas, with many illustrative case studies interspersed, students develop both physical intuition for how bioengineering materials behave, and the mathematics used to describe this behavior.

Introduction

How to introduce students to the complex mechanics of biomaterials? Introductory courses to engineering mechanics generally focus on the limiting cases of ideal solid or fluid behavior, while biological and biomimetic materials often violate the simplifying assumptions made in these cases [1].

It is thus logical to introduce biomechanics in a more inclusive context – that of **continuum mechanics** – in which the behaviors of traditional engineering materials are recognized as special cases, not as the norm. Complex biomaterials are not outliers. This approach has guided the development of graduate curricula in bioengineering [2, e.g.]. Designing an undergraduate bioengineering curriculum is itself a knotty problem, with nonunique solutions. Even defining the "fundamentals" can be a political and pedagogical challenge [3,4]. Continuum mechanics provides an interdisciplinary, inclusive foundation for further study of orthopedic, cardiovascular, and biomimetic mechanics.

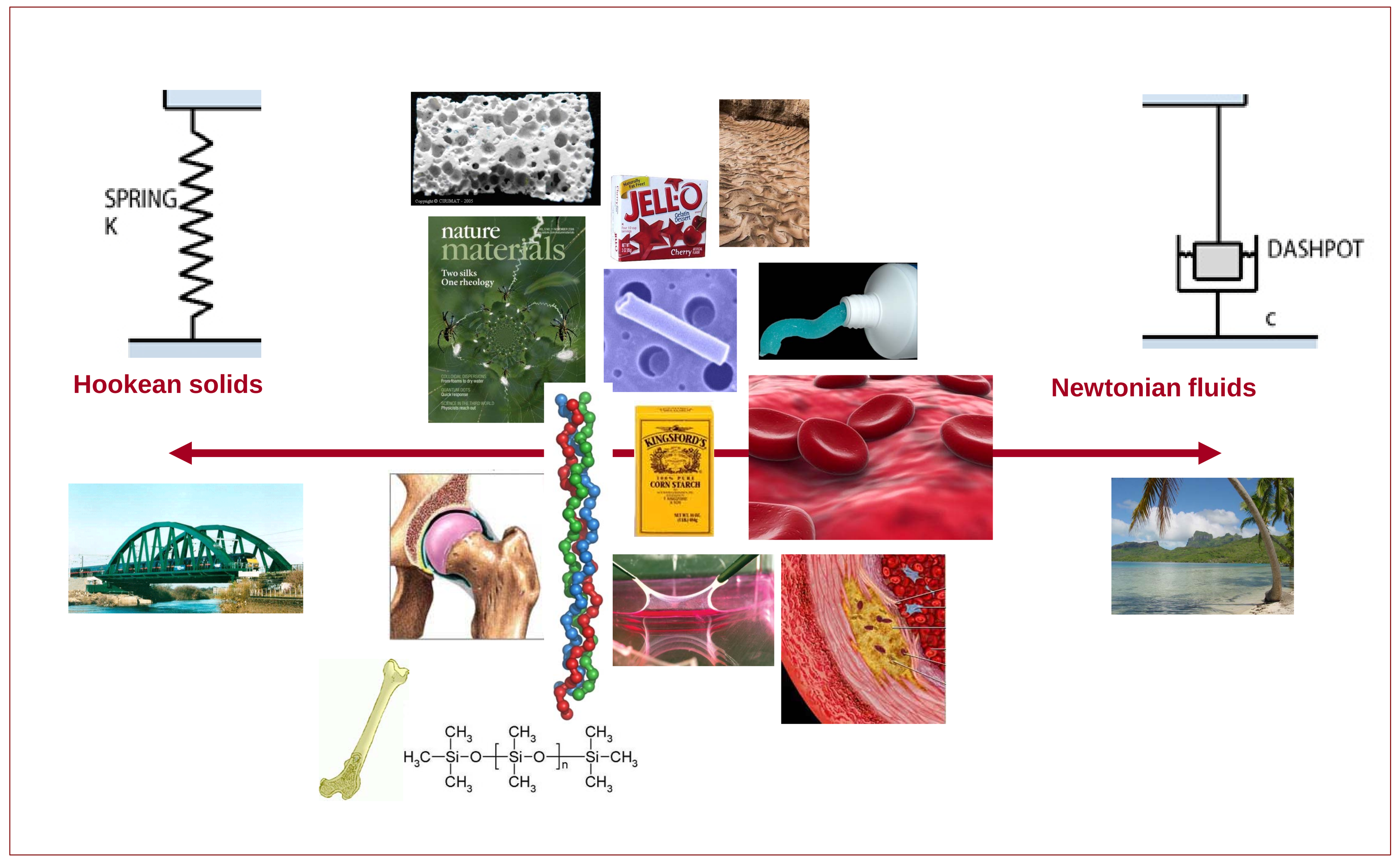


Stress-strain curve for bovine vein displays nonlinearity due to progressive recruitment of collagen fibers. Region I is dominated by elastin, while the steepening slope of Region II represents collagen's increasing contribution to vein's longitudinal stiffness.

From Rossmann, JMBBM 2010

A course in continuum mechanics is typically taken by graduate students or, less often, by advanced undergraduates. Continuum mechanics provides these students with a way to connect their previous work in solid and fluid mechanics: the revelation that the strong resemblance of the governing equations for the motion and deformation of both types of material was not coincidental. This perspective allows students to knit together disparate ideas from prior courses into one unified understanding of how materials respond to loading.

The course described here takes a continuum approach to strength of materials and introductory fluid mechanics, demonstrating the **connections** between solid and fluid mechanics and developing the larger mathematical issues shared by both fields.



Biomechanics is neither "solid" nor "fluid" mechanics

The revelatory *a-ha* of continuum mechanics can be an effective *beginning* for biomechanics instruction, introducing early undergraduate students to mechanics of all materials using a continuum approach. This type of preparation is especially useful for bioengineering students who will address problems that straddle disciplinary boundaries.

An integrated introduction to the mechanics of solids and fluids [3] demonstrates from the beginning the conceptual connections, as well as the larger mathematical issues shared, between types of materials. The context and foundation provided by this introduction is then available to students as they specialize. An integrated introduction is particularly useful in bioengineering – in which many materials are neither Hookean solids nor Newtonian fluids, but contain aspects of both types of behavior.

Students are encouraged to think about why the kinematics of a fluid are better described by a strain *rate* tensor than by pure strain; why viscosity is both similar to and different from Young's modulus; and how the constitutive laws for a solid "spring" and a viscous fluid "dashpot" can be seen as the extrema of a spectrum of material behavior. Most real materials, of course, have behaviors along the middle of that spectrum. Nonlinearity and viscoelasticity are a natural part of this integrative introduction to engineering mechanics.

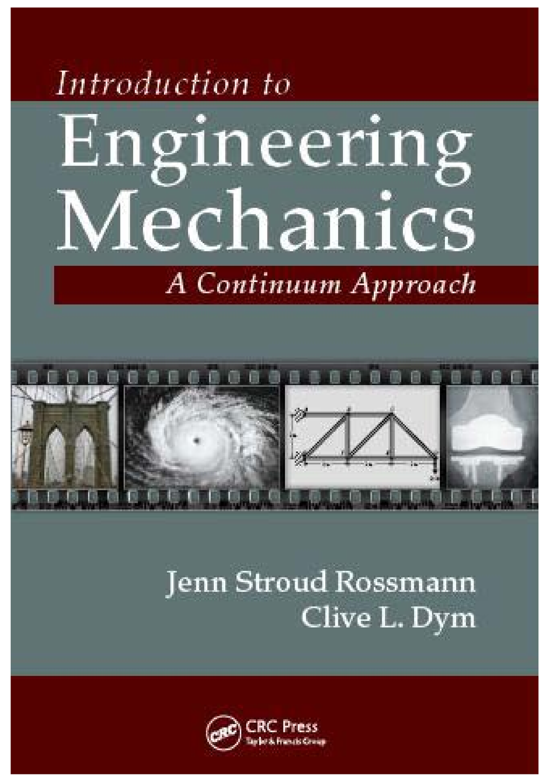
This approach puts biomaterials in the interesting middle of a spectrum of engineering materials, and makes their complex behavior an organic part of a rich picture.

Implementation

The course repeatedly emphasizes the continuum model, accounting for: the kinematics of deformation; the intensity of internal forces, or stress; constitutive laws that relate stress to deformation; and Newton's second law. There is a gradual progression from one- to higher-dimension cases. The introduction of second-order tensors to represent stress and strain, and of a stiffness matrix, facilitates the discussion of the anisotropy and heterogeneity of both biological and biomimetic materials.

The topic outline is closely tracked by the text prepared for the course [5]. This text is designed for undergraduates being introduced to solid and fluid mechanics, rather than the more advanced intended audience of other continuum mechanics textbooks.

Assessment



Both student survey- and instructor rubric-based data indicate students are achieving the course's desired learning outcomes. Rubrics have been developed to measure to what degree students develop particular skills. Student scores on learning outcomes such as their ability to calculate stresses, or to use mathematical models for viscoelasticity to predict a material's behavior, have been consistently high, averaging 4.4 on a five-point scale.

Conclusions

A continuum approach is well suited to introductory biomechanics instruction. This approach has made the complex and challenging subject of continuum mechanics accessible to early undergraduate students, allowing them to develop a method of analysis appropriate for the mechanics of biomaterials. The development of continuum mechanics from straightforward definitions in one dimension to higher dimensions has allowed us to introduce sophomore-level students to second-order tensors that make physical sense. We have endeavored to ensure that the mathematics enhances rather than obscures the elegance and value of our continuum approach. By including discussions of several case studies that showcase not just the details but the multi-faceted big picture, we have demonstrated to students the interdisciplinary applicability of their course material.

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References: [1] Fung, Y.C. Elasticity of soft tissues in simple elongation. *Am. J. Physiol., 213: 1532-1544, 1967;* [2] Bowen, R.M. 1989 Introduction to continuum mechanics for engineers, New York : Plenum Press; [3] Humphrey, J.D. Teaching Undergraduate (Students) Biomechanics, SBC2009 Workshop; [4] Roselli, R.J. and Brophy, S.P. 2001 Movement from a taxonomy-driven strategy of instruction to a challenge-driven strategy in teaching introductory biomechanics. Proceedings of the 2001 ASEE meeting, session 1309;
[5] Rossmann, J.S. and Dym, C.L. 2008 Introduction to engineering mechanics: a continuum approach: Taylor & Francis/CRC Press.

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