UNIAXIAL STRESS-STRAIN

Stress-Strain Curve for Mild Steel

The slope of the linear portion of the curve equals the modulus of elasticity.

DEFINITIONS

Engineering Strain

\[ \varepsilon = \frac{\Delta L}{L_0}, \]

\( \varepsilon \) = engineering strain (units per unit)
\( \Delta L \) = change in length (units) of member
\( L_0 \) = original length (units) of member

Percent Elongation

\[ \% \text{Elongation} = \left( \frac{\Delta L}{L_0} \right) \times 100 \]

Percent Reduction in Area (RA)

The % reduction in area from initial area, \( A_i \), to final area, \( A_f \), is:

\[ \% \text{RA} = \left( \frac{A_i - A_f}{A_i} \right) \times 100 \]

Shear Stress-Strain

\[ \gamma = \frac{\tau}{G}, \]

\( \gamma \) = shear strain
\( \tau \) = shear stress
\( G \) = shear modulus (constant in linear torsion-rotation relationship)

\[ G = \frac{E}{2(1 + v)}, \]

\( E \) = modulus of elasticity (Young's modulus)
\( v \) = Poisson's ratio

\[ v = \frac{-(lateral \ strain)}{(longitudinal \ strain)} \]

Uniaxial Loading and Deformation

\[ \sigma = \frac{P}{A}, \text{ where} \]

\( \sigma \) = stress on the cross section
\( P \) = loading
\( A \) = cross-sectional area

\[ \varepsilon = \frac{\delta}{L}, \text{ where} \]

\( \delta \) = elastic longitudinal deformation
\( L \) = length of member

\[ E = \frac{P}{\delta} \frac{A}{L} \]

\[ \delta = \frac{PL}{AE} \]

True stress is load divided by actual cross-sectional area whereas engineering stress is load divided by the initial area.

THERMAL DEFORMATIONS

\[ \delta_t = \alpha L (T - T_o), \]

\( \delta_t \) = deformation caused by a change in temperature
\( \alpha \) = temperature coefficient of expansion
\( L \) = length of member
\( T \) = final temperature
\( T_o \) = initial temperature

CYLINDRICAL PRESSURE VESSEL

Cylindrical Pressure Vessel

For internal pressure only, the stresses at the inside wall are:

\[ \sigma_t = P_i \frac{r_o^2 + r_i^2}{r_o - r_i} \]

\( \sigma_r = -P_i \)

For external pressure only, the stresses at the outside wall are:

\[ \sigma_t = -P_o \frac{r_o^2 + r_i^2}{r_o - r_i} \]

\( \sigma_r = -P_o \)

\( \sigma_t \) = tangential (hoop) stress
\( \sigma_r \) = radial stress
\( P_i \) = internal pressure
\( P_o \) = external pressure
\( r_i \) = inside radius
\( r_o \) = outside radius

For vessels with end caps, the axial stress is:

\[ \sigma_a = P_i \frac{r_i^2}{r_o - r_i} \]

\( \sigma_t, \sigma_r, \) and \( \sigma_a \) are principal stresses.

When the thickness of the cylinder wall is about one-tenth or less of inside radius, the cylinder can be considered as thin-walled. In which case, the internal pressure is resisted by the hoop stress and the axial stress.

\[ \sigma_t = \frac{P r}{t} \quad \text{and} \quad \sigma_a = \frac{P r}{2t} \]

where \( t \) = wall thickness and \( r = \frac{r_1 + r_2}{2} \).

**STRESS AND STRAIN**

**Principal Stresses**

For the special case of a two-dimensional stress state, the equations for principal stress reduce to

\[ \sigma_{\sigma}, \sigma_b = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

\[ \sigma_c = 0 \]

The two nonzero values calculated from this equation are temporarily labeled \( \sigma_a \) and \( \sigma_b \) and the third value \( \sigma_c \) is always zero in this case. Depending on their values, the three roots are then labeled according to the convention: 

*algebraically largest* = \( \sigma_1 \),  
*algebraically smallest* = \( \sigma_3 \),  
*other* = \( \sigma_2 \).  
A typical 2D stress element is shown below with all indicated components shown in their positive sense.

The circle drawn with the center on the normal stress (horizontal) axis with center, \( C \), and radius, \( R \), where

\[ C = \frac{\sigma_x + \sigma_y}{2}, \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

The two nonzero principal stresses are then:

\[ \sigma_a = C + R \]
\[ \sigma_b = C - R \]

The maximum *inplane* shear stress is \( \tau_{in} = R \). However, the maximum shear stress considering three dimensions is always

\[ \tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \]

**Hooke’s Law**

Three-dimensional case:

\[ \varepsilon_x = \frac{1}{E}(\sigma_x - v(\sigma_y + \sigma_z)) \]
\[ \gamma_{xy} = \tau_{xy}/G \]
\[ \varepsilon_y = \frac{1}{E}(\sigma_y - v(\sigma_z + \sigma_x)) \]
\[ \gamma_{yz} = \tau_{yz}/G \]
\[ \varepsilon_z = -\frac{1}{E}(v\sigma_x + v\sigma_y) \]
\[ \gamma_{zx} = \tau_{zx}/G \]

Plane stress case \((\sigma_z = 0)\):

\[ \varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) \]
\[ \varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x) \]
\[ \varepsilon_z = -\frac{1}{E}(v\sigma_x + v\sigma_y) \]

Uniaxial case \((\sigma_y = \sigma_z = 0)\):

\[ \sigma_x = E \varepsilon_x \text{ or } \sigma = E \varepsilon_x \]

\( E \) = modulus of elasticity

\( G \) = shear modulus

\( v \) = Poisson's ratio

TORSION
Torsion stress in circular solid or thick-walled (t > 0.1 r) shafts:
\[ \tau = \frac{T r}{J} \]
where \( J \) = polar moment of inertia

TORSIONAL STRAIN
\[ \gamma_{\phi z} = \lim_{\Delta \phi \to 0} r (\Delta \phi / \Delta z) = r (d\phi / dz) \]

The shear strain varies in direct proportion to the radius, from zero strain at the center to the greatest strain at the outside of the shaft. \( d\phi / dz \) is the twist per unit length or the rate of twist.

\[ \tau_{\phi z} = G \gamma_{\phi z} = Gr (d\phi / dz) \]
\[ T = G (d\phi / dz) \int_A r^2 dA = GJ (d\phi / dz) \]
\[ \phi = \int_0^L \frac{T}{GJ} dz = \frac{TL}{GJ}, \text{ where} \]

\( \phi \) = total angle (radians) of twist
\( T \) = torque
\( L \) = length of shaft

\( T/\phi \) gives the twisting moment per radian of twist. This is called the torsional stiffness and is often denoted by the symbol \( k \) or \( c \).

For Hollow, Thin-Walled Shafts
\[ \tau = \frac{T}{2A_m t}, \text{ where} \]
\( t \) = thickness of shaft wall
\( A_m \) = the total mean area enclosed by the shaft measured to the midpoint of the wall.

BEAMS
Shearing Force and Bending Moment Sign Conventions
1. The bending moment is positive if it produces bending of the beam concave upward (compression in top fibers and tension in bottom fibers).
2. The shearing force is positive if the right portion of the beam tends to shear downward with respect to the left.

\[ \sigma_x = -My/I, \text{ where} \]
\( M \) = the moment at the section
\( I \) = the moment of inertia of the cross section
\( y \) = the distance from the neutral axis to the fiber location above or below the neutral axis

The maximum normal stresses in a beam due to bending:
\[ \sigma_x = \pm Mc/I, \text{ where} \]
\( c \) = distance from the neutral axis to the outermost fiber of a symmetrical beam section.
\[ \sigma_x = -M/s, \text{ where} \]
\( s \) = \( I/c \): the elastic section modulus of the beam.

Transverse shear stress:
\[ \tau_{xy} = VQ/(Ib), \text{ where} \]
\( V \) = shear force
\( Q \) = \( A'y' \), where
\( A' \) = area above the layer (or plane) upon which the desired transverse shear stress acts
\( y' \) = distance from neutral axis to area centroid
\( B \) = width or thickness or the cross-section
Transverse shear flow:
\[ q = VQ/I \]

Deflection of Beams
Using $1/\rho = M/(EI)$,

$$EI \frac{d^2y}{dx^2} = M,$$

differential equation of deflection curve

$$EI \frac{d^3y}{dx^3} = dM(x)/dx = V$$

$$EI \frac{d^4y}{dx^4} = dV(x)/dx = -w$$

Determine the deflection curve equation by double integration
(apply boundary conditions applicable to the deflection and/or slope).

$$EI \left(\frac{dy}{dx}\right) = \int M(x) \, dx$$

$$EIy = \int \left[ \int M(x) \, dx \right] \, dx$$

The constants of integration can be determined from the
physical geometry of the beam.

Composite Sections
The bending stresses in a beam composed of dissimilar
materials (material 1 and material 2) where $E_1 > E_2$ are:

$$\sigma_1 = -nMy/I_1$$

$$\sigma_2 = My/I_1$$

$I_1$ = the moment of inertia of the transformed section

$n$ = the modular ratio $E_1/E_2$

$E_1$ = elastic modulus of material 1

$E_2$ = elastic modulus of material 2

The composite section is transformed into a section composed
of a single material. The centroid and then the moment of
inertia are found on the transformed section for use in the
bending stress equations.

COLUMNS
Critical axial load for long column subject to buckling:
Euler's Formula

$$P_{cr} = \frac{\pi^2EI}{(K\ell)},$$

where

$\ell$ = unbraced column length

$K$ = effective-length factor to account for end supports

Theoretical effective-length factors for columns include:
Pinned-pinned, $K = 1.0$
Fixed-fixed, $K = 0.5$
Fixed-pinned, $K = 0.7$
Fixed-free, $K = 2.0$

Critical buckling stress for long columns:

$$\sigma_{cr} = \frac{P_{cr}A}{Kr^2},$$

where

$r$ = radius of gyration $\sqrt{I/A}$

$Kr$ = effective slenderness ratio for the column

ELASTIC STRAIN ENERGY

If the strain remains within the elastic limit, the work done
during deflection (extension) of a member will be transformed
into potential energy and can be recovered.

If the final load is $P$ and the corresponding elongation of a
tension member is $\delta$, then the total energy $U$ stored is equal to
the work $W$ done during loading.

$$U = W = P\delta/2$$

The strain energy per unit volume is

$$u = U/AL = \sigma^2/2E$$

(for tension)
## MATERIAL PROPERTIES

### Table 1 - Typical Material Properties
(Use these values if the specific alloy and temper are not listed on Table 2 below)

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity, E [Mpsi (GPa)]</th>
<th>Modulus of Rigidity, G [Mpsi (GPa)]</th>
<th>Poisson's Ratio, ν</th>
<th>Coefficient of Thermal Expansion, α [10⁻⁶/°F (10⁻⁶/°C)]</th>
<th>Density, ρ [lb/in³ (Mg/m³)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>29.0 (200.0)</td>
<td>11.5 (80.0)</td>
<td>0.30</td>
<td>6.5 (11.7)</td>
<td>0.282 (7.8)</td>
</tr>
<tr>
<td>Aluminum</td>
<td>10.0 (69.0)</td>
<td>3.8 (26.0)</td>
<td>0.33</td>
<td>13.1 (23.6)</td>
<td>0.098 (2.7)</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>14.5 (100.0)</td>
<td>6.0 (41.4)</td>
<td>0.21</td>
<td>6.7 (12.1)</td>
<td>0.246–0.282 (6.8–7.8)</td>
</tr>
<tr>
<td>Wood (Fir)</td>
<td>1.6 (11.0)</td>
<td>0.6 (4.1)</td>
<td>0.33</td>
<td>1.7 (3.0)</td>
<td>–</td>
</tr>
<tr>
<td>Brass</td>
<td>14.8–18.1 (102–125)</td>
<td>5.8 (40)</td>
<td>0.33</td>
<td>10.4 (18.7)</td>
<td>0.303–0.313 (8.4–8.7)</td>
</tr>
<tr>
<td>Copper</td>
<td>17 (117)</td>
<td>6.5 (45)</td>
<td>0.36</td>
<td>9.3 (16.6)</td>
<td>0.322 (8.9)</td>
</tr>
<tr>
<td>Bronze</td>
<td>13.9–17.4 (96–120)</td>
<td>6.5 (45)</td>
<td>0.34</td>
<td>10.0 (18.0)</td>
<td>0.278–0.314 (7.7–8.7)</td>
</tr>
<tr>
<td>Magnesium</td>
<td>6.5 (45)</td>
<td>2.4 (16.5)</td>
<td>0.35</td>
<td>14 (25)</td>
<td>0.061 (1.7)</td>
</tr>
<tr>
<td>Glass</td>
<td>10.2 (70)</td>
<td>–</td>
<td>0.22</td>
<td>5.0 (9.0)</td>
<td>0.090 (2.5)</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.3 (2)</td>
<td>–</td>
<td>0.34</td>
<td>38.9 (70.0)</td>
<td>0.038 (1.05)</td>
</tr>
<tr>
<td>Polyvinyl Chloride (PVC)</td>
<td>&lt;0.6 (&lt;4)</td>
<td>–</td>
<td>–</td>
<td>28.0 (50.4)</td>
<td>0.047 (1.3)</td>
</tr>
<tr>
<td>Alumina Fiber</td>
<td>58 (400)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.141 (3.9)</td>
</tr>
<tr>
<td>Aramide Fiber</td>
<td>18.1 (125)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.047 (1.3)</td>
</tr>
<tr>
<td>Boron Fiber</td>
<td>58 (400)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.083 (2.3)</td>
</tr>
<tr>
<td>Beryllium Fiber</td>
<td>43.5 (300)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.069 (1.9)</td>
</tr>
<tr>
<td>BeO Fiber</td>
<td>58 (400)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.108 (3.0)</td>
</tr>
<tr>
<td>Carbon Fiber</td>
<td>101.5 (700)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.083 (2.3)</td>
</tr>
<tr>
<td>Silicon Carbide Fiber</td>
<td>58 (400)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.116 (3.2)</td>
</tr>
</tbody>
</table>

### Table 2 - Average Mechanical Properties of Typical Engineering Materials
(U.S. Customary Units)
(Use these values for the specific alloys and temperature listed. For all other materials refer to Table 1 above.)

<table>
<thead>
<tr>
<th>Materials</th>
<th>Specific Weight γ (lb/in³)</th>
<th>Modulus of Elasticity E (10³ ksi)</th>
<th>Modulus of Rigidity G (10³ ksi)</th>
<th>Yield Strength (ksi)</th>
<th>Ultimate Strength (ksi)</th>
<th>% Elongation in 2 in. specimen</th>
<th>Poisson's Ratio ν</th>
<th>Coef. of Therm. Expansion α (10⁻⁶/°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Metallic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum Wrought Alloys</td>
<td>0.101</td>
<td>10.6</td>
<td>3.9</td>
<td>60</td>
<td>60</td>
<td>25</td>
<td>68</td>
<td>68</td>
</tr>
<tr>
<td>Cast Iron</td>
<td></td>
<td>0.098</td>
<td>10.0</td>
<td>3.7</td>
<td>37</td>
<td>19</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>Stainless 304</td>
<td></td>
<td>0.295</td>
<td>10.4</td>
<td>3.9</td>
<td>37</td>
<td>19</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>Copper</td>
<td></td>
<td>0.283</td>
<td>10.0</td>
<td>3.4</td>
<td>37</td>
<td>19</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>Brass</td>
<td>0.316</td>
<td>14.8</td>
<td>5.4</td>
<td>11.4</td>
<td>11.4</td>
<td>14</td>
<td>35</td>
<td>35</td>
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<tr>
<td>Magnesium Alloy</td>
<td></td>
<td>0.319</td>
<td>15.0</td>
<td>5.6</td>
<td>50</td>
<td>30</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td><strong>Steel Alloys</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural A36</td>
<td></td>
<td>0.284</td>
<td>29.0</td>
<td>11.0</td>
<td>36</td>
<td>36</td>
<td>58</td>
<td>58</td>
</tr>
<tr>
<td><strong>Nonmetallic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete Low</td>
<td></td>
<td>0.086</td>
<td>3.20</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Concrete High</td>
<td></td>
<td>0.086</td>
<td>4.20</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Plastic Reinforced</td>
<td></td>
<td>0.0524</td>
<td>19.0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>104</td>
<td>70</td>
</tr>
<tr>
<td>Wood</td>
<td></td>
<td>0.0524</td>
<td>19.0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>104</td>
<td>70</td>
</tr>
<tr>
<td>Wood Select</td>
<td></td>
<td>0.017</td>
<td>1.90</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.30</td>
<td>3.7</td>
</tr>
<tr>
<td>White Spuce</td>
<td></td>
<td>0.130</td>
<td>1.40</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.36</td>
<td>5.1</td>
</tr>
</tbody>
</table>

* Specific values may vary for a particular material due to alloy or mineral composition, mechanical working of the specimen, or heat treatment. For a more exact value refer to books for the material should be consulted.
* The yield and ultimate strengths for ductile materials can be assumed equal for both tensile and compression.
* Measured perpendicular to the grain.
* Measured parallel to the grain.
* Deformation measured perpendicular to the grain when the load is applied along the grain.

**Simply Supported Beam Slopes and Deflections**

<table>
<thead>
<tr>
<th>BEAM</th>
<th>SLOPE</th>
<th>DEFLECTION</th>
<th>ELASTIC CURVE</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_{max} = \frac{-PL^2}{16EI}$</td>
<td>$v_{max} = \frac{-PL^3}{48EI}$</td>
<td>$v = \frac{Px}{48EI} (3L^2 - 4x^2)$</td>
</tr>
<tr>
<td>&amp; $0 \leq x \leq L/2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_1 = \frac{-Pab(L+b)}{6EIL}$</td>
<td>$v \bigg</td>
<td>_{x=a} = \frac{-Pba}{6EIL} (L^2 - b^2 - a^2)$</td>
</tr>
<tr>
<td>&amp; $0 \leq x \leq a$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_1 = \frac{-M_0L}{3EI}$</td>
<td>$v_{max} = \frac{-M_0L^3}{\sqrt{243EI}}$</td>
<td>$v = \frac{-M_0x}{6EIL} (x^2 - 3Lx + 2L^2)$</td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_2 = \frac{M_0L}{6EI}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_{max} = \frac{-wL^3}{24EI}$</td>
<td>$v_{max} = \frac{-5wL^4}{384EI}$</td>
<td>$v = \frac{-wx}{24EI} (x^3 - 2Lx^2 + L^2)$</td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_1 = \frac{-3wL^3}{128EI}$</td>
<td>$v \bigg</td>
<td>_{x=L/2} = \frac{-5wL^4}{768EI}$</td>
</tr>
<tr>
<td>&amp; $0 \leq x \leq L/2$</td>
<td>&amp; $0 \leq x \leq L/2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_2 = \frac{7wL^3}{384EI}$</td>
<td>$v_{max} = -0.006563 \frac{wL^4}{EI}$</td>
<td>$v = \frac{-wL}{384EI} (8x^3 - 24Lx^2 + 17L^2x - L^3)$</td>
</tr>
<tr>
<td>&amp; at $x = 0.4598L$</td>
<td>&amp; $L/2 \leq x &lt; L$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_1 = \frac{-7wL^3}{360EI}$</td>
<td>$v_{max} = -0.00652 \frac{wL^4}{EI}$</td>
<td>$v = \frac{-wp}{360EI} (3x^4 - 10Lx^2 + 7L^4)$</td>
</tr>
<tr>
<td><img src="image" alt="Beam Diagram" /></td>
<td>$\theta_2 = \frac{3wL^3}{45EI}$</td>
<td>at $x = 0.5193$</td>
<td></td>
</tr>
</tbody>
</table>

Cantilevered Beam Slopes and Deflections

<table>
<thead>
<tr>
<th>BEAM</th>
<th>SLOPE</th>
<th>DEFLECTION</th>
<th>ELASTIC CURVE</th>
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</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Beam Diagram" /></td>
<td>( \theta_{max} = \frac{-P L^3}{2 E I} )</td>
<td>( v_{max} = \frac{-P L^3}{3 E I} )</td>
<td>( v = \frac{-P x^3}{6 E I} (3 L - x) )</td>
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<tr>
<td><img src="image2.png" alt="Beam Diagram" /></td>
<td>( \theta_{max} = \frac{-P L^3}{8 E I} )</td>
<td>( v_{max} = \frac{-5 P L^3}{48 E I} )</td>
<td>( v = \frac{-P x^3}{6 E I} \left( \frac{3}{2} L - x \right) ) ( 0 \leq x \leq L/2 ) ( v = \frac{-P L}{24 E I} \left( 3 x - \frac{1}{2} L \right) ) ( L/2 \leq x \leq L )</td>
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<tr>
<td><img src="image3.png" alt="Beam Diagram" /></td>
<td>( \theta_{max} = \frac{-w L^3}{6 E I} )</td>
<td>( v_{max} = \frac{-w L^3}{8 E I} )</td>
<td>( v = \frac{-w x^3}{24 E I} (x^2 - 4 L x + 6 L^2) )</td>
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<tr>
<td><img src="image4.png" alt="Beam Diagram" /></td>
<td>( \theta_{max} = \frac{M_0 L}{E I} )</td>
<td>( v_{max} = \frac{M_0 L^2}{2 E I} )</td>
<td>( v = \frac{M_0 x^2}{2 E I} )</td>
</tr>
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<td><img src="image5.png" alt="Beam Diagram" /></td>
<td>( \theta_{max} = \frac{-w L^3}{48 E I} )</td>
<td>( v_{max} = \frac{-7 w L^3}{384 E I} )</td>
<td>( v = \frac{-w x^3}{24 E I} \left( x^2 - 2 L x + \frac{3}{2} L^2 \right) ) ( 0 \leq x \leq L/2 ) ( v = \frac{-w L^3}{192 E I} (4 x - L/2) ) ( L/2 \leq x \leq L )</td>
</tr>
<tr>
<td><img src="image6.png" alt="Beam Diagram" /></td>
<td>( \theta_{max} = \frac{-w_0 L^3}{24 E I} )</td>
<td>( v_{max} = \frac{-w_0 L^3}{30 E I} )</td>
<td>( v = \frac{-w_0 x^3}{120 E I L} (10 L^3 - 10 L^2 x + 5 L x^2 - x^3) )</td>
</tr>
</tbody>
</table>

DESIGN OF STEEL COMPONENTS
(ANSI/AISC 360-10)
LRFD, $E = 29,000$ ksi

BEAMS
For doubly symmetric compact I-shaped members bent about their major axis, the design flexural strength $\phi_b M_n$ is determined with $\phi_b = 0.90$ as follows:

**Yielding**

$$ M_n = M_p = F_y Z_x $$

where

$F_y =$ specified minimum yield stress

$Z_x =$ plastic section modulus about the x-axis

**Lateral-Torsional Buckling**

Based on bracing where $L_b$ is the length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section with respect to the length limits $L_p$ and $L_r$:

When $L_b \leq L_p$, the limit state of lateral-torsional buckling does not apply.

When $L_p < L_b < L_r$

$$ M_n = C_b \left[ M_p - \left( M_p - 0.7 F_y S_y \left( \frac{L_p - L_{1/4}}{L_p - L_p} \right) \right) \right] \leq M_p $$

where

$C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C}$

$M_{\text{max}} =$ absolute value of maximum moment in the unbraced segment

$M_A =$ absolute value of maximum moment at quarter point of the unbraced segment

$M_B =$ absolute value of maximum moment at centerline of the unbraced segment

$M_C =$ absolute value of maximum moment at three-quarter of the unbraced segment

**Shear**

The design shear strength $\phi_s V_n$ is determined with $\phi_s = 1.00$ for webs of rolled I-shaped members and is determined as follows:

$$ V_n = 0.6 F_y (d_{tw}) $$

COLUMNS
The design compressive strength $\phi_c P_n$ is determined with $\phi_c = 0.90$ for flexural buckling of members without slender elements and is determined as follows:

$$ P_n = F_{cr} A_g $$

where the critical stress $F_{cr}$ is determined as follows:

(a) When $\frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}$, $F_{cr} = \left[ 0.658 \pi \right] F_y$

(b) When $\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}}$, $F_{cr} = 0.877 F_y$

where

$KL/r$ is the effective slenderness ratio based on the column effective length ($KL$) and radius of gyration ($r$)

$KL$ is determined from AISC Table C-A-7.1 or AISC Figures C-A-7.1 and C-A-7.2 on p. 158.

$F_y$ is the elastic buckling stress = $\pi^2 E (KL/r)^2$

---

### VALUES OF $C_b$ FOR SIMPLY SUPPORTED BEAMS

<table>
<thead>
<tr>
<th>LOAD</th>
<th>LATERAL BRACING ALONG SPAN</th>
<th>$C_b$</th>
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</thead>
<tbody>
<tr>
<td>P</td>
<td>NONE LOAD AT MIDPOINT</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>AT LOAD POINT</td>
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<td></td>
<td>AT LOAD POINT</td>
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<td>AT LOAD POINTS LOADS SYMMETRICALLY PLACED</td>
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<td>AT FIFTH POINTS</td>
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<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>AT MIDPOINT</td>
<td>1.30</td>
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NOTE: LATERAL BRACING MUST ALWAYS BE PROVIDED AT POINTS OF SUPPORT PER AISC SPECIFICATION CHAPTER 7.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area A</th>
<th>Depth d</th>
<th>Web Flange tw</th>
<th>Axis X-X I S r Z</th>
<th>Axis Y-Y I r</th>
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</thead>
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</table>

### TABLE C-A-7.1
APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, $K$

<table>
<thead>
<tr>
<th>BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>THEORETICAL $K$ VALUE</td>
<td>0.5</td>
<td>0.7</td>
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<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED</td>
<td>0.65</td>
<td>0.80</td>
<td>1.2</td>
<td>1.0</td>
<td>2.10</td>
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</table>

**END CONDITION CODE**
- **(a)** ROTATION FIXED AND TRANSLATION FREE
- **(b)** ROTATION FREE AND TRANSLATION FIXED
- **(c)** ROTATION FIXED AND TRANSLATION FIXED
- **(d)** ROTATION FREE AND TRANSLATION FREE

**FOR COLUMN ENDS SUPPORTED BY, BUT NOT RIGIDLY CONNECTED TO, A FOOTING OR FOUNDATION, G IS THEORETICALLY INFINITY BUT UNLESS DESIGNED AS A TRUE FRICTION-FREE PIN, MAY BE TAKEN AS 10 FOR PRACTICAL DESIGNS. IF THE COLUMN END IS RIGIDLY ATTACHED TO A PROPERLY DESIGNED FOOTING, G MAY BE TAKEN AS 1.0. SMALLER VALUES MAY BE USED IF JUSTIFIED BY ANALYSIS.**

**AISC Figure C-A-7.1**
Alignment chart, sidesway inhibited (braced frame)

**AISC Figure C-A-7.2**
Alignment chart, sidesway uninhibited (moment frame)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Area &amp; Centroid</th>
<th>Area Moment of Inertia</th>
<th>(Radius of Gyration)²</th>
<th>Product of Inertia</th>
</tr>
</thead>
</table>
| ![Triangle](image1.png) | $A = \frac{bh}{2}$  
$x_c = \frac{2b}{3}$  
$y_c = \frac{h}{3}$ | $I_x = \frac{bh^3}{36}$  
$I_y = \frac{b^3h}{36}$  
$I_z = \frac{bh^3}{12}$  
$I_c = \frac{b^3h}{4}$ | $r_x^2 = \frac{h^2}{18}$  
$r_y^2 = \frac{b^2}{18}$  
$r_z^2 = \frac{h^2}{6}$  
$r_c^2 = \frac{b^2}{2}$ | $I_{x,y} = \frac{Abh}{36} = \frac{b^2h^2}{72}$  
$I_{xy} = \frac{Abh}{4} = \frac{b^2h^2}{8}$ |
| ![Triangle](image2.png) | $A = \frac{bh}{2}$  
$x_c = \frac{b}{3}$  
$y_c = \frac{h}{3}$ | $I_x = \frac{bh^3}{36}$  
$I_y = \frac{b^3h}{36}$  
$I_z = \frac{bh^3}{12}$  
$I_c = \frac{b^3h}{12}$ | $r_x^2 = \frac{h^2}{18}$  
$r_y^2 = \frac{b^2}{18}$  
$r_z^2 = \frac{h^2}{6}$  
$r_c^2 = \frac{b^2}{6}$ | $I_{x,y} = -\frac{Abh}{36} = -\frac{b^2h^2}{72}$  
$I_{xy} = \frac{Abh}{12} = \frac{b^2h^2}{24}$ |
| ![Rectangle](image3.png) | $A = bh$  
$x_c = \frac{b}{2}$  
$y_c = \frac{h}{2}$ | $I_x = \frac{bh^3}{12}$  
$I_y = \frac{b^3h}{12}$  
$I_z = \frac{bh^3}{3}$  
$I_c = \frac{b^3h}{3}$  
$I_p = \left[bh\left(b^2 + h^2\right)\right]/12$ | $r_x^2 = \frac{h^2}{12}$  
$r_y^2 = \frac{b^2}{12}$  
$r_z^2 = \frac{h^2}{3}$  
$r_c^2 = \frac{b^2}{3}$  
$r_p^2 = \left(b^2 + h^2\right)/12$ | $I_{x,y} = 0$  
$I_{xy} = \frac{Abh}{4} = \frac{b^2h^2}{4}$ |
| ![Triangle](image4.png) | $A = \frac{h(a+b)}{2}$  
$x_c = \frac{h(2a+b)}{3(a+b)}$  
$y_c = \frac{h}{3}$ | $I_x = \frac{h^3}{36(a+b)}\left(a^2 + 4ab + b^2\right)$  
$I_y = \frac{h^3}{12(3a+b)}$ | $r_x^2 = \frac{h^2}{18(a+b)}$  
$r_y^2 = \frac{h^2}{6(a+b)}$ | $I_{x,y} = 0$ |
| ![Triangle](image5.png) | $A = ab\sin\theta$  
$x_c = \frac{(b + a\cos\theta)/2}{2}$  
$y_c = \frac{(a\sin\theta)/2}{2}$ | $I_x = \frac{(a^3b\sin^3\theta)}{12}$  
$I_y = \frac{(ab\sin(3b^2\cos^2\theta))/12}{12}$  
$I_z = \frac{(a^2\sin^3\theta)}{3}$  
$I_p = \frac{(ab\cos^3\theta)}{3}$  
$I_c = \frac{-\left(a^2\sin^3\theta\sin^3\cos\theta\right)/6}{3}$ | $r_x^2 = \frac{(a\sin\theta)^2}{12}$  
$r_y^2 = \frac{(b^2 + a^2\sin^2\theta)}{12}$  
$r_z^2 = \frac{(a\sin\theta)^2}{3}$  
$r_p^2 = \frac{(b + a\sin^2\theta)}{3}$  
$r_c^2 = \frac{-\left(ab\cos\theta\right)}{6}$ | $I_{x,y} = \frac{(a^3b\sin^2\theta\cos\theta)}{12}$ |

<table>
<thead>
<tr>
<th>Figure</th>
<th>Area &amp; Centroid</th>
<th>Area Moment of Inertia</th>
<th>(Radius of Gyration)$^2$</th>
<th>Product of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Circle" /></td>
<td>$A = \pi a^2$</td>
<td>$I_x = I_y = \pi a^4/4$</td>
<td>$r_x^2 = r_y^2 = a^2/4$</td>
<td>$I_{x,y_c} = 0$ $I_{x,y} = Aa^2$</td>
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<tr>
<td><img src="image2" alt="Ellipse" /></td>
<td>$A = \pi(a^2 - b^2)$</td>
<td>$I_x = I_y = (\pi a^4 - \pi a^2 b^2 - \pi b^4)/4$</td>
<td>$r_x^2 = r_y^2 = (a^2 + b^2)/4$</td>
<td>$I_{x,y_c} = 0$ $I_{x,y} = Aa^2$</td>
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<tr>
<td><img src="image3" alt="Quarter Circle" /></td>
<td>$A = \pi a^2/2$</td>
<td>$I_x = \frac{a^4(9\pi^2 - 64)}{72\pi}$</td>
<td>$r_x^2 = \frac{a^2(9\pi^2 - 64)}{36\pi^2}$</td>
<td>$I_{x,y_c} = 0$ $I_{x,y} = 2a^4/3$</td>
</tr>
<tr>
<td><img src="image4" alt="Circular Sector" /></td>
<td>$A = a^2\theta$</td>
<td>$I_x = a^4(\theta - \sin\theta \cos\theta)/4$</td>
<td>$r_x^2 = \frac{a^2(\theta - \sin\theta \cos\theta)}{\theta}$</td>
<td>$I_{x,y_c} = 0$ $I_{x,y} = 0$</td>
</tr>
<tr>
<td><img src="image5" alt="Circular Segment" /></td>
<td>$A = a^2\left[\frac{\theta - \sin 2\theta}{2}\right]$</td>
<td>$I_x = Aa^2\left[\frac{1 - 2\sin^3\theta \cos\theta}{3\theta - 3\sin\theta \cos\theta}\right]$</td>
<td>$r_x^2 = Aa^2\left[\frac{1 - 2\sin^3\theta \cos\theta}{3\theta - 3\sin\theta \cos\theta}\right]$</td>
<td>$I_{x,y_c} = 0$ $I_{x,y} = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Area &amp; Centroid</th>
<th>Area Moment of Inertia</th>
<th>(Radius of Gyration)$^2$</th>
<th>Product of Inertia</th>
</tr>
</thead>
</table>
| ![Parabola](https://via.placeholder.com/150) | $A = 4ab/3$  
$x_c = 3a/5$  
$y_c = 0$ | $I_{x_c} = I_x = 4a^3b/15$  
$I_{y_c} = 16a^3b/175$  
$I_y = 4a^3b/7$ | $r_{x_c}^2 = r_x^2 = b^2/5$  
$r_{y_c}^2 = 12a^2/175$  
$r_y^2 = 3a^2/7$ | $I_{x,y_c} = 0$  
$I_{xy} = 0$ |
| ![Half a Parabola](https://via.placeholder.com/150) | $A = 2ab/3$  
$x_c = 3a/5$  
$y_c = 3b/8$ | $I_x = 2ab^3/15$  
$I_y = 2ba^3/7$ | $r_x^2 = b^2/5$  
$r_y^2 = 3a^2/7$ | $I_{xy} = Aab/4 = a^2b^2$ |
| ![Nth Degree Parabola](https://via.placeholder.com/150) | $A = bh/(n+1)$  
$x_c = n+1b/2$  
$y_c = h(n+1)/(2n+1)$ | $I_x = bh^3/3(n+1)$  
$I_y = h b^3/n+3$ | $r_x^2 = h^2(n+1)/3(n+1)$  
$r_y^2 = n+1b^2/n+3$ | |
| ![Nth Degree Parabola](https://via.placeholder.com/150) | $A = n+1b h/n+1$  
$x_c = n+1b/(2n+1)$  
$y_c = n+1h/(2n+2)$ | $I_x = n h^3/(3n+3)$  
$I_y = n b^3h/(3n+1)$ | $r_x^2 = n+1h^2/(3n+1)$  
$r_y^2 = n+1b^2/(3n+1)$ | |