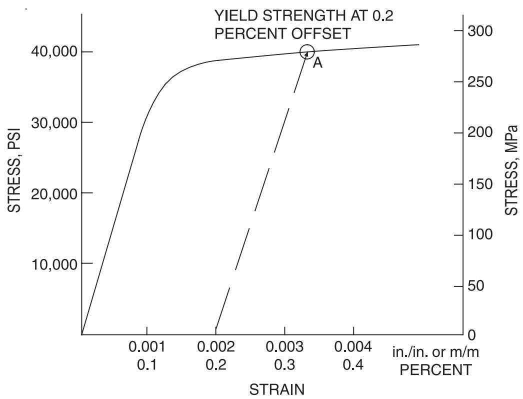
MECHANICS OF MATERIALS

UNIAXIAL STRESS-STRAIN

Stress-Strain Curve for Mild Steel

♦



The slope of the linear portion of the curve equals the modulus of elasticity.

DEFINITIONS

Engineering Strain

$\varepsilon = \Delta L/L_o$, where

ε = engineering strain (units per unit)

ΔL = change in length (units) of member

L_o = original length (units) of member

Percent Elongation

$$\%\text{ Elongation} = \left(\frac{\Delta L}{L_o}\right) \times 100$$

Percent Reduction in Area (RA)

The % reduction in area from initial area, A_i, to final area, A_f, is:

$$\%RA = \left(\frac{A_i - A_f}{A_i}\right) \times 100$$

Shear Stress-Strain

$\gamma = \tau/G$, where

γ = shear strain

τ = shear stress

G = *shear modulus* (constant in linear torsion-rotation relationship)

$$G = \frac{E}{2(1+v)}, \text{ where}$$

E = modulus of elasticity (Young's modulus)

v = *Poisson's ratio*

= – (lateral strain)/(longitudinal strain)

Uniaxial Loading and Deformation

$\sigma = P/A$, where

σ = stress on the cross section

P = loading

A = cross-sectional area

$\varepsilon = \delta/L$, where

δ = elastic longitudinal deformation

L = length of member

$$E = \sigma/\varepsilon = \frac{P/A}{\delta/L}$$

$$\delta = \frac{PL}{AE}$$

True stress is load divided by actual cross-sectional area whereas engineering stress is load divided by the initial area.

THERMAL DEFORMATIONS

$\delta_t = \alpha L(T - T_o)$, where

δ_t = deformation caused by a change in temperature

α = temperature coefficient of expansion

L = length of member

T = final temperature

T_o = initial temperature

CYLINDRICAL PRESSURE VESSEL

Cylindrical Pressure Vessel

For internal pressure only, the stresses at the inside wall are:

$$\sigma_t = P_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \quad \text{and} \quad \sigma_r = -P_i$$

For external pressure only, the stresses at the outside wall are:

$$\sigma_t = -P_o \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \quad \text{and} \quad \sigma_r = -P_o, \text{ where}$$

σ_t = tangential (hoop) stress

σ_r = radial stress

P_i = internal pressure

P_o = external pressure

r_i = inside radius

r_o = outside radius

For vessels with end caps, the axial stress is:

$$\sigma_a = P_i \frac{r_i^2}{r_o^2 - r_i^2}$$

σ_t, σ_r, and σ_a are principal stresses.

♦ Flinn, Richard A., and Paul K. Trojan, *Engineering Materials & Their Applications,* 4th ed., Houghton Mifflin Co., Boston, 1990.

When the thickness of the cylinder wall is about one-tenth or less of inside radius, the cylinder can be considered as thin-walled. In which case, the internal pressure is resisted by the hoop stress and the axial stress.

$$\sigma_t = \frac{P_i r}{t} \quad \text{and} \quad \sigma_a = \frac{P_i r}{2t}$$

where t = wall thickness and $r = \dfrac{r_i + r_o}{2}$.

STRESS AND STRAIN

Principal Stresses

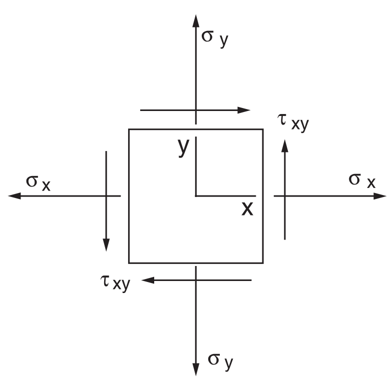
For the special case of a *two-dimensional* stress state, the equations for principal stress reduce to

$$\sigma_a, \sigma_b = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_c = 0$$

The two nonzero values calculated from this equation are temporarily labeled σ_a and σ_b and the third value σ_c is always zero in this case. Depending on their values, the three roots are then labeled according to the convention: *algebraically largest* = σ_1, *algebraically smallest* = σ_3, *other* = σ_2. A typical 2D stress element is shown below with all indicated components shown in their positive sense.

♦



Mohr's Circle – Stress, 2D

To construct a Mohr's circle, the following sign conventions are used.

1. Tensile normal stress components are plotted on the horizontal axis and are considered positive. Compressive normal stress components are negative.
2. For constructing Mohr's circle only, shearing stresses are plotted above the normal stress axis when the pair of shearing stresses, acting on opposite and parallel faces of an element, forms a clockwise couple. Shearing stresses are plotted below the normal axis when the shear stresses form a counterclockwise couple.

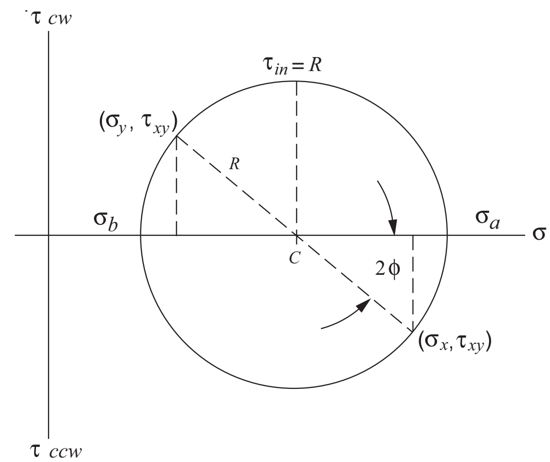
The circle drawn with the center on the normal stress (horizontal) axis with center, C, and radius, R, where

$$C = \frac{\sigma_x + \sigma_y}{2}, \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The two nonzero principal stresses are then:

♦

$\sigma_a = C + R$
$\sigma_b = C - R$



The maximum *inplane* shear stress is $\tau_{in} = R$. However, the maximum shear stress considering three dimensions is always

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}.$$

Hooke's Law

Three-dimensional case:

$\varepsilon_x = (1/E)[\sigma_x - v(\sigma_y + \sigma_z)]$ $\qquad \gamma_{xy} = \tau_{xy}/G$

$\varepsilon_y = (1/E)[\sigma_y - v(\sigma_z + \sigma_x)]$ $\qquad \gamma_{yz} = \tau_{yz}/G$

$\varepsilon_z = (1/E)[\sigma_z - v(\sigma_x + \sigma_y)]$ $\qquad \gamma_{zx} = \tau_{zx}/G$

Plane stress case ($\sigma_z = 0$):

$\varepsilon_x = (1/E)(\sigma_x - v\sigma_y)$

$\varepsilon_y = (1/E)(\sigma_y - v\sigma_x)$

$\varepsilon_z = -(1/E)(v\sigma_x + v\sigma_y)$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1 - v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \dfrac{1 - v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Uniaxial case ($\sigma_y = \sigma_z = 0$): $\quad \sigma_x = E\varepsilon_x$ or $\sigma = E\varepsilon$, where

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ = normal strain

$\sigma_x, \sigma_y, \sigma_z$ = normal stress

$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ = shear strain

$\tau_{xy}, \tau_{yz}, \tau_{zx}$ = shear stress

E = modulus of elasticity

G = shear modulus

v = Poisson's ratio

♦ Crandall, S.H., and N.C. Dahl, *An Introduction to Mechanics of Solids*, McGraw-Hill, New York, 1959.

TORSION

Torsion stress in circular solid or thick-walled (t > 0.1 r) shafts:

$$\tau = \frac{Tr}{J}$$

where J = polar moment of inertia

TORSIONAL STRAIN

$$\gamma_{\phi z} = \lim_{\Delta z \to 0} r(\Delta\phi/\Delta z) = r(d\phi/dz)$$

The shear strain varies in direct proportion to the radius, from zero strain at the center to the greatest strain at the outside of the shaft. $d\phi/dz$ is the twist per unit length or the rate of twist.

$$\tau_{\phi z} = G\gamma_{\phi z} = Gr(d\phi/dz)$$

$$T = G(d\phi/dz)\int_A r^2 dA = GJ(d\phi/dz)$$

$$\phi = \int_o^L \frac{T}{GJ}dz = \frac{TL}{GJ}, \text{ where}$$

ϕ = total angle (radians) of twist

T = torque

L = length of shaft

T/ϕ gives the *twisting moment per radian of twist*. This is called the *torsional stiffness* and is often denoted by the symbol k or c.

For Hollow, Thin-Walled Shafts

$$\tau = \frac{T}{2A_m t}, \text{ where}$$

t = thickness of shaft wall

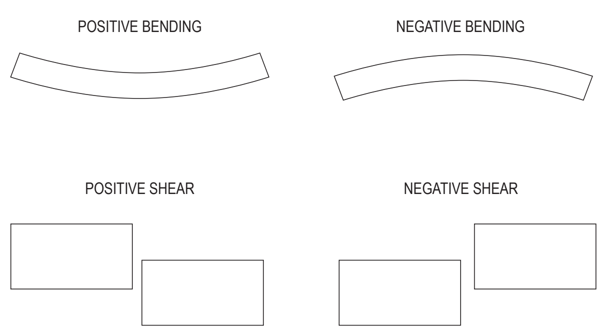
A_m = the total mean area enclosed by the shaft measured to the midpoint of the wall.

BEAMS

Shearing Force and Bending Moment Sign Conventions

1. The bending moment is *positive* if it produces bending of the beam *concave upward* (compression in top fibers and tension in bottom fibers).
2. The shearing force is *positive* if the *right portion of the beam tends to shear downward with respect to the left*.

♦



The relationship between the load (w), shear (V), and moment (M) equations are:

$$w(x) = -\frac{dV(x)}{dx}$$

$$V = \frac{dM(x)}{dx}$$

$$V_2 - V_1 = \int_{x_1}^{x_2}[-w(x)]dx$$

$$M_2 - M_1 = \int_{x_1}^{x_2} V(x)dx$$

Stresses in Beams

The normal stress in a beam due to bending:

$\sigma_x = -My/I$, where

M = the moment at the section

I = the *moment of inertia* of the cross section

y = the distance from the neutral axis to the fiber location above or below the neutral axis

The maximum normal stresses in a beam due to bending:

$\sigma_x = \pm Mc/I$, where

c = distance from the neutral axis to the outermost fiber of a symmetrical beam section.

$\sigma_x = -M/s$, where

s = I/c: the elastic section modulus of the beam.

Transverse shear stress:

$\tau_{xy} = VQ/(Ib)$, where

V = shear force

Q = $A'\overline{y'}$, where

A' = area above the layer (or plane) upon which the desired transverse shear stress acts

$\overline{y'}$ = distance from neutral axis to area centroid

B = width or thickness or the cross-section

Transverse shear flow:

$q = VQ/I$

♦ Timoshenko, S., and Gleason H. MacCullough, *Elements of Strengths of Materials*, K. Van Nostrand Co./Wadsworth Publishing Co., 1949.

Deflection of Beams
Using $1/\rho = M/(EI)$,

$$EI\frac{d^2y}{dx^2} = M, \text{ differential equation of deflection curve}$$

$$EI\frac{d^3y}{dx^3} = dM(x)/dx = V$$

$$EI\frac{d^4y}{dx^4} = dV(x)/dx = -w$$

Determine the deflection curve equation by double integration (apply boundary conditions applicable to the deflection and/or slope).

$$EI\,(dy/dx) = \int M(x)\,dx$$

$$EIy = \int\left[\int M(x)\,dx\right]dx$$

The constants of integration can be determined from the physical geometry of the beam.

Composite Sections
The bending stresses in a beam composed of dissimilar materials (material 1 and material 2) where $E_1 > E_2$ are:

$$\sigma_1 = -nMy/I_T$$
$$\sigma_2 = -My/I_T, \text{ where}$$

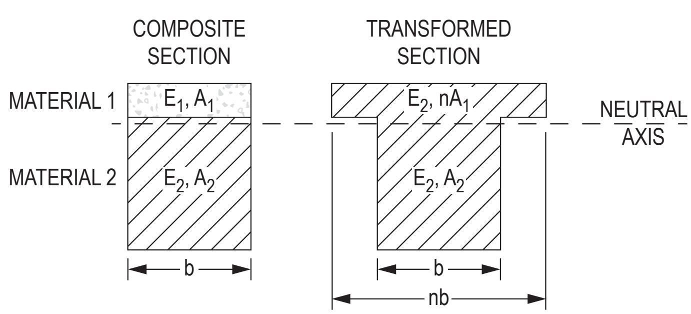
I_T = the *moment of intertia* of the *transformed* section

n = the modular ratio E_1/E_2

E_1 = elastic modulus of material 1

E_2 = elastic modulus of material 2

The composite section is transformed into a section composed of a single material. The centroid and then the moment of inertia are found on the *transformed section* for use in the bending stress equations.



COLUMNS
Critical axial load for long column subject to buckling:
Euler's Formula

$$P_{cr} = \frac{\pi^2 EI}{(K\ell)^2}, \text{ where}$$

ℓ = unbraced column length

K = effective-length factor to account for end supports

Theoretical effective-length factors for columns include:

Pinned-pinned, $K = 1.0$

Fixed-fixed, $K = 0.5$

Fixed-pinned, $K = 0.7$

Fixed-free, $K = 2.0$

Critical buckling stress for long columns:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(K\ell/r)^2}, \text{ where}$$

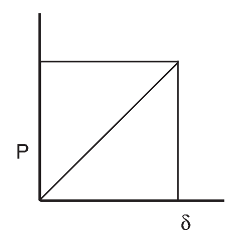
r = *radius of gyration* $\sqrt{I/A}$

$K\ell/r$ = effective *slenderness ratio* for the column

ELASTIC STRAIN ENERGY
If the strain remains within the elastic limit, the work done during deflection (extension) of a member will be transformed into potential energy and can be recovered.

If the final load is P and the corresponding elongation of a tension member is δ, then the total energy U stored is equal to the work W done during loading.

$$U = W = P\delta/2$$



The strain energy per unit volume is

$$u = U/AL = \sigma^2/2E \qquad \text{(for tension)}$$

MATERIAL PROPERTIES

Table 1 - Typical Material Properties
(Use these values if the specific alloy and temper are not listed on Table 2 below)

Material	Modulus of Elasticity, E [Mpsi (GPa)]	Modulus of Rigity, G [Mpsi (GPa)]	Poisson's Ratio, v	Coefficient of Thermal Expansion, α [10^{-6}/ºF (10^{-6}/ºC)]	Density, ρ [lb/in^3 (Mg/m^3)]
Steel	29.0 (200.0)	11.5 (80.0)	0.30	6.5 (11.7)	0.282 (7.8)
Aluminum	10.0 (69.0)	3.8 (26.0)	0.33	13.1 (23.6)	0.098 (2.7)
Cast Iron	14.5 (100.0)	6.0 (41.4)	0.21	6.7 (12.1)	0.246–0.282 (6.8–7.8)
Wood (Fir)	1.6 (11.0)	0.6 (4.1)	0.33	1.7 (3.0)	–
Brass	14.8–18.1 (102–125)	5.8 (40)	0.33	10.4 (18.7)	0.303–0.313 (8.4–8.7)
Copper	17 (117)	6.5 (45)	0.36	9.3 (16.6)	0.322 (8.9)
Bronze	13.9–17.4 (96–120)	6.5 (45)	0.34	10.0 (18.0)	0.278–0.314 (7.7–8.7)
Magnesium	6.5 (45)	2.4 (16.5)	0.35	14 (25)	0.061 (1.7)
Glass	10.2 (70)	–	0.22	5.0 (9.0)	0.090 (2.5)
Polystyrene	0.3 (2)	–	0.34	38.9 (70.0)	0.038 (1.05)
Polyvinyl Chloride (PVC)	<0.6 (<4)	–	–	28.0 (50.4)	0.047 (1.3)
Alumina Fiber	58 (400)	–	–	–	0.141 (3.9)
Aramide Fiber	18.1 (125)	–	–	–	0.047 (1.3)
Boron Fiber	58 (400)	–	–	–	0.083 (2.3)
Beryllium Fiber	43.5 (300)	–	–	–	0.069 (1.9)
BeO Fiber	58 (400)	–	–	–	0.108 (3.0)
Carbon Fiber	101.5 (700)	–	–	–	0.083 (2.3)
Silicon Carbide Fiber	58 (400)	–	–	–	0.116 (3.2)

Table 2 - Average Mechanical Properties of Typical Engineering Materials
(U.S. Customary Units)
(Use these values for the specific alloys and temperature listed. For all other materials refer to Table 1 above.)

Materials		Specific Weight γ (lb/in^3)	Modulus of Elasticity E (10^3 ksi)	Modulus of Rigidity G (10^3 ksi)	Yield Strength (ksi) σ_y			Ultimate Strength (ksi) σ_u			% Elongation in 2 in. specimen	Poisson's Ratio v	Coef. of Therm. Expansion α (10^{-6})/°F
					Tens.	Comp.	Shear	Tens.	Comp.	Shear			
Metallic													
Aluminum Wrought Alloys	2014-T6	0.101	10.6	3.9	60	60	25	68	68	42	10	0.35	12.8
	6061-T6	0.098	10.0	3.7	37	37	19	42	42	27	12	0.35	13.1
Cast Iron Alloys	Gray ASTM 20	0.260	10.0	3.9	—	—	—	26	97	—	0.6	0.28	6.70
	Malleable ASTM A-197	0.263	25.0	9.8	—	—	—	40	83	—	5	0.28	6.60
Copper Alloys	Red Brass C83400	0.316	14.6	5.4	11.4	11.4	—	35	35	—	35	0.35	9.80
	Bronze C86100	0.319	15.0	5.6	50	50	—	95	95	—	20	0.34	9.60
Magnesium Alloy	[Am 1004-T611]	0.066	6.48	2.5	22	22	—	40	40	22	1	0.30	14.3
Steel Alloys	Structural A36	0.284	29.0	11.0	36	36	—	58	58	—	30	0.32	6.60
	Stainless 304	0.284	28.0	11.0	30	30	—	75	75	—	40	0.27	9.60
	Tool L2	0.295	29.0	11.0	102	102	—	116	116	—	22	0.32	6.50
Titanium Alloy	[Ti-6Al-4V]	0.160	17.4	6.4	134	134	—	145	145	—	16	0.36	5.20
Nonmetallic													
Concrete	Low Strength	0.086	3.20	—	—	—	1.8	—	—	—	—	0.15	6.0
	High Strength	0.086	4.20	—	—	—	5.5	—	—	—	—	0.15	6.0
Plastic Reinforced	Kevlar 49	0.0524	19.0	—	—	—	—	104	70	10.2	2.8	0.34	—
	30% Glass	0.0524	10.5	—	—	—	—	13	19	—	—	0.34	—
Wood Select Structural Grade	Douglas Fir	0.017	1.90	—	—	—	—	0.30[c]	3.78[d]	0.90[d]	—	0.29[c]	—
	White Spruce	0.130	1.40	—	—	—	—	0.36[c]	5.18[d]	0.97[d]	—	0.31[c]	—

[a] SPECIFIC VALUES MAY VARY FOR A PARTICULAR MATERIAL DUE TO ALLOY OR MINERAL COMPOSITION, MECHANICAL WORKING OF THE SPECIMEN, OR HEAT TREATMENT. FOR A MORE EXACT VALUE REFERENCE BOOKS FOR THE MATERIAL SHOULD BE CONSULTED.

[b] THE YIELD AND ULTIMATE STRENGTHS FOR DUCTILE MATERIALS CAN BE ASSUMED EQUAL FOR BOTH TENSION AND COMPRESSION.

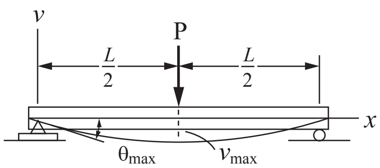
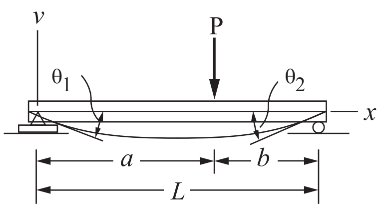
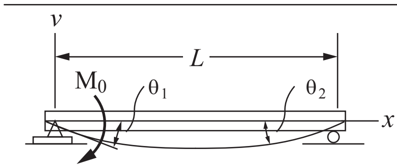
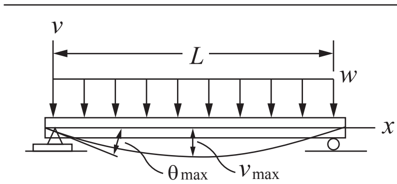
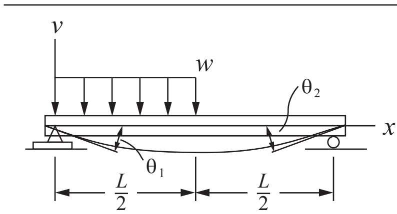
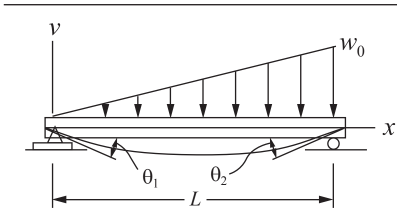
[c] MEASURED PERPENDICULAR TO THE GRAIN.

[d] MEASURED PARALLEL TO THE GRAIN.

[e] DEFORMATION MEASURED PERPENDICULAR TO THE GRAIN WHEN THE LOAD IS APPLIED ALONG THE GRAIN.

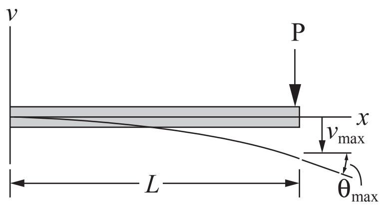
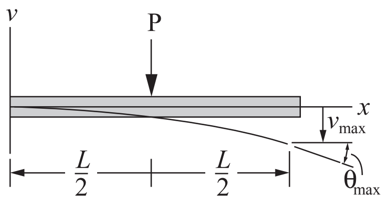
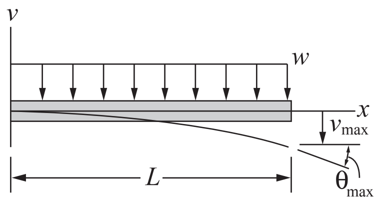
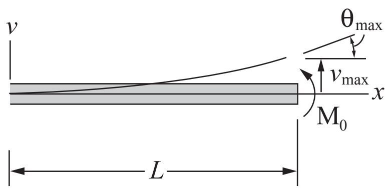
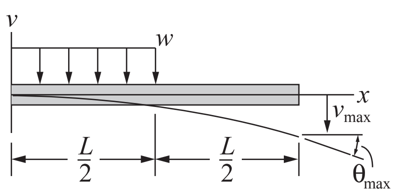
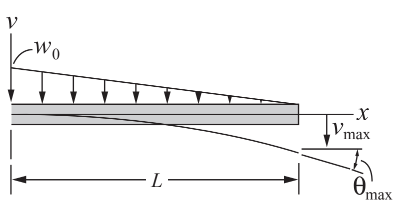
Hibbeler, R.C., *Mechanics of Materials*, 4th ed., Prentice Hall, 2000.

Simply Supported Beam Slopes and Deflections

BEAM	SLOPE	DEFLECTION	ELASTIC CURVE
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L+b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L+a)}{6EIL}$	$v\Big\rvert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0L}{3EI}$ $\theta_2 = \dfrac{M_0L}{6EI}$	$v_{max} = \dfrac{-M_0L^2}{\sqrt{243}EI}$	$v = \dfrac{-M_0x}{6EIL}(x^2 - 3Lx + 2L^2)$
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\rvert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0L^3}{360EI}$ $\theta_2 = \dfrac{w_0L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0L^4}{EI}$ at $x = 0.5193$	$v = \dfrac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

Hibbeler, R.C., *Mechanics of Materials*, 4th ed., Prentice Hall, 2000.

Cantilevered Beam Slopes and Deflections

BEAM	SLOPE	DEFLECTION	ELASTIC CURVE
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{6EI}\left(\dfrac{3}{2}L - x\right) \quad 0 \le x \le L/2$ $v = \dfrac{-PL^2}{24EI}\left(3x - \dfrac{1}{2}L\right) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{M_0L}{EI}$	$v_{max} = \dfrac{M_0L^2}{2EI}$	$v = \dfrac{M_0x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2 - 2Lx + \dfrac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \dfrac{-wL^3}{192EI}(4x - L/2)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0L^3}{24EI}$	$v_{max} = \dfrac{-w_0L^4}{30EI}$	$v = \dfrac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$

Hibbeler, R.C., *Mechanics of Materials*, 4th ed., Prentice Hall, 2000.

DESIGN OF STEEL COMPONENTS (ANSI/AISC 360-10) LRFD, E = 29,000 ksi

BEAMS

For doubly symmetric compact I-shaped members bent about their major axis, the *design flexural strength* $\phi_b M_n$ is determined with $\phi_b = 0.90$ as follows:

Yielding

$M_n = M_p = F_y Z_x$

where

F_y = specified minimum yield stress

Z_x = plastic section modulus about the x-axis

Lateral-Torsional Buckling

Based on bracing where L_b is the length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section with respect to the length limits L_p and L_r:

When $L_b \le L_p$, the limit state of lateral-torsional buckling does not apply.

When $L_p < L_b \le L_r$

$$M_n = C_b\left[M_p - \left(M_p - 0.7\,F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

where

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3M_A + 4M_B + 3M_C}$$

$M_{\max}$ = absolute value of maximum moment in the unbraced segment

M_A = absolute value of maximum moment at quarter point of the unbraced segment

M_B = absolute value of maximum moment at centerline of the unbraced segment

M_C = absolute value of maximum moment at three-quarter of the unbraced segment

Shear

The *design shear strength* $\phi_v V_n$ is determined with $\phi_v = 1.00$ for webs of rolled I-shaped members and is determined as follows:

$V_n = 0.6\,F_y\,(d\,t_w)$

COLUMNS

The *design compressive strength* $\phi_c P_n$ is determined with $\phi_c = 0.90$ for flexural buckling of members without slender elements and is determined as follows:

$P_n = F_{cr} A_g$

where the critical stress F_{cr} is determined as follows:

(a) When $\dfrac{KL}{r} \le 4.71\sqrt{\dfrac{E}{F_y}}$, $F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$

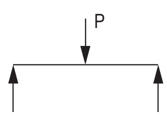
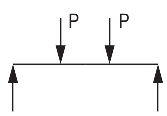
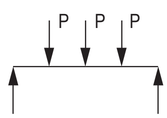
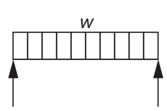
(b) When $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$, $F_{cr} = 0.877\,F_e$

where

KL/r is the effective slenderness ratio based on the column effective length (KL) and radius of gyration (r)

KL is determined from AISC Table C-A-7.1 or AISC Figures C-A-7.1 and C-A-7.2 on p. 158.

F_e is the elastic buckling stress = $\pi^2 E/(KL/r)^2$

VALUES OF C_b FOR SIMPLY SUPPORTED BEAMS		
LOAD	LATERAL BRACING ALONG SPAN	C_b
	NONE LOAD AT MIDPOINT	1.32
	AT LOAD POINT	1.67 1.67
	NONE LOADS AT THIRD POINTS	1.14
	AT LOAD POINTS LOADS SYMMETRICALLY PLACED	1.67 1.00 1.67
	NONE LOADS AT QUARTER POINTS	1.14
	AT LOAD POINTS LOADS AT QUARTER POINTS	1.67 1.11 1.11 1.67
	NONE	1.14
	AT MIDPOINT	1.30 1.30
	AT THIRD POINTS	1.45 1.01 1.45
	AT QUARTER POINTS	1.52 1.06 1.06 1.52
	AT FIFTH POINTS	1.56 1.12 1.00 1.12 1.56

NOTE: LATERAL BRACING MUST ALWAYS BE PROVIDED AT POINTS OF SUPPORT PER AISC *SPECIFICATION* CHAPTER F.

Adapted from *Steel Construction Manual*, 14th ed., AISC, 2011.

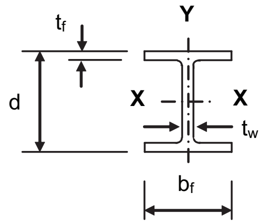
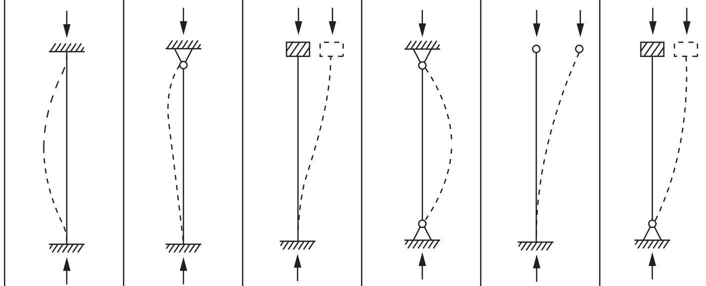


Table 1-1: W Shapes Dimensions and Properties

Shape	Area	Depth	Web	Flange		Axis X-X				Axis Y-Y	
	A	d	t_w	b_f	t_f	I	S	r	Z	I	r
	In.^{2}	In.	In.	In.	In.	In.^{4}	In.^{3}	In.	In.^{3}	In.^{4}	In.
W24X68	20.1	23.7	0.415	8.97	0.585	1830	154	9.55	177	70.4	1.87
W24X62	18.2	23.7	0.430	7.04	0.590	1550	131	9.23	153	34.5	1.38
W24X55	16.3	23.6	0.395	7.01	0.505	1350	114	9.11	134	29.1	1.34
W21X73	21.5	21.2	0.455	8.30	0.740	1600	151	8.64	172	70.6	1.81
W21X68	20.0	21.1	0.430	8.27	0.685	1480	140	8.60	160	64.7	1.80
W21X62	18.3	21.0	0.400	8.24	0.615	1330	127	8.54	144	57.5	1.77
W21X55	16.2	20.8	0.375	8.22	0.522	1140	110	8.40	126	48.4	1.73
W21X57	16.7	21.1	0.405	6.56	0.650	1170	111	8.36	129	30.6	1.35
W21X50	14.7	20.8	0.380	6.53	0.535	984	94.5	8.18	110	24.9	1.30
W21X48	14.1	20.6	0.350	8.14	0.430	959	93.0	8.24	107	38.7	1.66
W21X44	13.0	20.7	0.350	6.50	0.450	843	81.6	8.06	95.4	20.7	1.26
W18X71	20.8	18.5	0.495	7.64	0.810	1170	127	7.50	146	60.3	1.70
W18X65	19.1	18.4	0.450	7.59	0.750	1070	117	7.49	133	54.8	1.69
W18X60	17.6	18.2	0.415	7.56	0.695	984	108	7.47	123	50.1	1.68
W18X55	16.2	18.1	0.390	7.53	0.630	890	98.3	7.41	112	44.9	1.67
W18X50	14.7	18.0	0.355	7.50	0.570	800	88.9	7.38	101	40.1	1.65
W18X46	13.5	18.1	0.360	6.06	0.605	712	78.8	7.25	90.7	22.5	1.29
W18X40	11.8	17.9	0.315	6.02	0.525	612	68.4	7.21	78.4	19.1	1.27
W16X67	19.7	16.3	0.395	10.2	0.67	954	117	6.96	130	119	2.46
W16X57	16.8	16.4	0.430	7.12	0.715	758	92.2	6.72	105	43.1	1.60
W16X50	14.7	16.3	0.380	7.07	0.630	659	81.0	6.68	92.0	37.2	1.59
W16X45	13.3	16.1	0.345	7.04	0.565	586	72.7	6.65	82.3	32.8	1.57
W16X40	11.8	16.0	0.305	7.00	0.505	518	64.7	6.63	73.0	28.9	1.57
W16X36	10.6	15.9	0.295	6.99	0.430	448	56.5	6.51	64.0	24.5	1.52
W14X74	21.8	14.2	0.450	10.1	0.785	795	112	6.04	126	134	2.48
W14X68	20.0	14.0	0.415	10.0	0.720	722	103	6.01	115	121	2.46
W14X61	17.9	13.9	0.375	9.99	0.645	640	92.1	5.98	102	107	2.45
W14X53	15.6	13.9	0.370	8.06	0.660	541	77.8	5.89	87.1	57.7	1.92
W14X48	14.1	13.8	0.340	8.03	0.595	484	70.2	5.85	78.4	51.4	1.91
W12X79	23.2	12.4	0.470	12.1	0.735	662	107	5.34	119	216	3.05
W12X72	21.1	12.3	0.430	12.0	0.670	597	97.4	5.31	108	195	3.04
W12X65	19.1	12.1	0.390	12.0	0.605	533	87.9	5.28	96.8	174	3.02
W12X58	17.0	12.2	0.360	10.0	0.640	475	78.0	5.28	86.4	107	2.51
W12X53	15.6	12.1	0.345	9.99	0.575	425	70.6	5.23	77.9	95.8	2.48
W12X50	14.6	12.2	0.370	8.08	0.640	391	64.2	5.18	71.9	56.3	1.96
W12X45	13.1	12.1	0.335	8.05	0.575	348	57.7	5.15	64.2	50.0	1.95
W12X40	11.7	11.9	0.295	8.01	0.515	307	51.5	5.13	57.0	44.1	1.94
W10x60	17.6	10.2	0.420	10.1	0.680	341	66.7	4.39	74.6	116	2.57
W10x54	15.8	10.1	0.370	10.0	0.615	303	60.0	4.37	66.6	103	2.56
W10x49	14.4	10.0	0.340	10.0	0.560	272	54.6	4.35	60.4	93.4	2.54
W10x45	13.3	10.1	0.350	8.02	0.620	248	49.1	4.32	54.9	53.4	2.01
W10x39	11.5	9.92	0.315	7.99	0.530	209	42.1	4.27	46.8	45.0	1.98

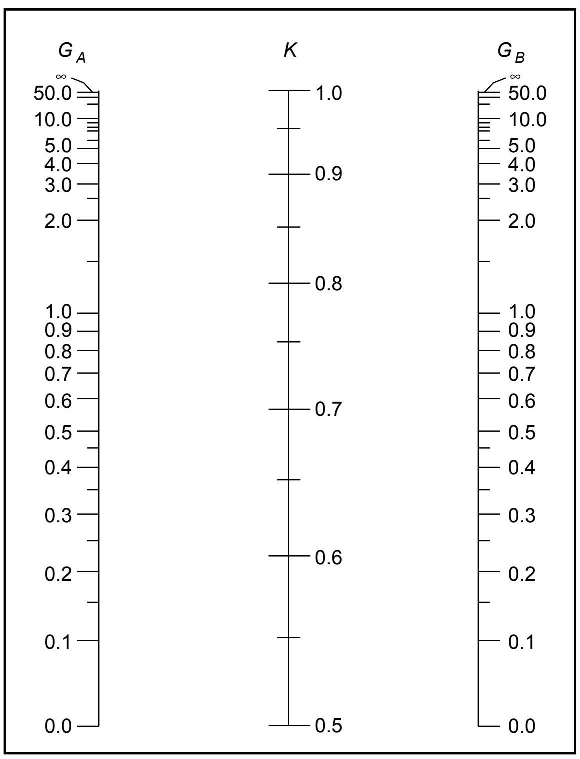
Adapted from *Steel Construction Manual*, 14th ed., AISC, 2011.

TABLE C-A-7.1
APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, *K*

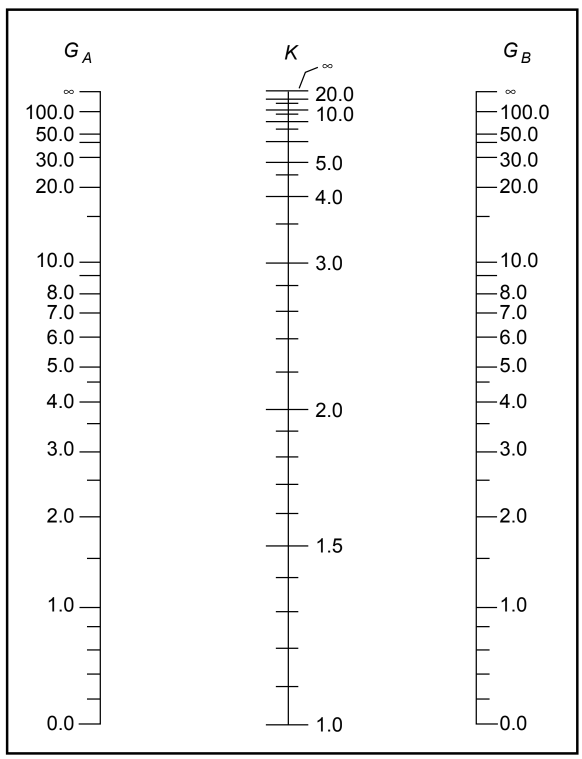
BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
						
THEORETICAL *K* VALUE	0.5	0.7	1.0	1.0	2.0	2.0
RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED	0.65	0.80	1.2	1.0	2.10	2.0
END CONDITION CODE	 *ROTATION FIXED AND TRANSLATION FIXED* *ROTATION FREE AND TRANSLATION FIXED* *ROTATION FIXED AND TRANSLATION FREE* *ROTATION FREE AND TRANSLATION FREE*					

FOR COLUMN ENDS SUPPORTED BY, BUT NOT RIGIDLY CONNECTED TO, A FOOTING OR FOUNDATION, G IS THEORETICALLY INFINITY BUT UNLESS DESIGNED AS A TRUE FRICTION-FREE PIN, MAY BE TAKEN AS 10 FOR PRACTICAL DESIGNS. IF THE COLUMN END IS RIGIDLY ATTACHED TO A PROPERLY DESIGNED FOOTING, G MAY BE TAKEN AS 1.0. SMALLER VALUES MAY BE USED IF JUSTIFIED BY ANALYSIS.

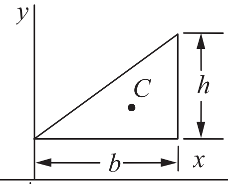
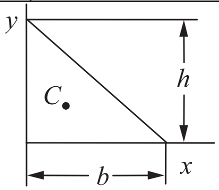
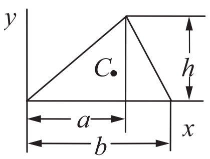
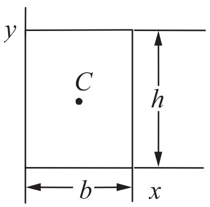
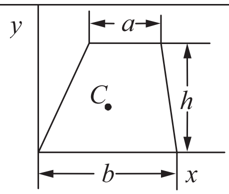
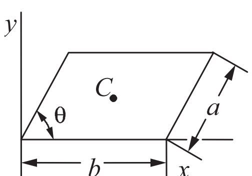
AISC Figure C-A-7.1
Alignment chart, sidesway inhibited (braced frame)



AISC Figure C-A-7.2
Alignment chart, sidesway uninhibited (moment frame)



Steel Construction Manual, 14th ed., AISC, 2011.

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r^2_{x_c} = h^2/18$ $r^2_{y_c} = b^2/18$ $r^2_x = h^2/6$ $r^2_y = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r^2_{x_c} = h^2/18$ $r^2_{y_c} = b^2/18$ $r^2_x = h^2/6$ $r^2_y = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a + b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = \left[bh\left(b^2 - ab + a^2\right)\right]/36$ $I_x = bh^3/12$ $I_y = \left[bh\left(b^2 + ab + a^2\right)\right]/12$	$r^2_{x_c} = h^2/18$ $r^2_{y_c} = \left(b^2 - ab + a^2\right)/18$ $r^2_x = h^2/6$ $r^2_y = \left(b^2 + ab + a^2\right)/6$	$I_{x_c y_c} = \left[Ah(2a - b)\right]/36$ $= \left[bh^2(2a - b)\right]/72$ $I_{xy} = \left[Ah(2a + b)\right]/12$ $= \left[bh^2(2a + b)\right]/24$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = \left[bh\left(b^2 + h^2\right)\right]/12$	$r^2_{x_c} = h^2/12$ $r^2_{y_c} = b^2/12$ $r^2_x = h^2/3$ $r^2_y = b^2/3$ $r^2_p = \left(b^2 + h^2\right)/12$	$I_{x_c y_c} = 0$ $I_{xy} = Abh/4 = b^2h^2/4$
	$A = h(a + b)/2$ $y_c = \dfrac{h(2a + b)}{3(a + b)}$	$I_{x_c} = \dfrac{h^3\left(a^2 + 4ab + b^2\right)}{36(a + b)}$ $I_x = \dfrac{h^3(3a + b)}{12}$	$r^2_{x_c} = \dfrac{h^2\left(a^2 + 4ab + b^2\right)}{18(a + b)}$ $r^2_x = \dfrac{h^2(3a + b)}{6(a + b)}$	
	$A = ab \sin\theta$ $x_c = (b + a \cos\theta)/2$ $y_c = (a \sin\theta)/2$	$I_{x_c} = \left(a^3 b \sin^3\theta\right)/12$ $I_{y_c} = \left[ab \sin\theta\left(b^2 + a^2\cos^2\theta\right)\right]/12$ $I_x = \left(a^3 b \sin^3\theta\right)/3$ $I_y = \left[ab \sin\theta(b + a \cos\theta)^2\right]/3$ $- \left(a^2 b^2 \sin\theta \cos\theta\right)/6$	$r^2_{x_c} = (a \sin\theta)^2/12$ $r^2_{y_c} = \left(b^2 + a^2\cos^2\theta\right)/12$ $r^2_x = (a \sin\theta)^2/3$ $r^2_y = (b + a \cos\theta)^2/3$ $- (ab \cos\theta)/6$	$I_{x_c y_c} = \left(a^3 b \sin^2\theta \cos\theta\right)/12$

Housner, George W., and Donald E. Hudson, *Applied Mechanics Dynamics*, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.

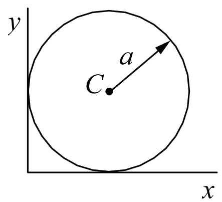
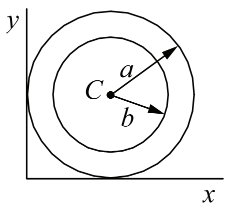
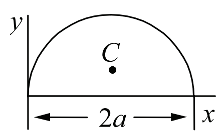
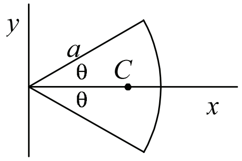
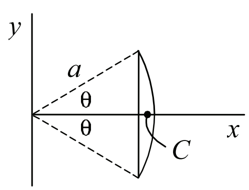
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = \pi a^2$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi a^4/4$ $I_x = I_y = 5\pi a^4/4$ $J = \pi a^4/2$	$r_{x_c}^2 = r_{y_c}^2 = a^2/4$ $r_x^2 = r_y^2 = 5a^2/4$ $r_p^2 = a^2/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$
	$A = \pi(a^2 - b^2)$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi\left(a^4 - b^4\right)/4$ $I_x = I_y = \frac{5\pi a^4}{4} - \pi a^2 b^2 - \frac{\pi b^4}{4}$ $J = \pi\left(a^4 - b^4\right)/2$	$r_{x_c}^2 = r_{y_c}^2 = \left(a^2 + b^2\right)/4$ $r_x^2 = r_y^2 = \left(5a^2 + b^2\right)/4$ $r_p^2 = \left(a^2 + b^2\right)/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$ $= \pi a^2\left(a^2 - b^2\right)$
	$A = \pi a^2/2$ $x_c = a$ $y_c = 4a/(3\pi)$	$I_{x_c} = \frac{a^4\left(9\pi^2 - 64\right)}{72\pi}$ $I_{y_c} = \pi a^4/8$ $I_x = \pi a^4/8$ $I_y = 5\pi a^4/8$	$r_{x_c}^2 = \frac{a^2\left(9\pi^2 - 64\right)}{36\pi^2}$ $r_{y_c}^2 = a^2/4$ $r_x^2 = a^2/4$ $r_y^2 = 5a^2/4$	$I_{x_c y_c} = 0$ $I_{xy} = 2a^4/3$
 CIRCULAR SECTOR	$A = a^2\theta$ $x_c = \frac{2a}{3}\frac{\sin\theta}{\theta}$ $y_c = 0$	$I_x = a^4(\theta - \sin\theta\cos\theta)/4$ $I_y = a^4(\theta + \sin\theta\cos\theta)/4$	$r_x^2 = \frac{a^2}{4}\frac{(\theta - \sin\theta\cos\theta)}{\theta}$ $r_y^2 = \frac{a^2}{4}\frac{(\theta + \sin\theta\cos\theta)}{\theta}$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
 CIRCULAR SEGMENT	$A = a^2\left[\theta - \frac{\sin 2\theta}{2}\right]$ $x_c = \frac{2a}{3}\frac{\sin^3\theta}{\theta - \sin\theta\cos\theta}$ $y_c = 0$	$I_x = \frac{Aa^2}{4}\left[1 - \frac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $I_y = \frac{Aa^2}{4}\left[1 + \frac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$r_x^2 = \frac{a^2}{4}\left[1 - \frac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $r_y^2 = \frac{a^2}{4}\left[1 + \frac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$I_{x_c y_c} = 0$ $I_{xy} = 0$

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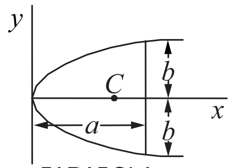
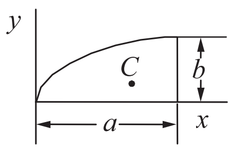
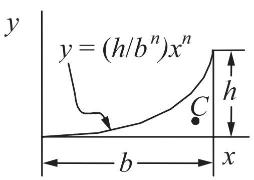
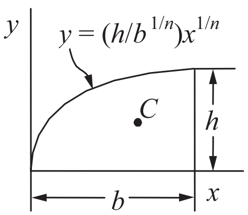
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
 PARABOLA	$A = 4ab/3$ $x_c = 3a/5$ $y_c = 0$	$I_{x_c} = I_x = 4ab^3/15$ $I_{y_c} = 16a^3b/175$ $I_y = 4a^3b/7$	$r_{x_c}^2 = r_x^2 = b^2/5$ $r_{y_c}^2 = 12a^2/175$ $r_y^2 = 3a^2/7$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
 HALF A PARABOLA	$A = 2ab/3$ $x_c = 3a/5$ $y_c = 3b/8$	$I_x = 2ab^3/15$ $I_y = 2ba^3/7$	$r_x^2 = b^2/5$ $r_y^2 = 3a^2/7$	$I_{xy} = Aab/4 = a^2b^2$
 nth DEGREE PARABOLA	$A = bh/(n+1)$ $x_c = \frac{n+1}{n+2}b$ $y_c = \frac{h}{2}\frac{n+1}{2n+1}$	$I_x = \frac{bh^3}{3(3n+1)}$ $I_y = \frac{hb^3}{n+3}$	$r_x^2 = \frac{h^2(n+1)}{3(3n+1)}$ $r_y^2 = \frac{n+1}{n+3}b^2$	
 nth DEGREE PARABOLA	$A = \frac{n}{n+1}bh$ $x_c = \frac{n+1}{2n+1}b$ $y_c = \frac{n+1}{2(n+2)}h$	$I_x = \frac{n}{3(n+3)}bh^3$ $I_y = \frac{n}{3n+1}b^3h$	$r_x^2 = \frac{n+1}{3(n+1)}h^2$ $r_y^2 = \frac{n+1}{3n+1}b^2$	

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