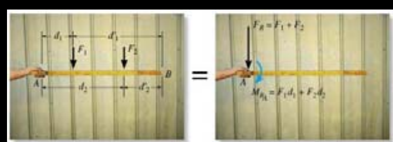
EQUIVALENT SYSTEMS

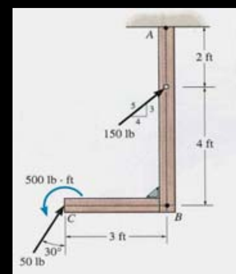
Today's Objectives:

Replace a 2D or 3D system of forces and couples with an equivalent system, consisting of a single resultant force and moment at a point

Replace a 2D system of forces and couples with an equivalent system, consisting of a single resultant force and specify the location of the force



IN-CLASS PROBLEM SOLVING



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A.

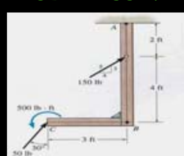
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A and add them to the 500 lb - ft free moment to find the resultant M_{RA} .

IN-CLASS PROBLEM SOLVING (cont.)

Summing the force components:



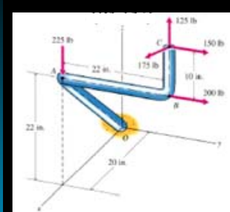
$+ \rightarrow \Sigma F_x$ = (4/5) 150 lb + 50 lb sin 30° = 145 lb

$+ \uparrow \Sigma F_y$ = (3/5) 150 lb + 50 lb cos 30° = 133.3 lb

Now find the magnitude and direction of the resultant.

$\|F_{RA}\| = (145^2 + 133.3^2)^{1/2} = 197$ lb and $\theta = \tan^{-1}$ (133.3/145) = 42.6 ° $\angle$

$+ M_{RA}$ = { (4/5)(150)(2) – 50 cos30° (3) + 50 sin30° (6) + 500 } = 760 lb·ft



Given: A 3-D force system.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F_{RO}} = \Sigma \boldsymbol{F_i}$

b) Find $\boldsymbol{M_{RO}} = \Sigma \boldsymbol{M_C} + \Sigma (\boldsymbol{r_i} \times \boldsymbol{F_i})$

$\boldsymbol{F_i}$ are the individual forces in Cartesian vector notation

$\boldsymbol{M_C}$ are any free couple moments in vector notation (none in this problem).

$\boldsymbol{r_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

DISTRIBUTED LOADING

Objective:

Determine the magnitude and location of the equivalent resultant force, F_R, for a given distributed load

EXAMPLE - DESIGN OF A FLOOR SYSTEM

Example of what a 50 psf live load would look like in an classroom area

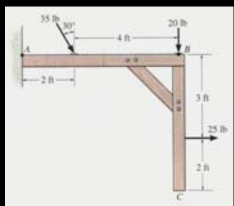
Structural engineers design floor and roof systems to carry required distributed area loads.

Design for the dead and live loads carried by the floor:

Dead – weight of material in floor (beams, plywood, tile)

Live – occupancy loads (people, furniture, books)

EXAMPLE #1 - FRAME



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location along the beam AB.

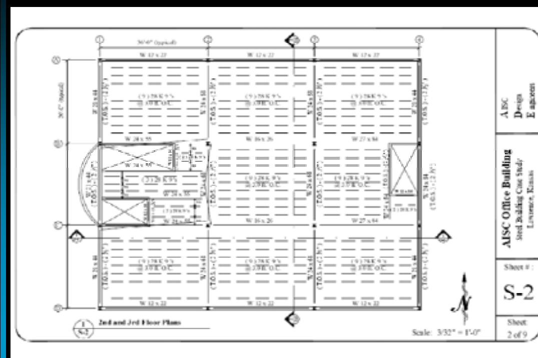
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A.

3) Shift the F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

STRUCTURAL PLAN OF A FLOOR SYSTEM



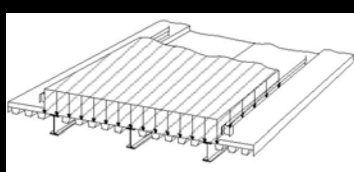
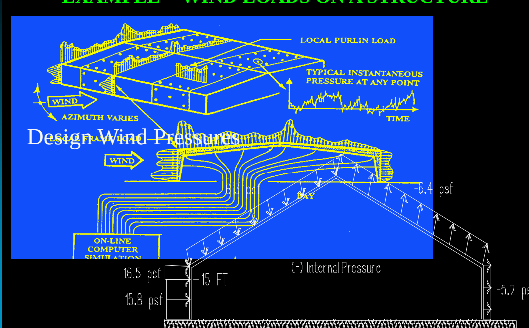


Figure 3 Uniformly distributed load on traffic lanes.

Metal Deck/Steel Joist Floor System

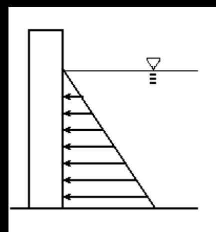
Metal Deck/Steel Joist Floor System

EXAMPLE - WIND LOADS ON A STRUCTURE

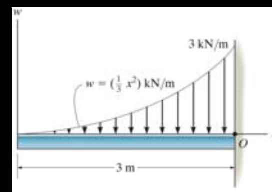


EXAMPLE - FLUID PRESSURE ON STRUCTURES

The pressure acts as a function of depth.



EXAMPLE 1



Given: Beam with distributed force

Find: Equivalent resultant force and couple moment at point *O*

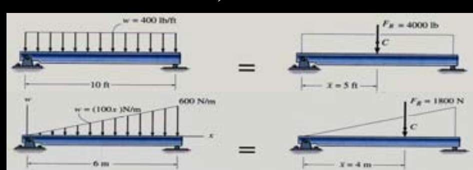
Plan:

1) Determine resultant force F_R by integration.
2) Determine the resultant moment M_{RO} by integration.

Extra) Determine where the resultant force F_R acts on the beam (d)

EXAMPLES

Until you learn more about centroids, we will consider primarily rectangular, triangular, and trapezoid distributed loads whose centroids are well defined (look at inside back cover of textbook).

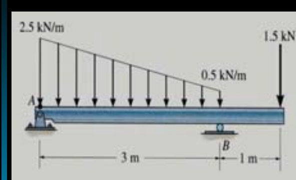


In a rectangular loading, F_R = 400 × 10 = 4,000 lb and $\bar{x}$ = 5 ft.

In a triangular loading ,
F_R = (0.5) (6000) (6) = 1,800 N and $\bar{x}$ = 6 – (1/3) 6 = 4 m.

Please note that the centroid in a right triangle is at a distance one third the width of the triangle as measured from its base.

PROBLEM SOLVING



Given: The loading on the beam as shown.

Find: The equivalent force F_R and its location d away from point A.

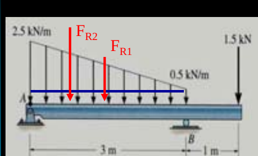
Plan:

1) Determine F_R for each distributed load (rect. and triangle)

2) Find the total F_R and M_{RA} considering all the loads

3) Determine the horz. distance d at which the single F_R force acts

PROBLEM SOLVING (continued)



Rectangular loading of height 0.5 kN/m and width 3 m,

F_{R1} = (0.5 kN/m)(3 m) = 1.5 kN

Located 1.5 m from point A

Triangular loading of height 2 kN/m and width 3 m,

F_{R2} = (1/2)(2 kN/m)(3 m) = 3 kN

Located 1 m from point A

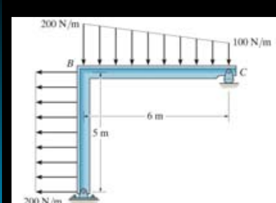
Combining all forces and moments:

F_R = 1.5 kN + 3 kN + 1.5 kN = 6 kN

M_{RA}= – (1.5 kN) (1.5m) – (3 kN)(1m) – (1.5 kN)(4m) = -11.25 kN-m

d = (11.25 kN-m) / (6 kN) = 1.88 m from A

EXAMPLE 2



Given: Frame with distributed forces

Find: Replace the distributed loading by a single equivalent resultant force and specify where its line of action intersects member BC, measured from C

Plan:

1) Break the trapezoidal load into rectangular and triangular loads
2) For each distributed load fin F_R and its location
3) Determine the total F_R and the M_{RC} considering all loads
4) Find d = M_{RC}/F_{Ry} where the single equivalent F_R force acts