Uncertainty Shocks, Mean-Variance Frontiers, and Business Cycles*

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Abstract

This paper constructs a model economy to explain how uncertainty shocks can cause business cycle fluctuations. In the model, during times of high uncertainty, the representative agent, due to risk aversion motives, moves resources from a high risk/high return production process to a low risk/low return production process. High uncertainty thus causes the firm level technology frontier to endogenously move inward and generate a recession. The uncertainty shock induced endogenous movements in the technology frontier closely mimic movements caused by conventional first moment technology shocks. This result highlights the possibility of mis-identification of uncertainty shocks in the data as first moment shocks. In addition, endogenous movements of the technology frontier in the model act to dampen the resulting volatility of macroeconomic variables thereby pointing to a systematic bias that can occur if the magnitude of uncertainty shocks is naively measured as the variance of the error term of the AR(1) total factor productivity process in data.

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1 Introduction

In a recession, output along with other macroeconomic variables decrease relative to their trends. In an expansion, each rises. This sequence of alternating expansions and recessions is called a business cycle. Several early studies, e.g. Frisch (1933) and Pigou (1927), proposed an impulse and propagation theory to explain these cycles. According to this theory, impulses, or external shocks, impact an economy and these impulses propagate through to macroeconomic variables by economic actors producing and interacting through markets. A more modern discussion of this impulse and propagation approach begins with Friedman and Schwartz (1963) and Lucas (1975).

Modern macroeconomics has relied mainly on dynamic stochastic general equilibrium (DSGE) models to explain how impulses or external shocks can translate into business cycle fluctuations (e.g. Kydland and Prescott (1982)). Surprisingly, however, even though these DSGE models are driven by external shocks that are stochastic in nature, not until recently has there been much interest in understanding how moments, other than the first moment, of these external shock processes can lead to economic fluctuations. Traditionally business cycle fluctuations have been the result of purely first moment exogenous shocks [1]. The idea, however, that higher moments can directly result in movements of first moment macroeconomic aggregates, and thus may be a more direct cause of business cycles, is not a new one. John Maynard Keynes (1936) in his work had hypothesized that 'animal spirits', changes in investor sentiment, could lead to recessions [2]. Encouraged by anecdotal evidence from the recent economic downturn macroeconomists have formalized the idea of higher moment shocks and started attempting to better understand the role of "uncertainty shocks" - or more generally second moments shocks and their effect on business cycle fluctuations.

It is a well-known result that the second moment of the stochastic driving process has little to no effect on the level of output in the quintessential model of real business cycle theory, the standard neoclassical model [3]. Thus, if we are to hypothesize that uncertainty matters

[1] It is important to note that this is not to say that there has not been interest in other moments of the shock process, but that this interest has mainly been confined to how higher moment changes result in changes in the higher moments of the economic aggregates. For example, a macroeconomist calibrating a RBC model in a DSGE framework will be sure to calibrate the volatility of any exogenous shock process to match the volatility of macroeconomic aggregates, but not the first moments of the aggregates.

[2] 'Animal spirits' change consumer confidence and thus may cause uncertainty.

[3] This important result is why we can safely linearize and study most macroeconomic business cycle models

for business cycles, then the first step is to develop a theory that links second moment variations in our models to first moment variations. This paper proposes such a theory based on endogenous technology choice. In our model exogenous movements in the second moment of the TFP process leads to changes in the first moment technology choice, which in turn leads to real first moment effects. The theory explored in this paper also sheds light on two possible empirical biases: (i) second moment shock induced movements in the first moment technology choice make it hard to determine when movements in the aggregate variables are the result of changes in the second moment of the technology process and when they are the result of changes in the first moment. (ii) the second moment shock induced movements also work to dampen the volatility of macroeconomic aggregates in the data. As a result naive measures of shock volatility from TFP data maybe downward bias.

Recessions have been empirically shown to be times of high uncertainty (see Bloom et. al. (2012), Bachmann & Bayer (2009a), and Alexopoloulos & Cohen (2009)). There is also anecdotal evidence that firms endogenously change their business practices in response to uncertainty. One key dimension that business practices change along is the mean-variance tradeoff frontier. I hypothesis that when aggregate volatility rises in the economy firms endogenously drop high risk-high return projects, which are now higher risk projects, in favor of low risk-low return projects. This hypothesis can be envisioned in two different ways: The first and most straight forward is that firms and entrepreneurs literally move resources from high risk-high return projects to low risk-low return projects. The best example at the macro-level of this would be investors that re-optimize their portfolios during times of high uncertainty to include more low risk-low return assets. This re-optimization represents a movement of capital at the aggregate level from high risk-high return projects to low risk-low return projects. Micro-level examples of the same phenomena would include businesses that cut down on research expenditures during recessions, research being a high risk-high return endeavor at the firm level. Restaurants, and other small businesses, have also been known to cut down on product offering during recessions and focusing purely on their core business products. Another way to interpret this movement along the mean-variance frontier during recessions is to appeal to the idea that during times of high uncertainty firms dedicate more resources towards mitigating the higher uncertainty, thereby lowering the net returns. Hiring consultancy firms during recessions to find ways to mitigate wastage would be an example of such behavior. The consultancy firm would reduce the variance of returns, but the cost associated with hiring the firm would also result in reduced net returns in the short run.

The key mechanism in this paper can understood by envisioning a simple economy where agents can choose to run their firms with one of two different technology processes. One of these processes is a low mean/low variance process and the other is a high mean/high variance process, i.e. two points on the mean-variance tradeoff frontier of endogenous technology choices. Now holding the means constant, and the variance of the low variance process constant, let us increase the variance of the high variance process (i.e. an increase in the slope of the tradeoff frontier), this change will cause the economic agents in this setup to endogenously change their technology process choice. A risk averse enough agent who previously chose to run the high mean/high variance technology in his firm, after the slope increase might instead choose to run the low mean-low variance technology. This movement represents an interesting result: Without appealing to real frictions, and remaining purely in the realm of exogenous shocks, in a model where firms can endogenously choose from a menu of mean-variance TFP processes a change in the menu along the variance dimension can cause changes in the optimal mean choice. In particular, in my model an increase in the variance causes a drop in the mean TFP, which in turn causes a recession. The model thus represents a mechanism for second moment shocks to have first moment effects.

In this paper second moment shocks to the exogenous driving process have direct effects on the first moments of the exogenous driving process. This is different than other models in this literature where uncertainty shocks have no direct effect on the first moment of the exogenous driving process [4]. For this reason, this paper highlights the problem with naively measuring uncertainty shocks using TFP data. A decrease in measured TFP could either be the result of a direct shock to the first moment of TFP or a movement in the first moment of TFP because of movements along the mean-variance frontier due to an uncertainty shock.

Uncertainty shock induced movements along the mean-variance technology frontier also act to dampen the effect of increases in the second moment. For example in our baseline calibration a shock that increases the variance of the high mean-high variance process by over eleven times only causes the standard deviation of measured TFP to rise by four times - a significant downward bias. This result takes an importance of its own in that as we economists continue

[4] For example in Bloom et. al. (2010) higher uncertainty causes hiring to stop, and thus an output recession. In their model the first moment of the exogenous driving process remains unchanged.

to study the effects of uncertainty shocks it becomes imperative to have accurate measures of the magnitude of uncertainty shocks. Naive measures of shock variances extracted from a TFP process can be downward bias.

Staying within the confines of technology shocks this paper can also more directly explain why we see declines in TFP without having to appeal to a story of technological regress. Increased uncertainty causes TFP to drop endogenously in the model, as opposed to TFP falling due to a exogenous negative shock to TFP. An important caveat here is that this result is not unique to our model, other papers in the uncertainty shock literature are also able to generate decreases in measured weighted aggregate productivity. Where our model is unique is that it is also additionally able to generate TFP decreases in the unweighted aggregate productivity. For example, in Bloom et. al.(2012) during times of high uncertainty productive firms expand less and unproductive firms contract less causing weighted TFP to fall, however, unweighted TFP, or more generally mean TFP at the firm level remains constant.

Even though this paper models uncertainty shocks as generating business cycle fluctuations, in the larger context I remain agnostic about this causation. Whereas there is ample evidence (e.g. Bloom et. al. (2012), Alexopoloulos & Cohen (2009), Bachmann & Bayer (2009a) that recessions are associated with higher uncertainty, it is arguable that the causation can run either way, i.e. high uncertainty causes recessions vs. recessions cause higher uncertainty. In the former ideological view this paper presents a theory of how second moment shocks can cause business cycle fluctuations, while in the latter causal relationship the paper can be reinterpreted as giving a theory of how uncertainty can amplify the business cycle fluctuations.

This paper is most closely related to a recent strand of literature that aims to understand the channel through which second moment shocks can have first moment effects in a general equilibrium real business cycle framework. Bloom et. al (2012) in their paper appeal to frictions in the capital accumulation process and labor market as causing a freeze in economic activity during times of high uncertainty. Firms in their model, due to costs of changing input decisions, during times of high uncertainty stop accumulating capital or hiring workers, and instead wait for the uncertainty to resolve thus causing a recession. A puzzling result of their framework is that on impact the higher uncertainty causes consumption to rise. On the other hand Bachmann & Bayer (2009a & 2009b) using a lumpy investment model conclude that the real effect of uncertainty is through its effect on expected future productivity. In their model higher uncertainty predicts lower future TFP, and thus through the bad news effect they are able to generate recessions in response to higher uncertainty. They further conclude a strong similarity between the effects of news shocks and uncertainty shocks, whereas I conclude a strong similarity between standard TFP shocks and uncertainty shocks. Both these papers also predict a rebound phase for most aggregate variable - as the uncertainty reverts to its steady state level the economy experiences a boom.

Bachmann & Moscarini (2011) present a model, which explores the opposite causation to mine. They argue that first moment shocks lead to higher uncertainty. In their model negative shocks lead to more risky choices by the firms, in particular, firms tend to be more likely to price experiment during bad times causing uncertainty to be higher. As previously pointed out this paper is not at odds with such a theory, in the context of their model this paper provides an amplification mechanism in the recessionary period. Fernandez-Villaverde et. al. (2009) in an international real business cycle model find that changes in the volatility of interest rates faced by small economies have real effects on economic aggregates. In their model higher volatility of interest rates causes output, consumption, investment, and labor hours to all fall. Sims (2007) builds a model with irreversible investment decision and finds that higher uncertainty leads to recessions. Illut & Schneider (2011) interpret changes in uncertainty as changes in ambiguity and show how changes in ambiguity can lead to business cycles fluctuations.

Part of the literature on uncertainty shocks also illustrates the role financial frictions play as a channel for second moment shocks to have first moment effects. Christiano et. al. (2009) model second moment shocks as "first moment" risk shocks that cause business cycle fluctuations due to financial frictions. Gilchrist et. al. (2009) find that high uncertainty causes the risk premium on bonds to rise, and thus causes an increase in the cost of capital, this in turn causes investment to fall. The inclusion of credit market frictions plays an important role in their proposed mechanism. Arellano et. al. (2010) also using financial frictions are able to construct a model where fluctuations in uncertainty cause business cycle fluctuations. In their model higher uncertainty causes the firms to reduce the size of their projects, whereas in my model the firms instead of decreasing the size of their project changes the type of their project.

The rest of this paper is organized as follows: In the next section I layout the main model of the paper, followed by the calibration of the model in the section following that. In section 4 I discuss the results of the paper along with the solution method. Section 5 concludes.

2 Model

To formalize the impact of uncertainty in a model with endogenous technology choices and also study the quantitative effects of uncertainty I construct a dynamic stochastic general equilibrium model with a representative agent whose preferences take the Epstein-Zin (1989) form. When compared to the standard CRRA utility formulation using Epstein-Zin preferences allows for greater flexibility in de-coupling the agents risk aversion from the intertemporal elasticity of substitution. My baseline model can be summarized by the following optimization problem faced by the social planner:

$$V(k,l,j;a) = \max_{c,i,k',l',j'} \{(1-\beta)\left[u(c,l)\right]^{1-\eta} + \beta\left[E(V(k',l',j';a')^{1-\sigma})\right]^{\frac{1-\eta}{1-\sigma}}\}^{\frac{1}{1-\eta}} \tag{2.1}$$

s.t.

$$c + i \leq f(a,k,l) \tag{2.2}$$

$$k' \leq i + (1-\delta)k \tag{2.3}$$

$$a \sim N(\bar{a}_j, \frac{\sigma_j}{\sqrt{1-\rho^2}}) \tag{2.4}$$

$$j \in J \subset \mathbb{N}\ ,\ k' \geq \underline{k}\ ,\ u(c,l) \geq \underline{u} \tag{2.5}$$

Figure 1 gives the per period timeline of events in the model economy. A representative agent of type j enters each period with some capital holdings, k, and a pre-determined labor supply, l. Upon entering the period the agent realizes the current period productivity, a, and the social planner now maximizes his lifetime utility. The agent's current period productivity draw is a function of his type. The agents type $j \in J \subset \mathbb{N}$ indicates the planners previous period choice of what technology process to run the production process with in the current period. Each type j is associated with a technology process that is defined by a mean-variance pair, $\{\bar{a}_j, \sigma_j\}$. Equation (2.4) gives the distribution of the i.i.d shocks for each type choice; this distribution is the long run ergodic distribution that results from a AR1 process with persistence, ρ, mean, $\bar{a}_j$, and per period conditional variance, σ_j.

Upon realizing the current period productivity the agent combines this productivity with his current capital holdings, k, and the pre-determined labor supply, l, to produce output using the production function, $f(a, k, l)$. The planner then optimally allocates the resulting output to consumption and investment. The consumption decision gives the agent utility, while the investment decision augments the agent's capital stock, k', for next period. As is standard old capital, k, depreciates by a fraction δ. The capital law of motion is thus given by equation (2.3).

My main departure from the standard model is that in addition to making the consumption - investment decisions, the planner also simultaneously makes two other decisions: (i) the planner decides next periods labor supply, l'. In the standard framework this decision is made within a period, instead of in the previous period. As my model is being driven by uncertainty shocks I need to ensure that the labor decision is made before the uncertainty is resolved. If the decision is made after the uncertainty is resolved then the planner would be risk loving in his labor choice because the marginal revenue product of labor is convex in productivity. A risk loving agent would thus undesirably increase labor. This result is commonly known as the Hartman-Abel effect (Hartman (1972) and Abel (1983)). (ii) the planner decides on what technology process, $j \in J \subset \mathbb{N}$, to run his firms with next period. This is the novel addition in my model. The menu of technology processes on offer to the planner differ in the mean-variance pairing, $\{\bar{a}_i, \sigma_i\}$, that they offer. As a result when choosing the technology process the social planner faces a tradeoff between choosing a process that has a high mean but comes with high variance, versus a process that has a low variance but is coupled with a low mean. The agents risk aversion plays a strong role in determining which process he chooses.

For numerical considerations I impose two additional constraints: $k' \geq \underline{k}$ and $u(c, l) \geq \underline{u}$. The first of these constraints for my parameterization, and other standard parameterizations, does not bind. The second constraint ensures that the agent always gets some positive utility and that this utility is always bounded away from a minimum of 0. My results are robust to changes in this bound.

3 Calibration

I calibrate the model as follows:

I need to choose the functional forms for the production process and the agents utility functions. For the production process I pick a standard Cobb Douglas production function $f(e,k,l) = ek^{\alpha}l^{\nu}$, and set $\alpha + \nu = 1$. The agents utility function is a standard Greenwood, Hercowitz, and Huffman (1988) utility function of the form $u(c,l) = (c - \psi \frac{l^{1+\gamma}}{1+\gamma})$ [5].

Table 1 contains the values of the parameters of the model.

To calibrate my model I set the subjective discount factor to a conventional $\beta = 0.985$ for a quarterly model. I pick the intertemporal elasticity of substitution, $\frac{1}{\eta}$ to be equal to 0.2 and the risk aversion parameter, σ to be equal to 6. These parameter values are estimates from Guvenen (2009) for a model with Epstein-Zin preferences, GHH utility, and other frictions. On the labor side of the utility, the Frisch labor elasticity is set to a standard 1.5 and ψ is calibrated to generate an average labor supply of 0.3 during expansions.

For the production process, given that $\alpha + \nu = 1$, only two parameters need to be chosen. The quarterly capital depreciation rate is set to equal 0.025, as is standard, and $\nu = 0.67$ to match the empirical observation that the labor share of income is approximately $\frac{2}{3}$.

Next, I calibrate the technology process parameters. As the paper's main goal is to illustrate a mechanism, without loss of generality, I allow for only two different technology process choices, setting $\|J\| = 2$. I normalize the mean of the high mean/high variance process to one, $\mu_2 = 1$, and the standard deviation of the low mean/low variance process to be approximately zero, $\sigma_1 = 0.0001$. I also pick the short run persistence parameter, which acts as a scale parameter in my i.i.d. setup to be equal to $0.859^{\frac{1}{4}}$ based on Khan & Thomas (2009).

[5] The main reason for using a GHH utility function is that it allows for more tractability.

I am now left with three parameters, the mean of the low mean/low variance process, μ_1, and the standard deviations of the high mean/high variance process during expansions and recessions, $\sigma_{2,E}$ and $\sigma_{2,R}$. I calibrate these parameters to match three moments in my model to the data: (i) GDP during recessions is approximately 2.5% lower than expansion, (ii) the variance of the ergodic distribution of TFP resulting from a short run AR1 process is $\frac{0.0081^2}{1-\rho^2}$ during expansions, (iii) the standard deviation of the TFP process is approximately four times higher during recessions. The resulting values of the calibrated parameters are contained in table 1. I will further discuss the values of these parameters for the two different regimes, expansions and recessions, in the results section.

For numerical ease I finally set $\underline{k} = 0.01$ and $\underline{u} = 0.0001$. These values ensure that I do not need to approximate in the neighborhood of where the value function would take on values close to infinity. My results are robust to small changes in these two parameters.

4 Results

4.1 Solution Method

To solve for the value function given by equation (2.1) I use standard function iteration. To make function iteration more accurate and "more" tractable I simplify the value function along a couple of dimensions. I assume that the stochastic process is fully described by a 9 state markov chain, this assumption along with the calibration of $\|J\| = 2$ implies that two of the four dimensions of the reduced value function sit in a space of finite cardinality, with the remaining dimensions in capital and labor, k, l, being defined over $[\underline{k}, \infty)$ and $[0, \infty)$. Given this structure and these assumptions I can use cubic splines to interpolate in the continuous dimensions, and then a pure discrete grid in the discrete dimensions to characterize a numerical equivalent of the true expected value function. Finally, I use golden section search for maximization over the continuous variables.

I define a steady state as a set of values for the set of endogenous state variables, $S = (k, l, j)$, such that first the steady state values are equal to the average value of S across multiple simulations, and second that this average value of S is insensitive to the periods of simulation. Alternative interpretations of this steady state would be that the across-simulation distribution over the state space is constant over time, or in the time dimension, the across-time distribution over the state space is constant across simulations.

To obtain the steady state, and the resulting mean and standard deviations reported in table 2 and 3, I use the expected value function defined above along with a value function defined on 100 knots in the capital dimension to obtain sufficiently accurate decision rules. I use these decision rules to simulate the economy for a 1,000,000 periods, and discard the first 100,000 periods, to obtain averages and standard deviations for various macroeconomic aggregates in the stochastic steady state. For the expected impulse response exercises in this paper I start the model off from the appropriate steady state value and then simulate the model for 50 periods, 2,000,000 times, and then report the average resulting responses. These results are sufficiently robust to the number of knots, the simulation periods, and the number of times a simulation is repeated to get the average response.

4.2 Simulation Results

Figure 2 plots the main result: in response to an exogenous increase in uncertainty, in the model economy of this paper, output falls. The mechanism of how and why this fall occurs can be understood by studying figures 3 through 5. The novel idea in this paper is that social planner can endogenously choose the type of technology with which to run the representative firm's production process; the choices available to the social planner lie along a mean-variance tradeoff frontier. In the model economy this is captured, without loss of generality, by allowing the social planner to either choose to run the representative firm with a low mean/low variance technology or a high mean/high variance technology as shown in figure 3. Recessions are times of higher uncertainty, that is, each technology process is associated with a higher variance, or alternatively recessions are times of a steeper mean-variance tradeoff frontier. The model captures this effect by imposing that during recessions the variance of the high mean/high variance process increases. During an expansion the social planner in the model always finds it optimal to choose the high mean/high variance technology. However, during a recession this tradeoff becomes steeper, and now due to the risk averse preferences of the representative agent, the social planner, depending on the state of the world, often also finds it optimal to choose the low mean/low variance technology. The ability to endogenously choose which technology to run the representative firm with, in response to changes in uncertainty, generates a set of equilibrium choices over time in the model. When exogenous uncertainty rises (i.e. the trade-off frontier becomes steeper) the social planner compensates by choosing the low mean/low variance technology more often. This in turn causes the mean level of technology to fall, and thus an output recession to result.

The dynamics of how the fall in the mean level of technology occurs over time as the social planner switches the technology types is shown in figures 4 and 5. Panel (a) of figure 4 plots a sample stochastic series for the two different types of technology processes available to the social planner. In the figure the economy is in an expansionary regime before time 0 and in a recessionary regime after time 0. The low mean/low variance technology remains stable across the two regimes; however, the high mean/high variance technology as can be seen in the figure experiences a substantial exogenous increase in variability. This increase in the variability during the recessionary regime causes the social planner to become more likely to choose the low mean/low variance technology as seen in panel (b) of the same figure. The actual type choice is dependent on the full state. In figure 5 we see how this endogenous movement in the technology type choice changes the mean level and standard deviation of the expected technology process. As the social planner becomes more and more likely to choose the low mean/low variance technology, as seen in panel (a), the technology level falls, and as seen in panel (b), the standard deviation falls relative to the high mean/high variance choice's variance, but rises relative to the expansionary state. The endogenous switching of technology dampens the effect of the uncertainty shock and in the process causes the mean level of technology to fall, and thus an output recession. Panels (a) and (c) in figure 6 further show that along with output, consumption and labor hours also fall, while panels (b) and (d) show that investment and capital initially rise, but eventually fall as well. The causes for the rise in investment will be discussed later. Tables 2 and 3 reinforce these findings, that in the baseline model output, consumption, investment, and labor all fall in the recessionary steady state. The presence of endogenous technology choices thus causes higher uncertainty to cause a fall in the macroeconomic aggregates.

A key implication of the model in this paper, that explains how second shocks can have first moment effect, is that it also suggests that measuring the variance of TFP in the data as a proxy for uncertainty shocks may result in measures of uncertainty shocks that are downward biased. As seen in tables 2 and 3 the standard deviation of technology only increases four times going from an expansionary steady state to a recessionary steady state, even though the underlying shock to uncertainty caused the high mean/high variance firm to have eleven times as high a standard deviation. In calibrating the model I attempted to match two key moments in the data, first that GDP drops 2.5% during recessions and second that the variance of TFP during recessions at the aggregate level is four times as high as during expansions. The model dynamics suggest that to match these moments in the data the underlying shock to uncertainty in the model must be such that the variance of the high mean/high variance technology increases eleven times. This raises an interesting question, if the baseline model is viewed as a good approximation to the endogenous technology choice process in the real economy, then the underlying uncertainty shocks driving the economy are much larger than the observed changes in the standard deviation of technology. The variance of TFP is a systematically downward biased measure of uncertainty shocks. Allowing for endogenous technology choices in the model results in an endogenous dampening of the high exogenous uncertainty.

Uncertainty shocks and negative technology shocks produce very similar dynamics in a model economy where the social planner can endogenously choose technology types. Figure 8 plots the impulse response for the baseline model, along with the model calibrated to ensure complete switching during recessions (i.e. no mixing of the technologies over time [6]), and the baseline model shocked, not with higher uncertainty, but a drop in technology (a standard negative technology shock). As can be seen in the figure the complete switching model driven by uncertainty shocks and the baseline model driven by a negative technology shock produce very similar responses. The baseline model, other than the slow decline in investment, also looks very similar to a model where technology is slowly decreasing. These similarities become even more glaring when we look at the steady state moments in table 2 and 3. As seen in table 2 the steady state means are virtually indistinguishable for a recession caused by low technology vs. high uncertainty. Table 3 shows that the standard deviation between the two models are different in that in the low technology case there is little change in the variance of the aggregates between a recession and expansion, whereas in the model driven by uncertainty shock there is a significant change. However, if we allow for the possibility of the negative technology shock to also have a higher standard deviation component - which is low enough that it doesn't cause any switching, but matches output volatility in recessions - then as can be seen in the same table the standard deviations between the baseline case and

[6]The complete switching model is calibrated to match the 2.5% drop in GDP during recessions, and instead of matching a four times as high TFP variance during recessions the model ensures that exactly one type is chosen in each regime. In equilibrium, this calibration result in the high mean/high variance firm as being the only choice during expansions, and the low mean/low variance firm being the only choice during recessions

this new model with a negative TFP shock line up completely. Both in terms of model dynamics and in terms of steady states, the standard technology shocks, that economists have been studying for years, look very similar to uncertainty shocks, and thus the model suggests that these two shocks may be indistinguishable in the data. It may not be possible to tell if the actual economy is driven by an uncertainty or a standard first moment technology shock.

A model with endogenous technology choices can explain how second moment shocks to technology can have first moment effects. Such a model can also generate two other interesting results, first it suggests that measures of uncertainty shocks might be biased downward, and second in such a model the effects of uncertainty shocks are possibly indistinguishable from standard first moment shocks, suggesting an identification problem between the two in the data.

It is curious to note that in figure 5 the mean level of the technology and the standard deviation of the technology do not seem to be asymptoting to the levels of the mean and variance of the low mean/low variance firm. This is confirmed by looking at tables 2 and 3, in the recessionary steady state, where the steady state is defined in the subsection above, the mean level of technology only falls 1.15% and the standard deviation only rises approximately 4 times, whereas as seen in table 1 the low mean/low variance technology is calibrated to have a mean that is 1.3% lower and a variance that is approximately $\frac{1}{11}$th of the high mean/high variance technology's variance. In the recessionary steady state the social planner is mixing the two types of technologies over time. The exact type choice at any given time depends on the state, $(k, j; a)$, as can be seen in figure 13. For my calibration the firm chooses the type 1 technology approximately 87.7% of the time, and the type 2 technology the remaining 12.3% of the time in the recessionary steady state. In figure 13 this would correspond to a steady state level of capital that is very close to the switching threshold. The reason for implementing such a calibration is as follows: in order to generate a technology series with mean and variance similar to that observed in the data, I must allow for a mixing of the technologies across time. The two different technology choices on the mean-variance frontier on their own respectively cannot match that moment of the data - the low mean/low variance technology would generate the correct mean for the technology series, but not the correct variance, and the high mean/high variance technology would do the opposite.

In panel (b) of figure 6 that plots the expected impulse responses for the baseline model, investment rises, instead of declining in response to higher uncertainty. Investment first rises in the period of the shock, and then slowly declines, eventually falling below its expansionary steady state level. The initial increase in investment in the period of the shock is because output is predetermined[7], and thus when the representative agent decreases consumption, in order to smooth it, investment must necessarily rise. It is optimal in this framework for the agent to increase investment over consumption due to the precautionary saving motive. One can interpret the increase in investment as an increase in inventory because output was predetermined and consumption fell.

The reason for why investment remains above the expansionary steady state level for several periods is much more subtle. The mixing of technology choices over time plays an important role in the post period zero dynamics of investment. In panel (b) of figure 4 we notice that on average the shift to the steady state mix of high mean/high variance and low mean/low variance technologies is gradual, meaning the agent lives very close to the choice threshold seen in figure 13. As a result if a representative firm is lucky enough to continually get high productivity draws then the social planner will continue to accumulate assets as a precautionary motive against future bad draws, and at the same time will continue to use the high mean/high variance technology. Figure 13 shows the optimal type choice, and as can be seen a high mean/high variance firm (type 2) is more likely to stay such a firm if it gets high productivity draws. Figure 7, that plots the responses in a complete switching model, makes this point even clearer. In this model the agent does not live close to the choice threshold and the planner always chooses to run the representative firm with the low mean/low variance technology during a recession. As can be seen in panel (c) of this figure after the initial "inventory" build-up investment falls and remains below the steady state. The slow decrease in investment in the baseline model accounts for the positive probability that the economy might take a path where the representative firm for some time continues to get high productivity draws after the uncertainty shock hits [8].

As robustness I make sure that a decrease in uncertainty does indeed cause an economic expansion (given the non-linear nature of the problem, symmetric responses between positive and negative shocks are not assured). Figure 9 plots the expected impulse responses for the baseline model when the uncertainty regime switches from an recessionary state to

[7]Capital and labor are both state variables and thus output is fixed.

[8]The figures in the paper give the expected impulse responses

an expansionary state. The social planner in response to the lower uncertainty switches from the mix of low mean/low variance and high mean/high variance technologies to the high mean/high variance technology. This is because the variance of the high mean/high variance technology has fallen raising the marginal benefit of using this technology. The choice of the high mean/high variance technology results in the mean TFP rising and thus increases in output, consumption, and labor hours. Investment falls in the period of the shock, because output is predetermined, but then rises and remains above the recessionary steady state. Negative uncertainty shocks in the model do cause expansions.

As further robustness figures 10 and 11 confirm the importance of the endogenous type choices in the model. Figure 10 plots the baseline model, but now with no endogenous technology choices. In this model there is some movement in investment, but for the most part output, consumption, and labor hours are virtually unaffected by the increase in uncertainty. The strong risk aversion due to the Epstein-Zin preference specification has little to no effect on any of the variables other than investment. When I shut down the risk aversion in figure 11, and use purely recursive preferences higher uncertainty has an almost negligible effect on all the aggregate variables, including investment. These robustness results illustrate that the key to the results in the model is the presence of endogenous technology choices in the model economy, and the interplay between these choices and the agent's risk aversion.

4.3 Transition along Growth Paths

The presence of endogenous technology choices in a model also has some interesting implications for understanding how countries accumulate capital along their growth paths towards the over-time stochastic steady state level of capital. In figure 12 I start my baseline economy of at a capital level that is 98.75% of the long run steady state level of capital to see how the economic aggregates transition to their long run stochastic steady state levels. The first observation from panel (e) of this figure is that at low levels of capital the economy 'plays it safe' choosing the low mean/low variance technology to accumulate capital. This would suggest that the model predicts that developing countries, that have capital levels much lower than their long run levels, generally tend to be very risk averse.

The second and more interesting observation is that along the growth path at some point the social planner starts becoming willing to take on risk, and overshoots his long run equilibrium level of optimal riskiness. This illustrates an interesting interplay between the risk aversion and marginal product of capital in the model. The optimal endogenous firm technology choice is a function of both these variables. The social planner dislikes risk and thus faces a cost of choosing the high mean/high variance technology, this dislike is governed by the risk aversion parameter and the level of capital. At low levels of the capital the risk aversion is higher and thus the cost is higher. The marginal benefit of the capital is given by the marginal product which is higher both if the social planner chooses the high mean/high variance technology, or if the capital level is low. As seen in panel (e) of figure 12 along the transition path at a particular level of capital the marginal benefit starts outweighing the marginal cost, that is the capital is high enough for the risk aversion to be weak but low enough for the marginal product to be high, causing the social planner to suddenly switch to the high mean/high variance technology, and thus overshoot. However, as the capital accumulates the marginal product falls once again causing the marginal cost, due to risk aversion, to rise relatively, and thus cause the social planner to choose a less risky option on the remaining path to the steady state. The full transition path thus predicts that along the growth path countries initially play it 'very safe', and then become very 'risky' and accumulate capital very quickly before once again becoming risk averse and gradually converging to their long run level of optimal risk.

5 Conclusion

Uncertainty shocks have the potential to be an important driver of business cycle fluctuations. In the standard neoclassical model, changes in the higher moments of the exogenous driving process have a negligible effect on the first moments of the economy's aggregates. In this paper I present a relatively simple modification to the standard model such that higher moments play a much larger role in generating first moment effects; I allow for a decoupling of risk aversion by using Epstein-Zin preferences and add the ability to make endogenous technology choices along a mean-variance frontier. The main result of the paper is that in this new model framework changes in uncertainty can generate large business cycle fluctuations while keeping the moments of technology consistent with the data.

This paper also highlights the need to further study the empirical identification of uncertainty shocks by illustrating two possible measurement problems. First, in the model the uncertainty shocks driving the business cycle are much larger than what is observed in the model generated data. This is because the endogenous technology choices along the mean-variance frontier dampen the effect of the uncertainty. Second, the data on moments from the model, and to a lesser extent the impulse responses from the model, are very similar for when the business cycle is driven by an uncertainty shock vs. when it is driven by a standard technology shock. The model thus hints towards the existence of an identification problem in the real data where we are unable to distinguish uncertainty shocks from standard first moment technology shocks. Both these puzzles point towards the possibility that higher moment shocks might be playing a much larger role than previously understood through the lens of other models.

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Table 1: Model Parameters

Parameter	Value	Description
β	0.985	Subjective discount factor (quarterly).
η^{-1}	0.2	Intertemporal elasticity of substitution, a'la Guvenen (2009).
σ	6	Risk Aversion Parameter, a'la Guvenen (2009).
γ	1.5	Frisch Labor Elasticity.
ψ	13.24	Relative disutility of labor, set to ensure that the average labor in expansions equals to 0.3
ν	0.67	Labor share in production.
α	0.33	Capital share in production, set such that $(1-\nu) = 0.67$.
δ	0.025	Depreciation rate of capital.
$\|J\|$	2	Number of different technology processes available.
μ_1	0.987	The mean of the first technology process, set to match an implied 2.5% drop in GDP during recessions in the model.
μ_2	1	The mean of the second technology process, normalized to 1.
σ_1	0.0001	The standard deviation of the first technology process, normalized to be close to 0.
$\sigma_{2,E}$	0.0081	The standard deviation of the second technology process during expansions, set to observed variance of TFP during expansions, a'la Bloom et. al. (2010).
$\sigma_{2,R}$	0.092	The standard deviation of the second technology process during recessions, set to match an implied 4 times higher standard deviation of TFP in the model during recessions, a'la Bloom et. al. (2010).
ρ	$0.859^{\frac{1}{4}}$	Implied short run persistence of an AR1 equivalent of the i.i.d process, a'la Khan & Thomas (2009).
$\underline{k}$	0.01	Lower Bound on Capital carried between periods in numerical solution.
$\underline{u}$	0.0001	Lower Bound on Consumption in numerical solution.

Table 2: Steady State Mean Values

	Output	Cons.	Inv.	Labor	Capital	Tech.
Recessionary Steady State						
Baseline	-2.52 %	-2.76 %	-2.26 %	-1.07 %	-2.26 %	-1.15 %
Model w/ Complete Switch	-2.87 %	-2.76 %	-3.19 %	-1.23 %	-3.19 %	-1.15 %
Only TFP Shock	-2.77 %	-2.79 %	-2.74 %	-1.10 %	-2.72 %	-1.15 %
TFP Shock + High Uncertainty, but no Type Choice	-2.49 %	-2.68 %	-2.00 %	-1.00 %	-1.98 %	-1.15 %
Model w/ Risk Aversion, but no Type Choice	0.28 %	0.11 %	0.70 %	0.10 %	0.73 %	0.00 %
Model w/o Risk Aversion, but no Type Choice	-0.03 %	-0.06 %	0.00 %	0.00 %	0.01 %	0.00 %
Expansionary Steady State						
Baseline	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
Model w/ Complete Switch	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
Only TFP Shock	0.00 %	0.00 %	0.00 %	0.00 %	-0.01 %	0.00 %
TFP Shock + High Uncertainty, but no Type Choice	-0.01 %	0.00 %	-0.04 %	0.00 %	-0.01 %	0.00 %
Model w/ Risk Aversion, but no Type Choice	0.01 %	0.01 %	0.00 %	0.03 %	0.02 %	0.00 %
Model w/o Risk Aversion, but no Type Choice	-0.04 %	-0.01 %	-0.11 %	0.00 %	-0.09 %	0.00 %

Note: The table above gives the percentage difference from the baseline model in the expansionary regime.

Table 3: Steady State Standard Deviations

	Output	Cons.	Inv.	Labor	Capital	Tech.
Recessionary Steady State						
Baseline	316.7 %	354.7 %	313.4 %	400.0 %	361.9 %	297.4 %
Model w/ Complete Switch	-98.7 %	-100.0 %	-98.6 %	-100.0 %	-98.8 %	-98.7 %
Only TFP Shock	-1.6 %	-1.9 %	-1.7 %	0.0 %	-1.5 %	0.0 %
TFP Shock + High Uncertainty, but no Type Choice	281.4 %	275.5 %	281.8 %	300.0 %	274.7 %	296.7 %
Model w/ Risk Aversion, but no Type Choice	288.2 %	286.8 %	288.7 %	300.0 %	286.1 %	297.0 %
Model w/o Risk Aversion, but no Type Choice	286.6 %	283.0 %	287.0 %	300.0 %	281.6 %	297.0 %
Expansionary Steady State						
Baseline	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %
Model w/ Complete Switch	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %
Only TFP Shock	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %
TFP Shock + High Uncertainty, but no Type Choice	-0.3 %	-1.9 %	0.0 %	0.0 %	-1.6 %	0.0 %
Model w/ Risk Aversion, but no Type Choice	0.0 %	0.0 %	0.0 %	0.0 %	-0.6 %	0.0 %
Model w/o Risk Aversion, but no Type Choice	-0.3 %	-1.9 %	0.0 %	0.0 %	-1.7 %	0.0 %

Note: The table above gives the percentage difference from the baseline model in the expansionary regime.

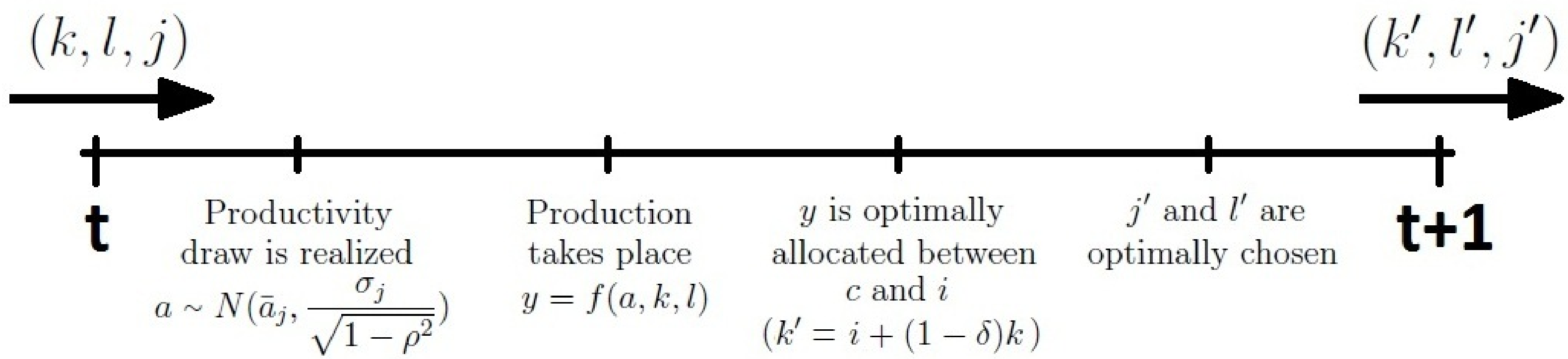

Figure 1: Model Timeline

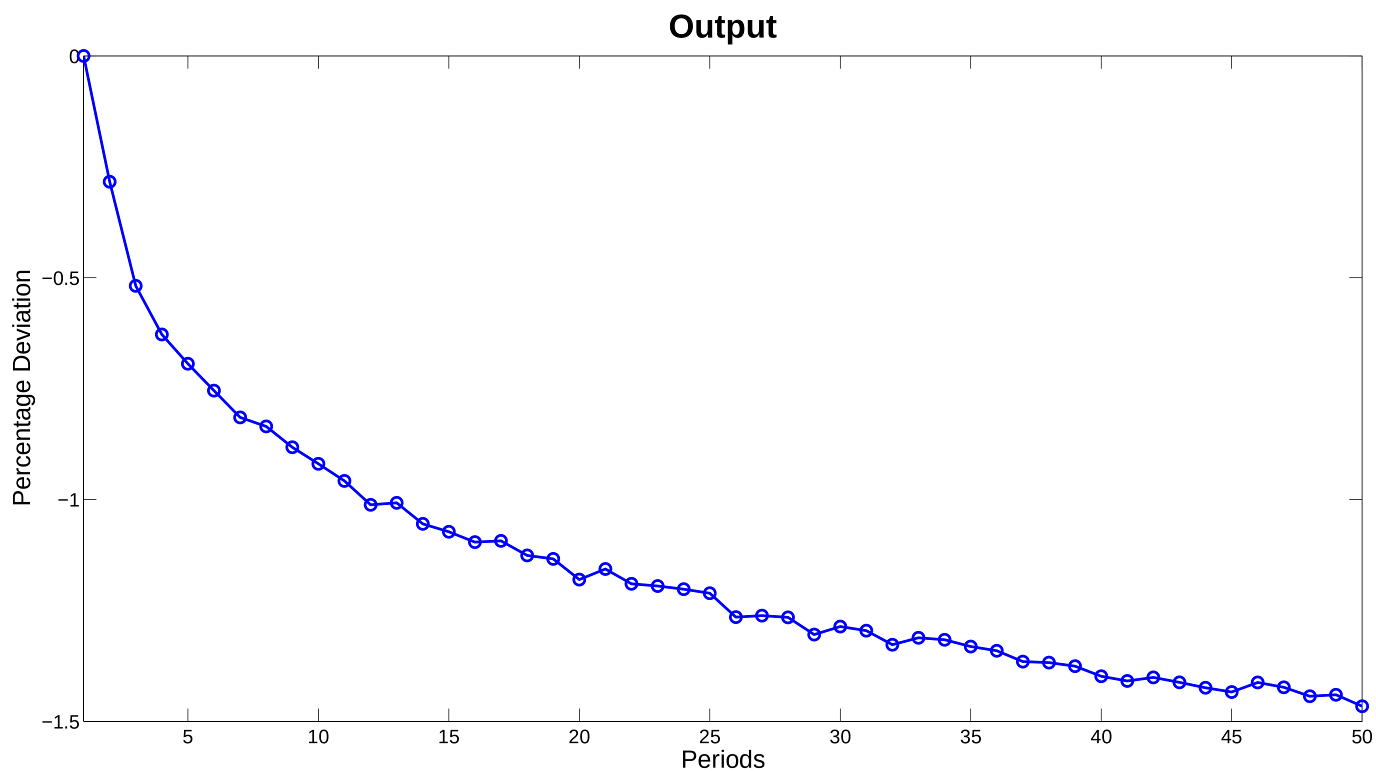

Figure 2: Expected Impulse Response to an Uncertainty Shock: Output in the Baseline Model

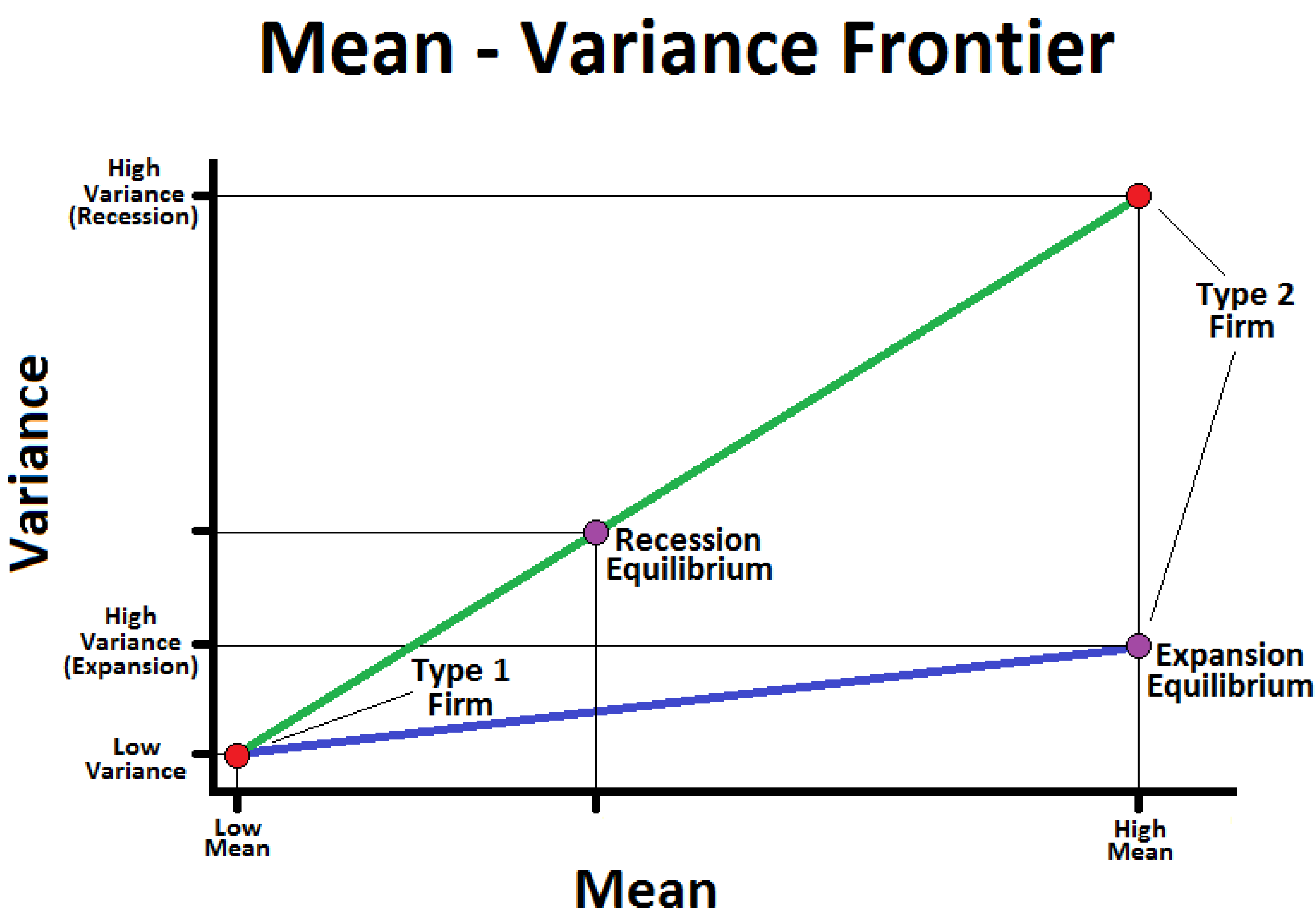

Figure 3: The Mean-Variance Frontier

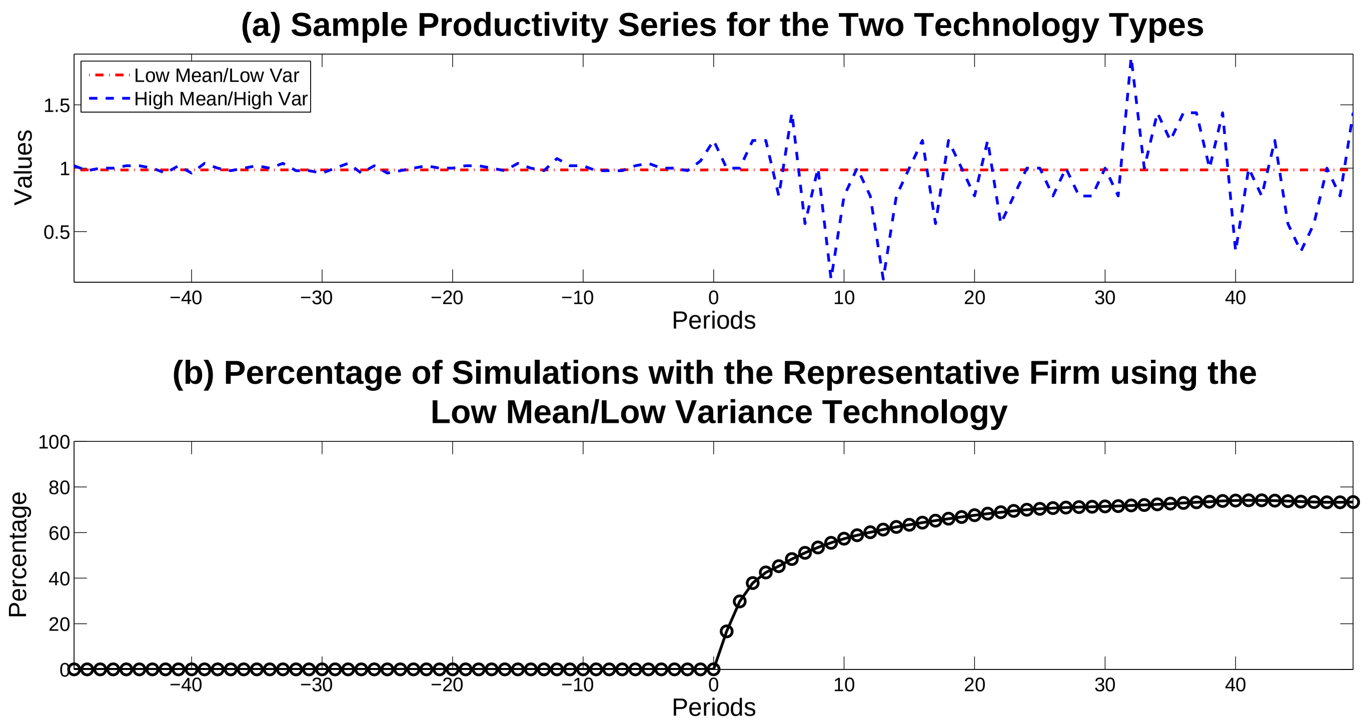

Figure 4: Expected Impulse Response to an Uncertainty Shock: The Technology Type Choices

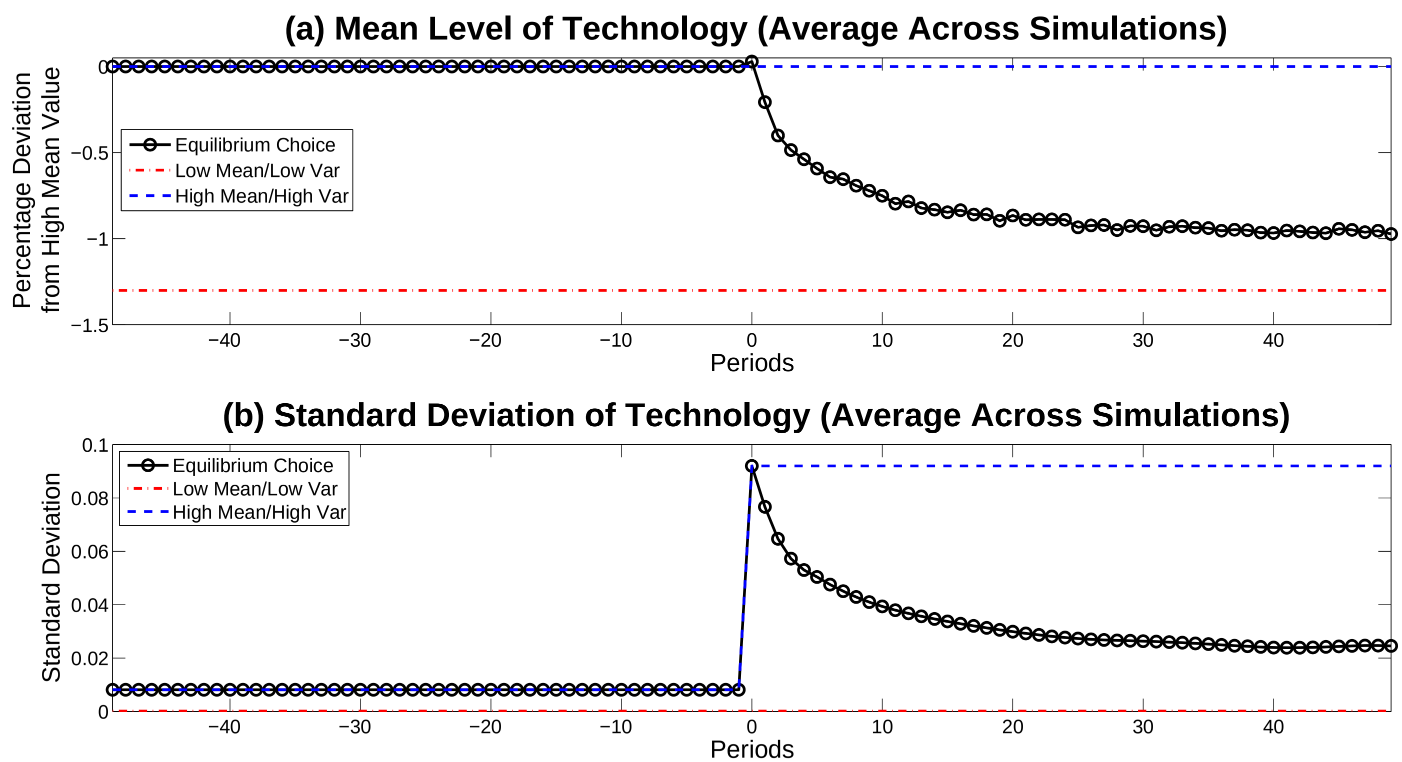

Figure 5: Expected Impulse Response to an Uncertainty Shock: Tech. Level & Standard Dev. in the Baseline Model

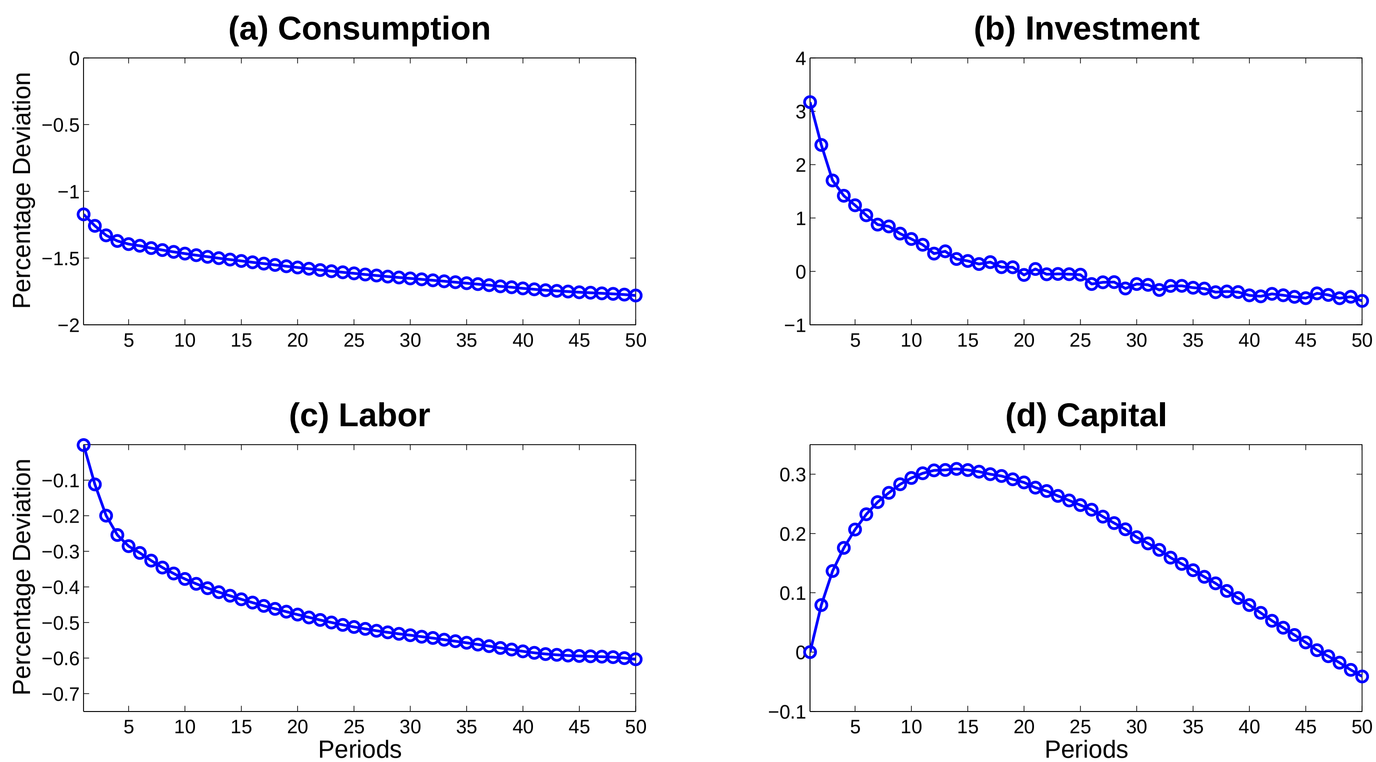

Figure 6: Expected Impulse Response to an Uncertainty Shock: Baseline Model

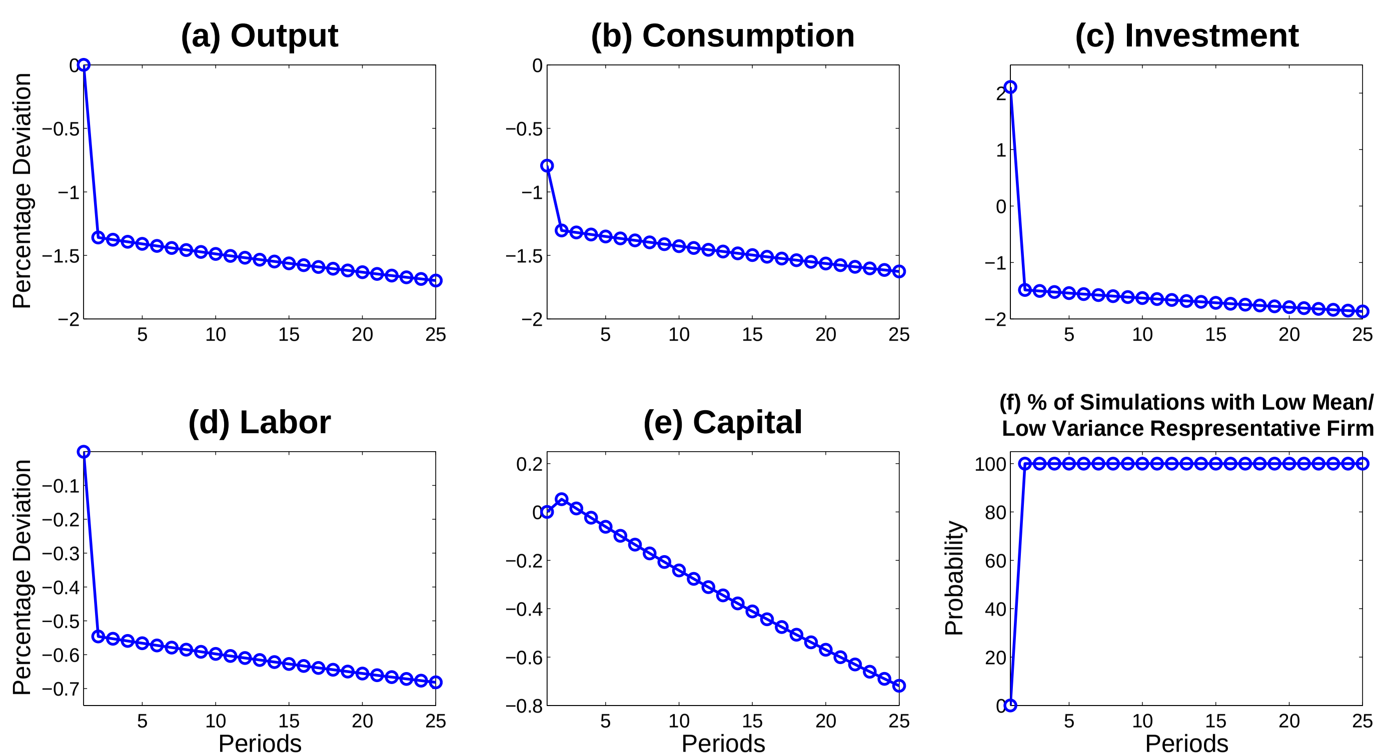

Figure 7: Expected Impulse Response to an Uncertainty Shock: Model w/ Complete Switching

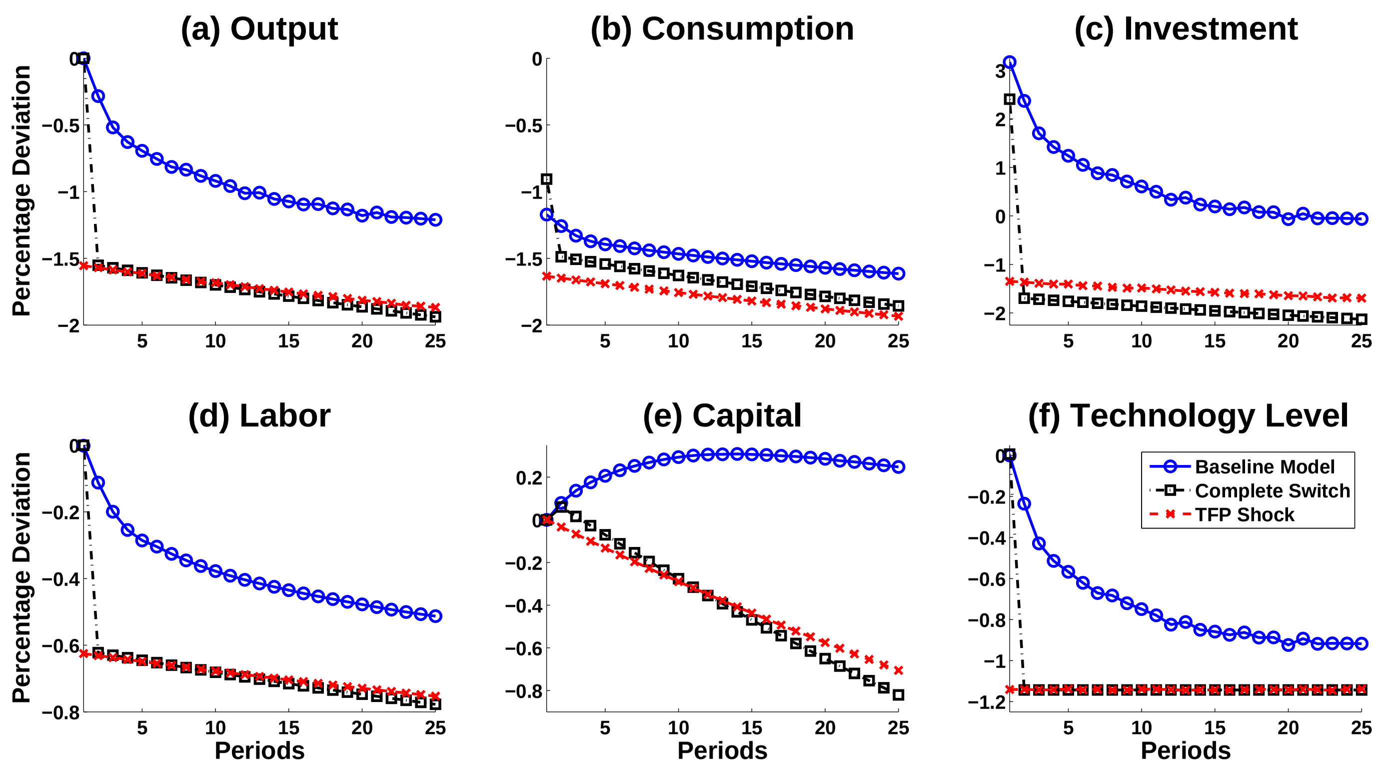

Figure 8: Expected Impulse Response: Comparison of a TFP Shock and an Uncertainty Shock

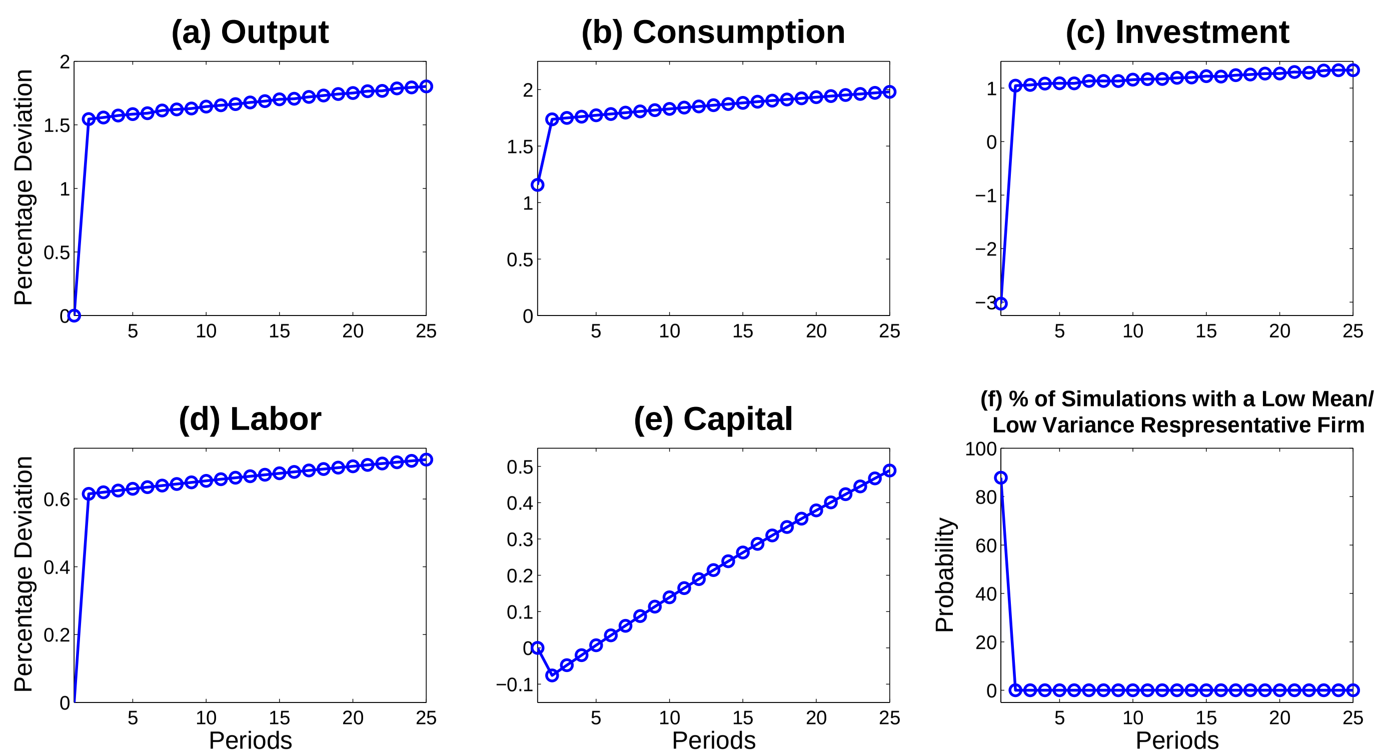

Figure 9: Expected Impulse Response to a Negative Uncertainty Shock: Baseline Model

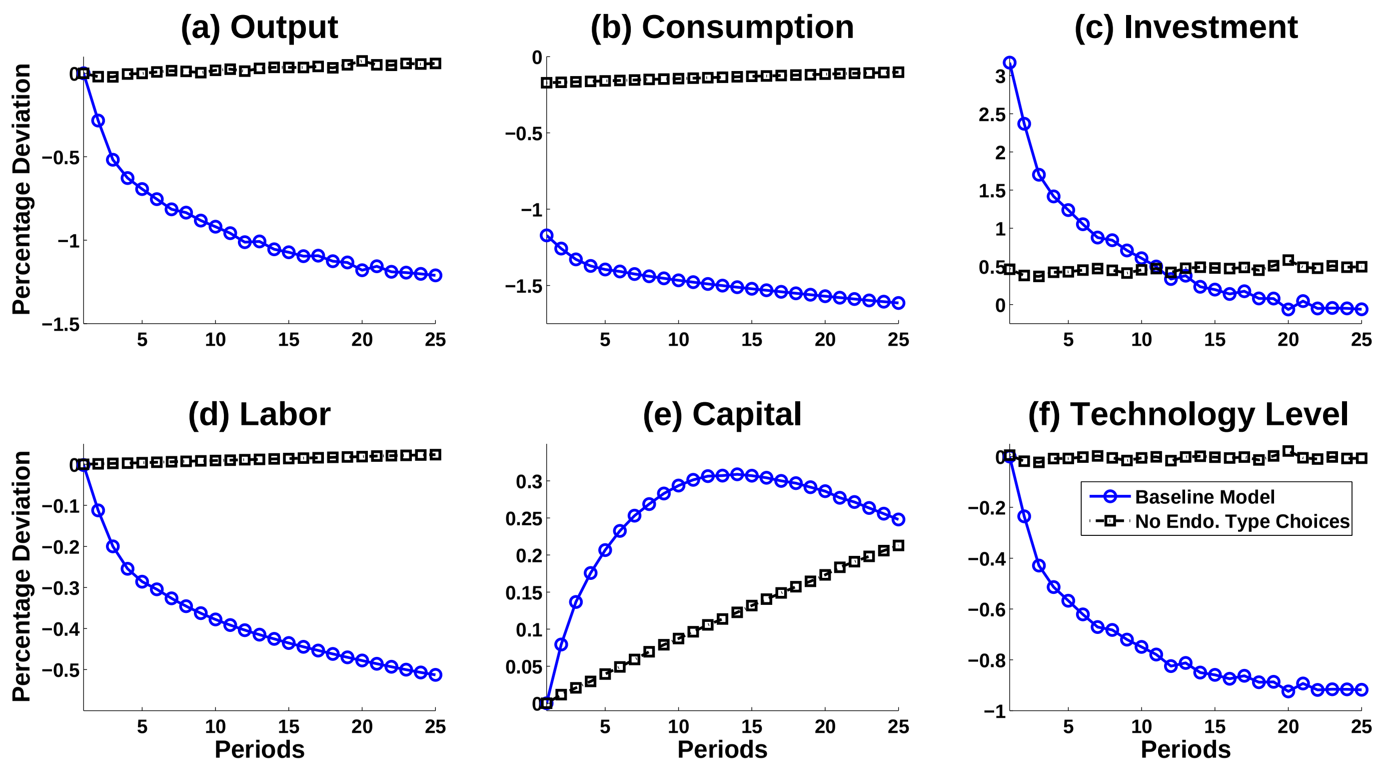

Figure 10: Expected Impulse Response to an Uncertainty Shock: Baseline Model vs. Model w/ No Endogenous Type Choices

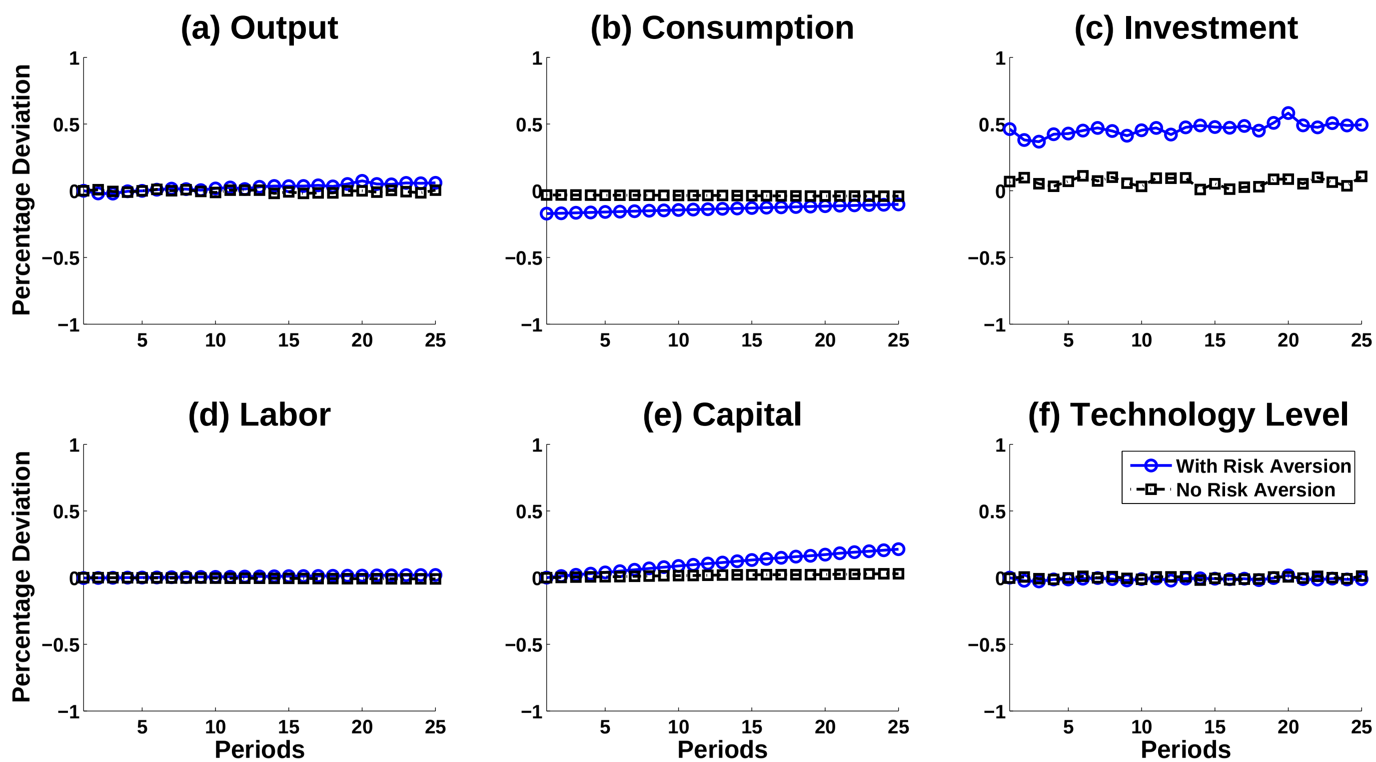

Figure 11: Expected Impulse Response to an Uncertainty Shock: Model w/ Risk Aversion vs. w/o Risk Aversion

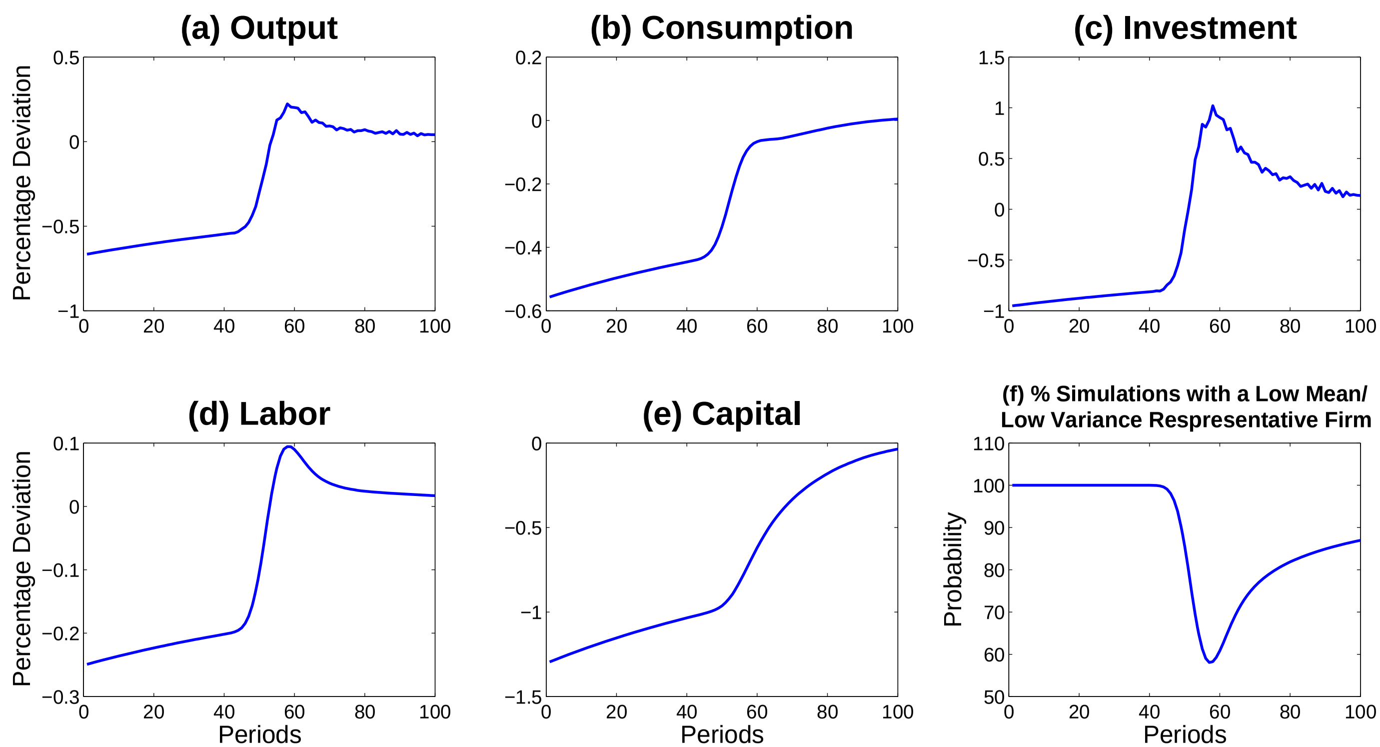

Figure 12: Transition Path from Below Steady State Capital

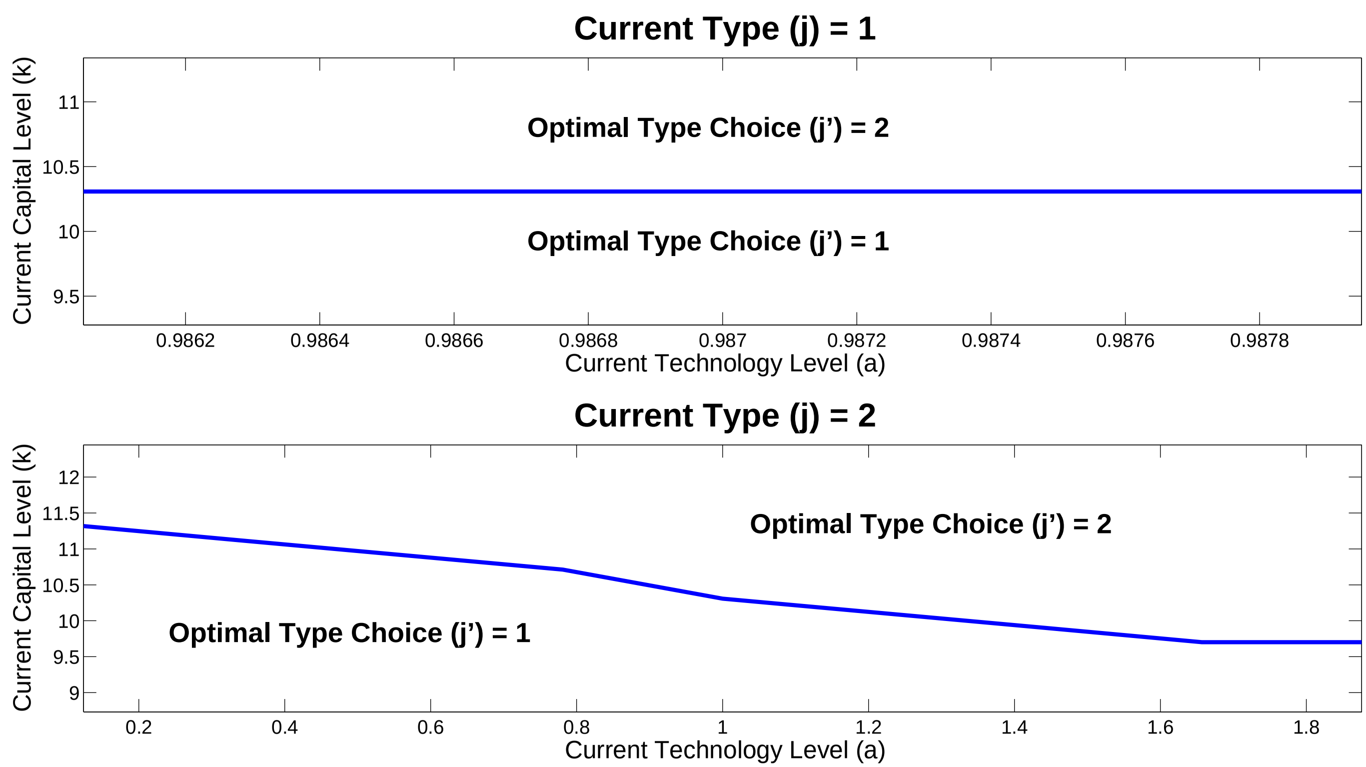

Figure 13: Optimal Type Choice in a Recessionary Phase