Chapter 5
Norms of Representation and Voter Welfare: Simulations

ABSTRACT: This chapter uses a simulation model to generate predictions about the effects of gerrymandering institutions on four different measures of voter welfare, with interactions for partisan tides and party polarization. Four representation norms are defined: policy median, compositional, personal, and discursive. The simulation generates three major predictions with respect to these norms: (a) bipartisan maps generate higher average personal and discursive utility than nonpartisan maps, and sometimes generate higher policy median utility; (b) partisan maps occasionally generate higher policy median utility than bipartisan or partisan maps, and compromise between the two on the other three measures; and (c) while extreme polarization is always deleterious to all measures of voter welfare, moderate polarization can sometimes improve discursive utility for many maps under all tides, and can improve other utility measures under strong partisan tides.

In an extension, the simulations find that majority-minority districting mandated by the 1982 VRA amendments benefits ideologically extreme minority voters more under the electoral conditions in which the amendments were passed (frequent Democratic majorities, low polarization) than under those in which they were implemented.

I. Competing Norms in Redistricting Research

Since the advent of the Baker v. Carr line of Supreme Court cases mandating equal population in legislative districts, scholars, politicians, and interest groups have supported numerous different measures and standards for what constitutes fair districting. Implicit in these different standards lie different conceptions for how voters should best feel represented by their government. In advocating for a particular standard, politicians and scholars often broach their own conception of representation norms, speaking past the advocates of other norms, typically leaving it to courts to decide between competing fundamental and non-cumulative values. This chapter uses a simulation model to predict the effects of districting institutions on these competing fundamental values, engaging them each equally and in parallel.

Some scholars argue that the most important goal in “fair” districting is to maximize the number of unbiased, competitive seats (e.g. Niemi & Deegan 1978). This goal has been favored by “good government” groups such as Common Cause, and codified into several successful
efforts at redistricting reform (e.g. Arizona’s 2000 Independent Redistricting Commission initiative). More recently, others have claimed that maximizing competitive districts does not best serve the interest of voters, and that mapmakers should instead seek to improve the match between legislators and their constituents by drawing “safe” seats for both parties (e.g. Buchler 2005, Brunell 2008). And in the wake of the amendments to the Voting Rights Act in 1982, an additional line of research has focused on the specific ability of racial minorities to have their interests addressed in the legislature through districting criteria (e.g. Canon 1999, Cameron et al. 1996, but see Chapter 4 for comprehensive discussion).

Yet, the districting system for the U.S. Congress, while determined on a state-by-state level, is still most frequently done through the normal state legislative process (see McDonald 2004), and thus very often partisan motivations and partisan power determine its shape. And the potential for one-party rule over districting seems appalling to scholars, media, and good-government groups across the board, with rare exceptions. It is described as a “pathology of democracy” (Shapiro 1985 p. 239) and an “anti-democratic practice that can effectively poison democratic institutions” (Polsby & Popper 1991 p. 305). Recent ballot initiative reforms have tried to eliminate partisan motives in redistricting even while keeping the power to district with the legislature (e.g. Florida Amendment 5 and Amendment 6 in 2010). But while disgust with partisan gerrymanders may be almost universal on an intuitive level, the empirical evidence that a biased map actually makes voters worse off is much sparser (Gelman and King 2003, Kang 2004).\footnote{Gelman and King (1993) argue that partisan and bipartisan redistricting benefits voters with respect to both responsiveness and bias, but this is in contrast to no redistricting, as opposed to a nonpartisan procedure. Kang (2004) also defends “offensive” partisan gerrymandering in contrast with “defensive” partisan gerrymanders. These correspond broadly to “aggressive” and “moderate” partisan gerrymanders respectively in my simulation model. Kang’s assumptions about the competitiveness/responsiveness of elections under these regimes are also somewhat similar to those found in Chapter 3, although his assumptions with regard to polarization and representation are largely different than what is predicted by the Gerrymandering model.} The paper will assess whether these partisan gerrymanders are as harmful to good
representation as critics claim, or whether they can actually advance some measures of voter welfare over bipartisan or nonpartisan institutions.

Further complicating these arguments, increased ideological party polarization has become an important area of research into the representation connection between voters and legislators in recent years (McCarty et al. 2006). While a gerrymander can potentially increase or decrease the polarization of a resulting legislature, McCarty, Poole, and Rosenthal (2008) find that this has not been a major contributing factor in the increased polarization of Congress in recent decades. Other scholars find similar levels of polarization in competitive and non-competitive districts (McGhee 2008, Brunell & Grofman 2005). Yet researchers still find that legislators are much more polarized than voters (Lauderdale 2010, Fiorina et al. 2006). Thus, it appears that the polarization of legislative candidates is largely independent of changes in either the voter distribution or in gerrymandering institutions. In assessing the impact of gerrymandering on redistricting institutions, this paper will also examine how exogenous party polarization can interact with these institutions to exaggerate or substitute for the representation of extreme voters.

This chapter uses a simulation model to measure the impact of various gerrymandering regimes on four competing “norms” of democratic representation, including interactions of each regime with various partisan tides and party polarization conditions. In doing so, we address the following questions:

1.) Which democratic norms, and under what conditions, are best served by “nonpartisan” gerrymander, which draws competitive, heterogeneous districts, as compared to a “bipartisan” gerrymander, that attempts to draw as many safe seats for members of both parties as possible?

2.) Are any democratic norms under any conditions best served by a “partisan” gerrymander, in which one party draws a biased map attempting to win a large majority of the seats?
3.) Under what conditions can exogenous party polarization improve some norms of voter welfare; e.g. are there any gerrymanders in which polarization serves as a substitute for drawing districts to create diverse representation?

The simulation model is also extended to address a fourth question related to minority representation broached in Chapter 4:

4.) Under what democratic representation norms are members of an ideological extreme minority well-served by a gerrymander that draws a proportionate number of “majority-minority” districts?

Note that this paper includes only simulation results, which generate testable predictions. The next chapter will test several of these predictions empirically.

II. Simulation Specifications

The results in this chapter are generated from the same simulation model employed throughout the dissertation. In addition to the parameters manipulated in Chapter 3, this chapter also allows for variation in the polarization parameter $\delta$, which before was not relevant when we were only interested in the party, and not the ideology, of the resulting legislators. Additionally, this chapter looks at effects on the four measures of voter welfare discussed in the introduction. For ease of references, these four measures are:

- Policy Median Utility: Distance of voter from median delegation members
- Personal Utility: Distance of voter from legislator elected in voter’s district
- Compositional Utility: Average distance of voter from each legislator
- Discursive Utility: Distance of voter from closest legislator
For all figures in the results section, average utility is calculated across the entire range of voters for each of the four norms. The model never averages, sums, or compares values across different norms. Simulation results are shown below for the following parameter values:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>number of voters</td>
<td>435</td>
</tr>
<tr>
<td>$d$</td>
<td>number of districts</td>
<td>15</td>
</tr>
<tr>
<td>$x$</td>
<td>voter ideology</td>
<td>$\sim U_{\text{discrete}}[-217,217]$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Vote/seat responsiveness</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>ideological partisanship</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>candidate polarization</td>
<td>Integer values 0 to 100</td>
</tr>
<tr>
<td>$\tau$</td>
<td>partisan tides</td>
<td>-20, 0, 20$^2$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>gerrymander (packed seats)</td>
<td>0 (nonpartisan gerrymander)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 (aggressive partisan)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 (moderate partisan)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15 (bipartisan)</td>
</tr>
<tr>
<td>$s$</td>
<td>simulation iterations</td>
<td>5,000 each value</td>
</tr>
</tbody>
</table>

The simulation was run for all values of $\delta$ between 0 and 100 (in intervals of 10), crossed with all values of $\tau$ from -25 to 25 (in intervals of 5), for all values of $\gamma$ from 0 to 5 and 15, with 5,000 simulation iterations for each combination of values. For ease of presentation purposes, a selection from these simulations is shown in the figures. The $\gamma$ value of 2 is chosen to represent an “aggressive partisan” gerrymander, while a $\gamma$ value of 5 is chosen to represent a “moderate partisan” gerrymander. Similarly, a $\tau$ value of -20 is shown to represent strong Democratic tides (adverse to a gerrymandering Republican party), $\tau = 0$ to represent neutral tides, and $\tau = 20$ to represent strong Republican tides. Results depicting other values for these variables are

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2 For both the tides ($\tau$) and gerrymandering ($\gamma$) parameters, a wide range of values were simulated in addition to those shown in the figures, which have been limited to three values of $\tau$ and four values of $\gamma$ for each of display. Other possible values of these parameters display similar trends to those shown.
available from the author, and show similar conclusions, with one caveat: the $\tau$ value to represent strong adverse partisan tides was deliberately chosen to be past the inflection point of backfire for the aggressive partisan map, but not past this inflection point for the moderate partisan map (see Chapter 3 for description of backfire inflection points in partisan maps).

III. Simulation Results

A. Summary of Key Predictions

In the interest of completeness, the subsequent sections of this chapter present a broad array of simulation results; given the large number of findings, only some of which are surprising or interesting, it can be easy to overlook which results are most important to current debates about redistricting reform. Therefore, a few key findings are previewed here, indexed to where they are explained in greater detail. We have seen in the introduction that the balance of reformers, legal scholars, and political scientists advocate for nonpartisan reforms and competitive districts, while denouncing partisan gerrymander and (separately) increased party polarization. So we will concentrate on predictions addressing three specific questions:

- First, do nonpartisan institutions, particularly those that mandate or facilitate drawing competitive districts, improve the quality of representation?
  - We find that nonpartisan institutions, designed to generate competitive elections, do extremely poorly with respect to personal and discursive representation, the latter particularly when polarization is low (see right half of Figure 1). With respect to policy median representation, they generally perform well, particularly when tides are high (see right half of Figure 1). We might generally conclude that nonpartisan reforms will help voters who care more about the overall composition of the legislature, and hurt voters who care about being represented by an
individual; they will also help voters when parties are polarized and tides are volatile.

• Second, do partisan gerrymanders that attempt to maximize representation for one party degrade the quality of representation?
  o We find that partisan institutions only perform very badly on the policy median measure (upper left of Figure 3). And even on this measure, aggressive maps can perform well when tides are strong in either direction (Figure 4). We might thus be most concerned with partisan gerrymanders when they are “moderate”, i.e. they reinforce an existing majority rather than attempting to win as many new seats as possible, or we expect very few shifts in the political climate.

• Third, how does the quality of representation change as party polarization increases? Are there any conditions under which greater polarization leads to higher voter welfare?
  o We find that extreme polarization, where there is complete ideological separation by party, is harmful to all measures of voter welfare (see e.g. Appendix Figure A2). Moderate polarization can be helpful to discursive representation generally (Figure 5), and other norms when partisan tides are large, as some polarization makes delegations more responsive (Figure 6).

B. Partisan Composition of the Legislature

Before generating predictions with respect to voter welfare, it is useful to quickly review the findings of previous chapters with respect to party composition. Table 2 depicts the percentage of total seats won by Republicans for the chosen values of \( \tau \) and \( \gamma \) (where Republicans are the gerrymandering party for the partisan maps). As can be seen from this table, Republicans still win a bare majority of seats under adverse tides (\( \tau = -20 \)) in the case of the
moderate partisan gerrymander (γ = 5), but see a backfire under adverse tides in the case of the aggressive partisan gerrymander (γ = 2). At the same time, the aggressive partisan gerrymander is more favorable to Republicans under neutral and favorable tides. Chapter 3 describes these results in more detail, supported by empirical evidence.

Table 2. Percentage of Seats Won by Republicans

<table>
<thead>
<tr>
<th>τ (tide)</th>
<th>γ (Ideologically Packed Liberal Seats)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>-20</td>
<td>20.8%</td>
</tr>
<tr>
<td>0</td>
<td>50.0%</td>
</tr>
<tr>
<td>20</td>
<td>79.2%</td>
</tr>
</tbody>
</table>

C. Nonpartisan Maps v. Bipartisan Maps

The first question we will address is under what conditions the bipartisan gerrymander, drawing safe, ideologically homogenous districts, is preferable to the nonpartisan gerrymander, drawing all heterogeneous, competitive districts. Figure 1 below shows the average utility achieved under bipartisan and nonpartisan maps under neutral tides (τ = 0). Figure A1 in the appendix shows the same graphs for strong partisan tides (τ = 20). For all measures except policy median utility, which will be discussed further below, the results are very similar across tides conditions.

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3 Because both the populations and the gerrymanders are symmetric, results with respect to bipartisan and nonpartisan maps are identical regardless of the direction of tides, given the same absolute value. For partisan maps, and once an asymmetry in the population is introduced as in the extension in Section IV, the direction of tides is distinguished.
For two utility measures, the results are very consistent, and somewhat obvious. The bipartisan map performs better with respect to personal utility, regardless of the value of $\tau$ or $\delta$, while the nonpartisan map performs better with respect to compositional utility. This result is quite intuitive: the bipartisan map places all voters in districts with like-minded constituents, so an agreeable representative will likely be elected in every voter’s district, particularly when polarization is low, leading to high personal utility. But that same propensity to allow extreme voters to elect extreme legislators harms compositional utility, where every voter cares about the ideology of every legislator equally. Under neutral tides and low polarization, the nonpartisan map elects all moderate legislators, which is extremely pleasing to moderate voters, and makes extreme voters ambivalent from a compositional utility standpoint. But when legislators from across the ideological spectrum are elected, compositional utility among the extreme voters...
remains ambivalent (since they now face some legislators that make them extremely happy, but others that make them unhappy), but the compositional representation of moderate voters is reduced (they go from extremely happy with all legislators to moderately unhappy with some).

With respect to discursive utility, results similar to the personal utility graph are seen in the bipartisan map. But under the nonpartisan map, the results are heavily dependent on polarization. Under the bipartisan map, all interests are represented by some legislator (mostly by their own legislator). When polarization is low, extreme voters are not represented at all in a legislature composed entirely of moderates. But as polarization increases to $\delta = 50$, extreme voters can now find discursive representation through the more partisan legislators still elected from those same moderate districts, resulting in an increase in average utility. Still, the bipartisan map is always superior on this measure, as it is largely designed to be.

With respect to policy median utility, the bipartisan and nonpartisan lines track each other closely both with and without strong partisan tides. But the differences we do see depend entirely on tides conditions. Figure 2 below isolates policy median utility when $\delta<50$ under neutral tides and strong partisan tides.

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4 As discussed in the section on polarization, average utility declines again with increased polarization when $\delta>50$, as moderate voters are now hurt more by their lack of discursive representation than extreme voters are helped.
Under neutral tides, policy utility is very high when $\delta = 0$, as both maps will produce a policy median equal to the ideology of the statewide median voter. As polarization increases, the bipartisan map performs slightly better than the nonpartisan map, for a not entirely obvious reason. With high polarization, the nonpartisan map, with moderate medians in every district, will produce a legislature where every legislator is approximately $\delta\%$ away from the median (i.e. every legislator is polarized); thus the median legislator is equally likely to be a liberal Democrat or a conservative Republican, but will never be a moderate. In contrast, under the bipartisan maps, the election of a moderate (a legislator with ideology close to 0) is still possible, through a Democrat winning a conservative seat or a Republican winning a liberal seat. Although such outcomes are not probable at the level of the individual seat, only one such “underdog” candidate needs to win in order to be the median legislator, leading to some likelihood of a moderate median under the bipartisan map, versus no such possibility under the nonpartisan maps.\footnote{Note that this reflects reality at least anecdotally in that the most moderate Republicans in Congress (such as Connie Morella or Joseph Cao) represent strongly liberal districts, not moderate ones, while the most moderate Democrats (such as Charlie Stenholm) represent strongly conservative districts.}

In contrast, the nonpartisan map performs better as polarization increases under strong partisan tides ($\tau = 20$). This reflects the greater responsiveness of the nonpartisan maps to shifts
in public opinion. In the bipartisan case, because so many seats are safe for each party even when $\tau$ is large, a small number of seats determine which party will control the median, and thus there is a much higher likelihood that Republicans will be in control in the case of Democratic tides, and vice versa, which is a disaster for the average policy median utility measurement. Under the more responsive nonpartisan map, the chance the legislature will be controlled by a party adverse to tides, with an ideology against the majority opinion, is much lower, and thus the nonpartisan map performs better under the policy median measurement.

Table 3 summarizes how the choice of the nonpartisan or bipartisan institution depends on which norm of representation one values most highly. If one values personal or discursive representation, in which voters care more about having a single voice in the legislature, the bipartisan map is clearly better. But if one weighs all legislators equally in assessing representation, as the compositional measure does, the nonpartisan map is the winner by an equally clear margin. If one cares most about the median, and thus the likely policy outcome, which map is better depends on the electorate’s propensity for large swings: the bipartisan map is superior when swings are small and close elections are the norm, while the nonpartisan map proves more dependably responsive in reaction to strong tides.

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6 See Figure 2 in Chapter 3. When $\tau = 20$ (a strong Republican tide), there is approximately a 10% chance that Democrats will control the legislature under the bipartisan map, versus a 0.5% chance under the nonpartisan map.
**D. Partisan Gerrymanders**

The popular debate about redistricting institutions has very consistently assumed that allowing partisan motivation to enter into the process is an almost universal public bad, and that no matter what norms one values most, allowing a single party to draw maps to further their own ends must be among the worst solutions. This section will evaluate this conventional wisdom using the simulation model. As discussed above, two forms of the “partisan” gerrymander will be tested. The “aggressive” gerrymander packs the opposing party’s extremists into only 2 of the 15 seats, attempting to win the remaining 13 seats for themselves, while the “moderate” gerrymander packs opposing partisans into 5 seats, leading to a more secure 10 seats for the gerrymandering party. Under the “strong partisan tides” condition ($\tau = 20$; in this case Republican tides), the “aggressive” Democratic gerrymander backfires, leading to Republican majorities, while the “moderate” Democratic gerrymander holds fast for the Democrats.

Figure 3 below shows all utility measures for partisan gerrymanders under neutral tides (it also includes the nonpartisan and bipartisan gerrymanders, identical to Figure 1). For three of the four measures, both partisan maps yield average utilities somewhere in between the bipartisan and nonpartisan maps. The exception is the policy median norm. In this case, the median legislators generated under the partisan maps will consistently be more ideologically extreme than the median legislator elected under other methods. On this measure, the partisan gerrymanders are clearly the worst option in the case of a close election. If one cares little about policy median utility, then perhaps the partisan maps represent a good compromise between the extreme outcomes of the other maps on the other measures. But is there any case in which a partisan gerrymander actually performs the best of any of the maps?
Figure 3. Partisan, Bipartisan, and Nonpartisan Gerrymanders under Neutral Tides

Figure 4 shows that the answer is yes, under strong partisan tides conditions. The left-hand side of this figure depicts policy median utility under strong Republican tides for six different gerrymanders, including both the aggressive and moderate versions of partisan gerrymanders for both parties; the right-hand side drops the moderate partisan lines, and limits the x-axis to polarization values below 50 for ease of visualization.\(^7\)

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\(^7\) Graphs for all utility measures for these six maps under strong partisan tides are displayed in Figure 6 in the polarization section.
From the left-hand side, we observe that the aggressive forms of the partisan gerrymander yield average utilities that are very close to those of their nonpartisan and bipartisan counterparts. The only map that performs extremely badly is the moderate Democratic gerrymander, because it still usually produces a liberal Democrat as the median legislator despite the Republican tilt of the electorate. In contrast, the backfiring aggressive Democratic map, as well as both Republican maps, yield Republican medians in conformity with public opinion.

Additionally, the right-hand side clarifies that there are conditions under which the aggressive partisan map for both parties is actually the utility-maximizing map among the six choices: the Republican map when polarization is low, and the Democratic map when polarization is high. But more importantly, among these four maps, utility is quite similar when tides are strong enough that the partisan maps backfire and produce responsiveness (as was seen in the Pennsylvania map in the 2000s). It is only when partisan mapmakers limit their ambitions to produce robust but slim majorities that policy median utility suffers across the range of tides.
Table 4. Ranking of Partisan, Nonpartisan, and Bipartisan Gerrymanders
Based on utility measure and tides

<table>
<thead>
<tr>
<th>Utility Measure</th>
<th>Bipartisan</th>
<th>Nonpartisan</th>
<th>Aggressive Partisan</th>
<th>Moderate Partisan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compositional</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Personal</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Discursive</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Policy Median (Neutral Tide)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Policy Median (Strong Tides)</td>
<td>2/3*</td>
<td>1/2*</td>
<td>1/3*</td>
<td>4</td>
</tr>
</tbody>
</table>

* Ranking depends on polarization parameter

As summarized in Table 4, partisan maps prove most problematic when we care more about utility derived from the median legislator than other forms of utility, and even then mostly when electoral tides tend to be close, or partisan maps are sufficiently reinforced so as to not backfire in the face of adverse tides. Is there a case in which we might not care about the median? If we are looking at maps of individual states that will make up a national body (as is the case for congressional district maps), the median legislator in a state’s delegation probably does not hold much more importance than any other marginal legislator in the delegation. Additionally, when aggregating results across many states, we might be less concerned when one party controls the process in a single state, if overall control of the process is roughly balanced at the national level.

Of course, if one particular group is systematically poorly represented, we might not regard average utility across the entire population as an appropriate measure of a map’s effectiveness regardless of the norm we are measuring. This is likely the case in evaluating how well a map meets the requirements of the Voting Rights Act for dealing with the representation of “discrete and insular minorities”. Section V extends the model to evaluate each of the norms for a minority of the population only.
E. Party Polarization

We have already observed that the party polarization parameter ($\delta$) has a significant influence on average utility under all measures. As discussed in the introduction, the focus of recent literature on the problems associated with increased polarization make this an important topic to explore. This section summarizes the impact of the polarization parameter on each measure of average utility. From a cursory glance at the graphs, it does appear that increasing polarization generally reduces average utility in most cases. Therefore, this section will focus on isolating those exceptional conditions in which increased polarization increases some measure of average utility. We find this occurs under two general conditions:

- First, increased polarization can sometimes increase discursive utility when the redistricting regime does not guarantee representation for a wide range of ideologies.
- Second, increased polarization can increase all measures of utility under strong partisan tides by making election outcomes more responsive to changes in public opinion.
- However, in all cases, once polarization has passed a certain threshold, all further increases in polarization reduce all measures of average utility under all conditions.

We observed that for all specifications (i.e. all four utility measures for all values of $\gamma$ and $\tau$), average utility is strictly decreasing in $\delta$ once $\delta$ exceeds 50. Thus, once party polarization hits a particularly moderate threshold, further polarization is always harmful to voter utility under all measures and gerrymanders, under all tides conditions.

The value of 50 for $\delta$ is an important threshold here, in that it indicates the highest polarization value under which all voters in the uniform distribution can be perfectly represented discursively in the legislature. Assume a uniform population range $[-K, K]$ (where $K$ represents the more conservative/Republican voter, and $-K$ is the most liberal voter). When $\delta<50$, a Republican who wins in a district with median $-K$ will have an ideology$<0$ (i.e. Republicans
elected in the most liberal districts will be to the left of the population median voter), while a Democrat elected in a median K district will have ideology > 0. But when δ > 50, a Republican elected from a district with median –K will still have ideology > 0; all Republicans will locate to the right of all Democrats, and there will be no possible overlap between the parties in the legislatures. Additionally, it will be impossible for any legislator to locate at 0, and thus impossible for the median voter to be perfectly discursively represented. Translated into real-world terms: as long as there is still potential ideological overlap between the parties in the legislature, further polarization can still benefit average utility under certain conditions or measures. But once such overlap is impossible, further polarization is always harmful to average utility.

Although extreme polarization is always harmful to average utility, there are certain conditions under which moderate polarization can improve utility. As shown in Figure 3 in the previous section, policy median, compositional, and personal utility are always decreasing with increased polarization when tides are neutral. But discursive utility can sometimes be improved with increased party polarization, depending on the extent to which the gerrymander already ensures diverse representation. Figure 5 below depicts discursive utility under neutral tides for various gerrymanders. When the gerrymander already ensures that a wide range of ideologies will be represented, as in the bipartisan map, polarization reduces even discursive utility. But for the nonpartisan and partisan maps, increased polarization increases utility. In the case of the nonpartisan map, this is because it allows extreme voters to gain some representation (as discussed in section B). In the case of the partisan maps, polarization allows both moderate voters and extreme voters favorable to the gerrymandering party to gain greater discursive representation, subject to vote outcomes.

8 For the remainder of this section, all figures will be limited to a [0,50] range for δ.
The other general condition under which polarization can increase utility is under strong partisan tides. In this case, polarization makes the legislature responsive to tides in terms of ideology. When $\delta = 0$ and $\tau = 20$ (a strong Republican tide), the bipartisan and nonpartisan maps will overwhelmingly elect a Republican majority, but this majority will have the same ideology as if a Democratic majority had been elected, with a median ideology of 0, despite the fact that the statewide median voter now desires a policy of 20. As polarization increases toward $\delta = 20$, the Republicans elected in median districts now begin to reflect the opinion of the voters under Republican tides.

Figure 6 depicts all four utility measures under strong Republican tides. As previously observed, discursive utility is improved by increased polarization under several gerrymanders to a maximum of $\delta = 50$. But unlike under neutral tides, increased polarization also increases personal utility under several maps and compositional utility under the nonpartisan and aggressive Democratic maps, at least when polarization is already very low. And the most
dramatic difference is seen in the policy median utility graph: when $\delta = 0$, increased polarization would improve average utility under all maps except the moderate Democratic map. In all cases except the $\gamma = 5$ Republican map, the likely result of an election when $\tau = 20$ is a Republican majority; moderately increasing polarization increases the conservatism of this Republican majority, making it more responsive to the will of the voters under Republican tides. It is only under the moderate Republican map, which already generates a more than sufficiently conservative legislature under no polarization, that policy utility uniformly decreases in $\delta$.

**Figure 6. Utility under Republican Tides (Effects of Moderate Polarization)**

So what does all this tell us about what we should feel about increased polarization based on which norms of representation we value most? First, it will always be harmful to any norm if polarization gets so high that there can be no theoretical overlap between the parties. Given the
recent decline in moderates that cross party lines in both houses of Congress, it is possible that our current political climate is approaching this point. But neither should we strictly desire no exogenous party polarization. Even in the case of perpetually close national elections, party polarization can increase average discursive utility (the extent to which all voters have some voice in the legislature), unless the gerrymander already robustly assures a wide range of representation through the creation of many safe seats as in the bipartisan map. And under conditions in which we observe wide electoral tides and swings in public opinion, moderately polarized parties can assure that both legislative composition and policy outcomes are sufficiently responsive to the swings, except in cases (as in high-γ gerrymanders) where the gerrymander had implemented such a reinforced partisan majority that the increased polarization will just as often work against the will of the voters as it works for it.

IV. Extension: Majority-Minority Districting

As detailed in Chapter 4, the representation of racial and language minorities, especially in jurisdictions where there has been a history of systematic discrimination, has been a major subject in the jurisprudence and literature on redistricting since the passage of the 1982 amendments to the Voting Rights Act. That chapter alters both the population distribution and the available redistricting institutions in the simulation model to generate insight into how these amendments have interacted with partisan tides to influence the partisan make-up of Southern congressional delegations. This section will use the same alterations to assess the representation of “discrete and insular minorities” with respect to the four norms of representation.

For this extension, I assign 20% of the voters an ideology equal to the 90th percentile most liberal voter in the original model, while the remaining rightmost 80% remains the same;

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9 In fact, the empirical analysis in the Chapter 6 suggests that polarization in Congress around 2008 might be best estimated around $\delta = 65$. 
this 20% might be thought to be African-American voters in the South, being both internally homogenous and substantially more liberal than even the white Democratic population. Thus, while the original model has 435 voters with an ideology range $U[-217, 217]$, the “discrete minority” population has 87 voters with ideology -174, and the remaining 348 voters are uniformly distributed $U[-130, 217]$. In other words both the median and the mean ideology of the population remains unchanged from the distribution specified in the basic model, but this distribution is more polarized.

I also simulate utilities generated under the “VRA compliant” gerrymander for a population with a discrete minority from Chapter 4. This procedure also takes a $\gamma$ gerrymandering parameter, but rather than using $\gamma$ to create ideologically packed districts, the procedure creates $\gamma$ “majority-minority” districts that will contain a bare majority of the ideologically extreme population, while all other voters are spread as evenly as possible among all other available district slots.\(^\text{10}\) For the state with 15 districts and a 20% minority, I have used gerrymander parameter $\gamma = 3$ (proportionate minority districts). The result is that 3 districts have a minority (-174) median, while 12 districts have a median slightly right-of-center [21,32].

<table>
<thead>
<tr>
<th>$\tau$ (tide)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>15</th>
<th>VRA (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>20.8%</td>
<td>25.9%</td>
<td>51.2%</td>
<td>40.0%</td>
<td>40.3%</td>
</tr>
<tr>
<td>0</td>
<td>50.0%</td>
<td>65.9%</td>
<td>58.8%</td>
<td>50.0%</td>
<td>64.1%</td>
</tr>
<tr>
<td>20</td>
<td>79.2%</td>
<td>74.4%</td>
<td>64.3%</td>
<td>59.8%</td>
<td>71.1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma$ (Ideologically Packed Liberal Seats)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
</tbody>
</table>

Appendix Figure A2 shows the average utilities under all measures and gerrymanders for the population with the discrete minority (comparable to figures X and Y for the uniformly distributed population). These figures also include an additional line for the “VRA-compliant”

\(^{10}\) See Chapter 4, Section V for the reasoning and background behind this alignment of voters.
gerrymander with three majority-minority seats. Table 5 shows the mean number of Republican-held seats for each of these gerrymanders under neutral, Republican, and Democratic tides. Reproducing the findings with respect to partisan tides in Chapter 4, the VRA gerrymander is majority-controlled by Republicans under neutral and Republican tides, but controlled by Democrats under Democratic tides. The results for the remaining gerrymanders are very close to those for the uniform population. This is echoed in the appendix utility figures: adding the asymmetric minority to the population, while not altering the mean or median ideology of the population, has very little effect on any of the average utility measures for any gerrymander. All of the conclusions drawn with respect to polarization and partisan gerrymanders hold true when we make this change in the population distribution. Additionally, the VRA-compliant map tends to run close to the aggressive partisan map for all utility measurements.

But for this case, we are particularly concerned about which redistricting institution best serves the interests of the discrete minority. Therefore, I have also run the simulations averaging all utilities among this one-fifth of the population (those with ideology -172) only. Because this population is asymmetric, we must evaluate all maps separately under Democratic and Republican tides (i.e. the effects on Democratic maps of Republican tides are no longer the same as the effects of Republican maps on Democratic tides). The complete results can be seen in Appendix Figure A3; the most notable results are highlighted here.

The central questions we wish to answer from these simulations is: Under what conditions does the VRA-compliant gerrymander, with majority-minority districts, benefit the minority? Where it does not, what options are better?
Figure 7 displays utilities for the minority under neutral tides. Under neutral tides, the VRA map mostly performs poorly, largely because it will usually generate a Republican majority while the minority of course prefers a liberal Democratic legislature. We see this in particular in the policy median and compositional measures, where along with the Republican gerrymanders, the VRA map is among the worst and becomes increasingly bad as polarization rises. In the case of personal utility, where we might expect a map that guarantees that most members of the minority will be in a district able to elect a like-minded legislator, the map proves far inferior to the moderate Republican and bipartisan maps, where all minority members are in districts composed entirely of minority members. And as polarization increases, the VRA map becomes inferior also to the Democratic maps, as the increasing liberalism of the Democratic legislators elected in moderately liberal districts compensates for the lack of direct minority control.
Unsurprisingly, the measure on which the VRA gerrymander performs well is discursive utility, particularly in comparison to the Democratic gerrymander. Here, we perhaps see a major motivation for the VRA amendments; prior to the 1990’s maps passed in the South, where state legislatures were still dominated by Democrats, were typically meant to maximize Democratic seats, but did not provide any guaranteed African-American representation. On this measure alone (electing at least one African-American legislator) the majority-minority districts drawn under the VRA amendments were certainly an improvement.

Appendix Figure A3 also shows the average utility comparisons for the minority under VRA-compliant map for strong Republican ($\tau=20$) and strong Democratic ($\tau=-20$) tides. In most of these cases, the VRA map represents a mediocre result for the discrete minority (e.g., under Republican tides, it is among the worst maps with respect to policy median, and about average with respect to personal representation; under Democratic tides, it falls somewhere in the middle on both of these measures.) But there is one condition under which the VRA map performs uniquely well among those tested: composition utility under Democratic tides.
This particular situation is shown in Figure 8 above, for δ ranging from 0 to 50. This condition represents something of a “perfect storm” for the interests of a liberal minority: they get to elect a proportionate number of districts with legislators reflecting their own very liberal ideology, and at the same time the remaining (somewhat conservative) districts, elect moderate Democrats in response to the national tide. So from the standpoint of compositional utility, minorities are very happy with 20% of their legislators, and somewhat happy with the remaining 80%. This situation does not occur under any other gerrymander, either because they fail to draw enough seats that minorities have majority control over (e.g. the nonpartisan and aggressive partisan maps), or they also draw several seats that still elect conservatives even in cases of strong Democratic tides (e.g. the bipartisan or moderate Republican maps). We might see this situation reflected in the 2008 congressional election results in states such as North Carolina (2 African-American Democrats, 6 white Democrats, 5 Republicans) and Mississippi (1 African-
American Democrat, 2 white Democrats, 1 Republican), where Democrats were able to take advantage of leftward tides to win several conservative-leaning seats while still maintaining roughly proportionate black-plurality districts.

**Majority-Minority Districting Summary**

We now turn to summarizing under what general conditions the VRA-compliant gerrymander benefits the discrete minorities it was created to serve. First, it appears that, relative to other maps, minorities suffer under the VRA gerrymander as polarization increases. As shown in Figures 7 and A3, average utility for minorities is decreasing in \( \delta \) for all four measures of utility under neutral tides, while it is increasing or steady under Democratic, nonpartisan, and bipartisan gerrymanders. Under Democratic tides (\( \tau < 0 \)), average utility is generally increasing in \( \delta \) for all four measures under most maps, but with respect to personal and discursive measures, it is increasing at a faster rate under Democratic or nonpartisan maps than under the VRA map. Under Republican tides (\( \tau > 0 \)), utility is decreasing in \( \delta \) at a relatively similar rate for all but the discursive measure, with the exception of the moderate Democratic map, where it is mostly steady as \( \delta \) increases. Finally, in the case of discursive representation, utility is increasing in \( \delta \) when \( \tau = 20 \) for the Democratic and nonpartisan maps, but idiosyncratic under the VRA map.

So overall, increased polarization hurts minorities under the VRA gerrymander more than under other maps (except for the Republican maps, which it resembles in most cases).

Second, minorities benefit more from strong Democratic tides under the VRA maps than under other gerrymanders (at least when tides are strong enough to generate Democratic majorities under this map). As mentioned above, the only condition measured under which the VRA map outperforms all the other maps shown is compositional utility when \( \tau = -20 \). But the conclusion is broader than that. For the policy median utility measure, the VRA map is among
the worst under neutral and Republican tides, but falls in the middle under Democratic tides (depending on $\delta$). With respect to discursive utility, the VRA gerrymander is always tied among the best maps under Democratic tides, but is among the worst maps (depending on $\delta$) under Republican tides.

Finally, when $\delta$ is reasonably low, the VRA map tends to perform better with respect to discursive and compositional utility measures, and very poorly with respect to policy median.

From these conclusions, it is interesting to note that the VRA map seems to suit the national conditions that were prevalent at the time the amendments were enacted more than those that have been prevalent during the past two decades that the amendments have been in effect. In comparison to the post-1990 era, congressional elections in the 1970’s and 1980’s featured both less party polarization and more frequent Democratic advantages in the national popular vote, and both of these trends were particularly true in the South. Figure 9 depicts the national congressional popular vote, and that same vote in the South (the former Confederacy plus Oklahoma) for congressional elections 1972 to 2010. In the first half of this era, Democrats won the popular vote every year, and won by greater margins in the South than the nation as a whole. But elections from 1992 onward featured several cycles that were very close, as well as several elections with Republican or Democratic tides, while Republicans performed better in the South than the rest of the nation.\(^{11}\)

\(^{11}\) The rapid shift in the South from 1990 to 1994 is also partially due to gerrymandering itself. Democrats were more than twice as likely to run unopposed in Southern congressional districts prior to 1992, thereby understating their potential vote during this era (27.4% ran unopposed from 1972-1990, compared to 13.8% from 1992-2008).
Similarly with respect to polarization, Figure 10 below depicts party polarization in Congress from 1972 to 2008, measured by the difference in mean DW-NOMINATE scores between Democrats and Republicans. The figure shows that polarization has risen dramatically over the past 40 years, and has risen more in the South than the rest of the nation. So the year in which the VRA amendments were passed was part of an era that saw much less polarization and much more prevalent Democratic tides than the era during which these amendments were implemented. In this context, it is understandable why the “paradox of representation” advanced by Canon and other scholars became apparent only several years after the VRA amendments were implemented: had they been implemented in an earlier time, there would perhaps have not been a paradox at all.
V. Discussion and Conclusion

The gerrymandering simulation model has yielded predictions prompted by important questions about the relationship between districting and representation, in response to changes in polarization and partisan tides. On several of these questions, we uncovered predictions counter to the conventional wisdom about the best way to structure redistricting institutions to satisfy competing representations, interests, and dimensions.

First, in what might be seen as fodder for the critics of nonpartisan standards favoring competitive elections, we found that the bipartisan map was superior to the nonpartisan map not only in the case of personal and discursive utility (where its advantages are obvious), but also in the case of policy median utility when tides fell within a narrow, neutral range. Perhaps more surprisingly, the simulations makes a viable case for partisan gerrymanders. In the case of personal, compositional, and discursive utility, the maps provide a compromise between the
respective strengths of the bipartisan and nonpartisan maps. And while the partisan map is weakest in terms of policy median utility when tides are neutral (because they produce a biased median outcome), they can actually be the most responsive map when tides are strong and polarization moderate, at least in the case of aggressive maps.

Additionally, we found that increased party polarization can increase vote utility both by contributing to the discursive representation of extreme voters when they are not already assured representation through a bipartisan map, and by improving all measures of utility under several maps by making elections more responsive to voter preferences when partisan tides are strong. Finally, we modified the model to test whether majority-minority districting benefits ideologically extreme minority voters, finding that such districts perform poorly under many specifications. However, mandating such districts is helpful to minority voters under more narrow conditions, conditions that were more prevalent during the time the VRA amendments were enacted.

With the exception of the last finding, all of these conclusions are drawn in reference to average voter utility across some measure, without consideration for which particular voters benefit. It is important to acknowledge that this is not the only way to judge the merits of an electoral institution. If one side is systematically excluded from having a voice in government, we may discount the representational benefits that a map has to the other side, valuing a fair outcome over a utility-maximizing one. So we might accept the benefits of aggressive partisan gerrymanders only so long as both parties have roughly equal opportunity to draw partisan maps.

As with any model, simplifying assumptions (e.g. lack of geographic limitations, exogenous polarization, etc.) can cast doubt on the real-world applicability of the simulation results. With these caveats in mind, Chapter 6 tests several of the predictions from the model from an empirical data set of congressional elections and survey responses.
We have now simulated predictions about how each gerrymandering regime should perform on each of the four welfare measures; the next chapter will take a first cut at testing these predictions. Unlike Chapter 3, where a large data set could allow tests over four decades, data that permits us to estimate ideologies, and thus welfare measures, of individual voters in each congressional district is much more limited in availability. Therefore, we will mostly be testing the predictions with respect to a specific period in time, the 2006-2008 elections that swept Democrats into control despite the prevalence of Republican gerrymanders in several large states.

In Chapter 3, we specifically tested these “wave” elections for the effects of institutions on competitiveness, finding that nonpartisan maps and Republican maps (adverse to tides) generated more competition and turnover than bipartisan or Democratic maps, also supporting the predictions of the model. In the next chapter, we will test whether these trends also generated the sort of difference in representation quality that the model predicts here. Do the states drawing nonpartisan maps actually greater policy median utility for their voters under strong tides but less personal utility than the bipartisan maps? And do the aggressive partisan maps actually produce greater policy median utility as tides increase? These are the sorts of questions, specifically addressing the public and legal debate over redistricting reform, that our simulation model has generated, and the questions that the final chapter aims to answer.
Appendix Figures

Figure A1. Partisan vs. Bipartisan Maps under Strong Partisan Tides
Figure A2. Population with Liberal Minority under Neutral Tides
Table A3. Utility of Liberal Minority under Democratic & Republican Tides