**Boat Model Description**

Our boat model consists of two main parts: a linear approximation of the boat's angular acceleration, angular velocity, and angular position and a non-linear approximation of acceleration, velocity, and position.

In this model, we consider our system states as: angular position, boat angular velocity, forward/sideways velocity, and x/y position. (Fig. 1) These states allow for simple modeling of the boat while retaining the ability to easily fuse x-y measurement data from sensors.

The rotation of the boat jet drives changes the calculation of the boat position based on the trigonometric functions which adds non-linearity into our system. This non-linearity in the movement model could require the use of a particle-filter rather than the suggestion of the Kalman Filter.
The boat is driven by 4 motors, two sideways thrusters at the boat's bow and stern that point perpendicular to the center line and two forward/backwards thrusters that point parallel to the center line. (Fig. 2)

![Fig. 2 Boat motor configuration](image)

To complete our boat model analysis we preformed analysis on the frictional forces caused by the boats movement and force that our motors would need to exert to maintain our specification (see boat frictional analysis and motor force analysis).

From this information we derived the linear boat movement model which is as follows:

\[ \begin{bmatrix}
    x\text{Vel} \\
    \text{forVel} \\
    \text{yVel} \\
    \text{sideVel} \\
    \theta
\end{bmatrix}
= 
\begin{bmatrix}
    \cos(\theta) & 0 & \sin(\theta) & 0 & 0 \\
    \frac{m}{\cos(\theta)} & 0 & \frac{m}{\sin(\theta)} & 0 & 0 \\
    0 & -f_s & 0 & 0 & 0 \\
    0 & 0 & 0 & -f_\theta & 0 \\
    0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x\text{Pos} \\
    \text{forVel} \\
    \text{yPos} \\
    \text{sideVel} \\
    \theta
\end{bmatrix}
+ 
\begin{bmatrix}
    0 & 0 & 0 & 0 & -1 \\
    0 & m & 0 & 0 & 0 \\
    0 & 0 & m & 0 & 0 \\
    0 & 0 & 0 & m & -L_2 \\
    0 & 0 & 0 & 0 & L_1
\end{bmatrix}
\begin{bmatrix}
    F1 \\
    F2 \\
    F3 \\
    F4 \\
    \text{Rvel}
\end{bmatrix} \]
Using this state-space model a simulation was then created using matlab and simulink.

Fig. 4 shows the boat model in its entirety. In the top left the motor inputs that correspond to the input vector of the state space model. On the top right is the boat rotation model and on the bottom right is the boat linear movement model.

Fig. 5 shows the boat rotational model. It is possible to see the motor inputs in block A, once again these correspond to the input vector in the state space model. Block B calculates the angular acceleration caused by each of the motor input. This corresponds to the bottom two rows in the B matrix of the state space model. In block C it is possible to see where the input acceleration is summed and integrated into an angular velocity. Also in this block are the calculations of the frictional force based on angular velocity and the calculation of the acceleration due to this. This acceleration is summed in the same block as the accelerations due to motor force. Also in this block the angular velocity is integrated into an angular position. Finally, block D calculates the sine and cosine of the angular position for use in the linear movement portion of the model.

Fig. 6 shows the boat linear movement model. Block A shows the inputs from the motors. These can be traced in Fig. 4 back to their respective motors. Block B shows the conversion of the motor force into acceleration in both forward and sideways directions. Block C shows the calculation of the drag accelerations based on velocity and the integrations of the accelerations into their respective velocities. Block D shows the decomposition of the forward and sideways movement vectors into their absolute x and y components and their integration into x and y position. Also in this block is the summing of the constant river velocity into the boat’s x velocity. This corresponds to the RVel variable in the state space model.

Simulations based on a small kayak and two 200 watt forward motors, supplying 21 Newtons of force, and two 100 watt sideways/turning motors indicated that at full power the top speed of the boat is 2.01m/s. We believe that due to systematic over estimation of the frictional forces. Thus as we continue to characterize our boat we can obtain better estimations of the forces and better estimation of the motor force required.
Fig. 3 Boat movement model top level

Fig. 5. Boat Rotational Model