LPARD-TDF-2012
ACCELEROMETER ERROR ANALYSIS

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Introduction

An examination of the accelerometer provided in the Ardupilot kit (Analog Devices ADXL335\(^1\)). Said examination focuses on the potential error due to integrating from the conversion of acceleration to position data.

Analysis

Determining position error for an accelerometer is difficult due to the presence of a double integral. We will use the following equation developed by Kionix\(^2\) (which describes the conversion from acceleration to position):

\[
x(t) = \int_{0}^{t} \int_{0}^{t} a(t) dt dt + \delta S \int_{0}^{t} \int_{0}^{t} a(t) dt dt + \frac{\delta V_b}{S} \int_{0}^{t} \int_{0}^{t} dt dt
\]

The second and third terms will give us our integrated intrinsic error, so we’ll neglect the first term. Focusing first on the third term due to its independence from the actual acceleration, we need the values for the voltage bias error \(\delta V_b\) and the sensitivity \(S\). Going by the datasheet we find that the sensitivity is 300mV/g and that our voltage bias error varies based primarily on temperature (1mg/°C), noise error (measured in \(\mu g/\sqrt{Hz}\)), and ratiometric errors. In this case we’ll be assuming a 1% voltage bias error, which comes out to 0.015V (as our voltage bias is 1.5V as per the datasheet). After running the double integration, we find that our third term coefficient is 0.05m/s\(^2\). This makes our third term \(e_3(t) = 0.025t^2\). This gives us the following:

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>error (m)</td>
<td>0.025</td>
<td>0.1</td>
<td>0.225</td>
<td>0.4</td>
<td>0.625</td>
</tr>
</tbody>
</table>

Note that our error is 1 meter at approximately \(6\frac{1}{3}\) seconds with only a 1% error.

The second term is based on the systems acceleration as well as sensitivity error \(\delta S\). If we again assume a 1% error, we find that term to be 0.036m/s\(^2\). If we then assume a worst

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case constant acceleration of 1.4m/s² this term becomes $e_2(t) = 0.0252t^2$. This means our worst case second term is roughly equivalent to our 1% error third term. The two error terms combined gives us:

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<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>combined error (m)</td>
<td>0.0502</td>
<td>0.2008</td>
<td>0.4518</td>
<td>0.8032</td>
<td>1.255</td>
</tr>
</tbody>
</table>

When combined these two error terms have an error of 1 meter at approximately 4.46 seconds.

To get our combined error terms to a useful standard deviation we’ll map it to a normal distribution - specifically, setting the combined error to the $3\sigma$ point (or 99.7% off the normal distribution). We assume that all points possibly covered by our combined error will be within that portion of a normal distribution. To avoid having our error compound to a large extent, we’ll reset the accelerometer every two seconds. Doing this gives us a maximum combined error of approximately 0.2008 meters, which then gives us $\sigma = 0.0669m$, or 6.69cm.