Exam 3 Review

1. Dimensional Analysis & Similitude (Ch 8)

Dimensional Analysis: dimensional homogeneity – used to determine the form of equations; exponent method $F = M^1L^1T^{-2}$

Similitude: properly scaled models of real physical systems must have the same ratios of dominant forces as in the prototype

Reynolds # = inertial F/viscous F (pipe flow, friction, boundary layer flow)

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p \qquad \left(\frac{V L}{\nu}\right)_m = \left(\frac{V L}{\nu}\right)_p$$

Froude # = inertial F/gravity F (open-channel flow, dams, waves)

$$\left(\frac{V}{\sqrt{g L}}\right)_m = \left(\frac{V}{\sqrt{g L}}\right)_p$$

Euler # a.k.a. Pressure Coefficient = pressure F/inertial F (pressure changes)

$$\left(\frac{\Delta p}{\rho V^2}\right)_m = \left(\frac{\Delta p}{\rho V^2}\right)_p$$

Inertial force represented dimensionally as $F = \rho V^2 L^2$

Note that some modeling situations may involve more than one π-group. Often not possible to satisfy *both* Re and Fr at the same flow rate

2. Boundary Layers, Friction, and Drag (Ch 9/11)

- review handouts on boundary layers – friction drag vs. pressure (form) drag

Friction and drag coeffs are functions of Re, because the B.L. changes as flow velocity increases – separation point and size of wake in particular can shift

Friction Drag (flow parallel to a boundary): $F_{Df} = C_f \rho \left(\frac{U^2}{2} \right) A_{shear}$

- A_{shear} is the area parallel to flow (often two-sided)
- for C_f, Re_{crit} = 500,000
- use upper dashed line *only* if B.L. is tripped <u>and</u> Re < 10^7

Total Drag (2-D & 3-D objects, e.g., spheres, cars): $F_D = C_D \rho \left(\frac{U^2}{2} \right) A_{proj}$

- Fig 11.5 & 11.11, Table 11.1 provided for drag coeff values ($C_D >> C_f$)
- A_{proj} is the projected area of the object (perpendicular to flow)

For *laminar* flow around a sphere (*Re* < 0.5) we have "Stokes' Law":

$F_D = 3\pi\mu V d$ $\quad$ $C_D = 24/Re$

- Note, if you use the laminar flow assumption, you must check *Re* at the end

Applications:

- calculate friction drag along a flat object
- calculate drag force on a 3-D object
- determine terminal velocity – Iterative – best to guess C_D

3. Friction in Pipe Flow (Ch 10)

Pipe flow is BL flow – since velocity varies across the pipe

Moody diagram and k_s values (Table 10.4) will be provided

$Re = \frac{\rho V D}{\mu}$ where *D* = pipe diameter

Re < 2000 → laminar flow $\quad$ Re > 4000 → turbulent flow

Head Loss:

For either laminar *or* turbulent flow:

Darcy-Weisbach equation: $h_L = f \frac{L}{D}\frac{V^2}{2g}$

f = f (k$_s$/d, Re) $\rightarrow$ use Moody Diagram!

Note that relative roughness lines curve upward to the left

If flow is laminar, you can use: $f = \frac{64}{\text{Re}}$

Energy Equation:

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} + h_p - h_T = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_L$$

Notes:

- if flow is turbulent, $\alpha \approx 1$; if laminar, $\alpha = 2$
- Case 1 problem – direct solution – find h_L using Moody diagram for f
- Case 2 problem – indirect solution because you are solving for Q but f depends on V and thus Q
- Case 3 problem – also indirect, won't be on the exam

- if you have to iterate, its usually best to start by assuming an f value (0.01 to 0.03)
- iterate until the solution converges

You will also have some conceptual questions (multiple-choice) on Exam 3.